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Optimization of a Triangular Fin with Variable Fin Base Thickness

Hyung Suk Kang

***Abstract*—**A triangular fin with variable fin base thickness is analyzed and optimized using a two-dimensional analytical method. The influence of fin base height and fin base thickness on the temperature in the fin is listed. For the fixed fin volumes, the maximum heat loss, the corresponding optimum fin effectiveness, fin base height and fin tip length as a function of the fin base thickness, convection characteristic number and dimensionless fin volume are represented. One of the results shows that the optimum heat loss increases whereas the corresponding optimum fin effectiveness decreases with the increase of fin volume.

***Keywords*—**A triangular fin, Convection characteristic number, Heat loss, Fin base thickness.

I. Introduction

EXTENDED surfaces or fins are used to increase the heat dissipation in many engineering and industrial applications such as the cooling of combustion engines, electronic equipments, compressors, aircraft and so on. Many papers for the various fin shapes using many kinds of methods have been presented. For example, Sfeir applied the heat balance integral method to solve for the heat flow and temperature distribution in extended surfaces of different shapes and boundary conditions [1]. Ma *et al.* investigated a two-dimensional rectangular fin with arbitrary variable heat transfer coefficient on the fin surface using a Fourier series approach [2]. Kang and Look analyzed the trapezoidal fins of various slopes using the analytical method [3]. Abrate and Newnham presented heat conduction in an array of triangular fins with an attached wall using the finite element method [4].

All these papers analyzed but not optimized the fin. There are many papers that deal with the fin optimum design. For example, Laor and Kalman studied the optimization of the three shapes (rectangular, triangular and parabolic) for the three types of fins (longitudinal, spine and annular) using the general heat balance differential equation [5]. Chung *et al.* dealt with the optimum design of convective longitudinal fins of a trapezoidal profile using the general differential equation based on the energy balance [6]. Yeh investigated the optimum dimensions of rectangular fins and cylindrical pin fins [7]. Considering different uniform heat transfer coefficients on the fin faces and on the tip, Casarosa and Franco approached the optimum design of single longitudinal fins with a constant thickness by means of an accurate mathematical method [8]. All these optimizations are based on the one-dimensional analysis.

For the two-dimensional optimization, Chung and Iyer presented an extended integral approach to determine the optimum dimensions for rectangular longitudinal fins and pin fins by incorporating traverse heat conduction [9]. Kang and Look present the optimum heat loss and dimensions based on the fixed fin base height for a thermally and geometrically asymmetric trapezoidal fin using the analytical method [10]. Kundu and Das determined the optimum dimensions for eccentric annular fins using Lagrange multiplier technique [11].

In all these papers, the fin base temperature is given as a constant for the boundary condition and the effect of fin base thickness is not considered. In this study, by using a two-dimensional analytical method, a straight triangular fin with variable fin base thickness is analyzed and optimized for the fixed fin volume.

II. 2-D Analytical Method

For a straight triangular fin with variable fin base thickness as shown in Fig. 1, dimensionless two-dimensional governing differential equation under steady state is

$$\frac{\partial^2\theta}{\partial X^2}+\frac{\partial^2\theta}{\partial Y^2}=0\cdot \tag{1}$$

Four boundary conditions are required to solve the governing differential equation and these conditions are shown as (2) through (5).

$$-\left.\frac{\partial\theta}{\partial X}\right|_{X=L_b}=\frac{1-\left.\theta\right|_{X=L_b}}{L_b} \tag{2}$$

$$-\left.\frac{\partial\theta}{\partial Y}\right|_{Y=0}=0 \tag{3}$$

$$-\left.\frac{\partial\theta}{\partial X}\right|_{X=L_e}+M\cdot\left.\theta\right|_{X=L_e}=0 \tag{4}$$

$$-\int_0^{L_h}\left.\frac{\partial\theta}{\partial X}\right|_{X=L_b}dY=\frac{M\sqrt{L_h^2+(L_e-L_b)^2}}{L_h}\int_0^{L_h}\theta dY \tag{5}$$

The solution for the temperature distribution $\theta(X, Y)$ within the triangular fin obtained using separation of variables method with (1) through (4) is

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H. S. Kang is with the Kangwon National University, Chuncheon 200-701, Korea (phone: 82-33-250-6316; fax: 82-33-242-6013; e-mail: hkang@ kangwon.ac.kr).

$$\theta(X,Y)=\sum_{n=1}^{\infty}\frac{g_1(\lambda_n)f(X)\cos(\lambda_n Y)}{g_2(\lambda_n)+g_3(\lambda_n)}\cdot \quad (6)$$

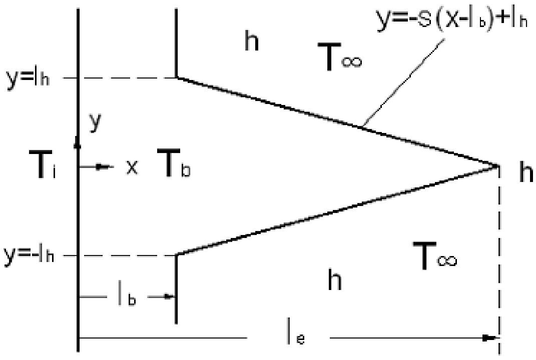

Fig. 1 Geometry of a triangular fin with variable wall thickness

where,

$$f(X)=\cosh(\lambda_n X)+g_4(\lambda_n)\sinh(\lambda_n X) \quad (7)$$

$$g_1(\lambda_n)=\frac{4\sin(\lambda_n L_h)}{2\lambda_n L_h+\sin(2\lambda_n L_h)} \quad (8)$$

$$g_2(\lambda_n)=\cosh(\lambda_n L_b)-L_b\lambda_n\sinh(\lambda_n L_b) \quad (9)$$

$$g_3(\lambda_n)=g_4(\lambda_n)\{\sinh(\lambda_n L_b)-L_b\lambda_n\cosh(\lambda_n L_b)\} \quad (10)$$

$$g_4(\lambda_n)=-\frac{\lambda_n\tanh(\lambda_n L_e)+M}{\lambda_n+M\tanh(\lambda_n L_e)} \quad (11)$$

The eigenvalues λ_n can be calculated using (12) that is arranged from (5).

$$0=g_5(\lambda_n)+g_6(\lambda_n)+\frac{M}{\lambda_n\sqrt{1+s^2}}[\{g_7(\lambda_n)+g_8(\lambda_n)\}$$
$$\cdot\{g_9(\lambda_n)+g_{10}(\lambda_n)\}-\{g_{11}(\lambda_n)+g_{12}(\lambda_n)\}$$
$$\cdot\{g_{13}(\lambda_n)+g_{14}(\lambda_n)-1\}] \quad (12)$$

where,

$$g_5(\lambda_n)=\sinh(\lambda_n L_b)\sin(\lambda_n L_h) \quad (13)$$

$$g_6(\lambda_n)=g_4(\lambda_n)\cosh(\lambda_n L_b)\sin(\lambda_n L_h) \quad (14)$$

$$g_7(\lambda_n)=\cosh(\frac{\lambda_n L_h}{s}+\lambda_n L_b) \quad (15)$$

$$g_8(\lambda_n)=g_4(\lambda_n)\sinh(\frac{\lambda_n L_h}{s}+\lambda_n L_b) \quad (16)$$

$$g_9(\lambda_n)=\cos(\lambda_n L_h)\sinh(\frac{\lambda_n L_h}{s}) \quad (17)$$

$$g_{10}(\lambda_n)=s\cdot\sin(\lambda_n L_h)\cosh(\frac{\lambda_n L_h}{s}) \quad (18)$$

$$g_{11}(\lambda_n)=\sinh(\frac{\lambda_n L_h}{s}+\lambda_n L_b) \quad (19)$$

$$g_{12}(\lambda_n)=g_4(\lambda_n)\cosh(\frac{\lambda_n L_h}{s}+\lambda_n L_b) \quad (20)$$

$$g_{13}(\lambda_n)=\cos(\lambda_n L_h)\cosh(\frac{\lambda_n L_h}{s}) \quad (21)$$

$$g_{14}(\lambda_n)=s\cdot\sin(\lambda_n L_h)\sinh(\frac{\lambda_n L_h}{s}) \quad (22)$$

The heat loss conducted into the fin through the fin base is calculated by (23).

$$q=-2\int_0^{l_h}k\left.\frac{\partial T}{\partial x}\right|_{x=l_b}l_w\,dy \quad (23)$$

Dimensionless heat loss from the fin is written as

$$Q=\frac{q}{k\phi_i l_w}=-2\sum_{n=1}^{\infty}\frac{g_1(\lambda_n)\{g_5(\lambda_n)+g_6(\lambda_n)\}}{g_2(\lambda_n)+g_3(\lambda_n)}\ . \quad (24)$$

Fin Effectiveness

Fin effectiveness is defined as the ratio of heat loss from the fin to that from the outside wall. With assuming that heat is transferred from the inside wall to the outside wall along x direction only, the energy balance equation can be written in dimensionless form as

$$\frac{d^2\theta}{dX^2}=0\cdot \quad (25)$$

Two boundary conditions are given in (26) and (27).

$$\theta|_{X=0}=1 \quad (26)$$

$$\left.\frac{\partial\theta}{\partial X}\right|_{X=L_b}+M\cdot\theta|_{X=L_b}=0 \quad (27)$$

When (25) is solved using these boundary conditions, the dimensionless temperature distribution between inside wall and outside wall is

$$\theta=\frac{-M}{1+M\cdot L_b}X+1\cdot \quad (28)$$

The heat loss from the outside wall is calculated as

$$q_w=-2kl_h l_w\left.\frac{dT}{dx}\right|_{x=l_b}\cdot \quad (29)$$

The dimensionless heat loss from the outside wall can be expressed as

$$Q_w=\frac{q_w}{k\phi_i l_w}=\frac{2M\cdot L_h}{1+M\cdot L_b}\cdot \quad (30)$$

Fin effectiveness is then expressed as

$$\varepsilon=Q/Q_w\cdot \quad (31)$$

Fin Volume

The triangular fin volume, as shown in Fig. 1, can be calculated by (32).

$$v=2\int_{l_b}^{l_e}\{l_h-\frac{l_h(x-l_b)}{l_e-l_b}\}l_w\,dy \quad (32)$$

The dimensionless fin volume is expressed as

$$V = \frac{v}{l_c^2 \cdot l_w} = L_h (L_e - L_b) \cdot \quad (33)$$

III. Results and Discussion

The dimensionless temperature profile along the normalized Y position (i.e. $NY=2Y/L_h$) for different values of fin base height at $X=(L_b+L_e)/2$ is shown in Fig. 2. It is observed that the temperature decreases as the fin base height decreases for the same value of NY. It also shows that the decreasing rate of temperature along the normalized Y position becomes more remarkable as the fin base height increases.

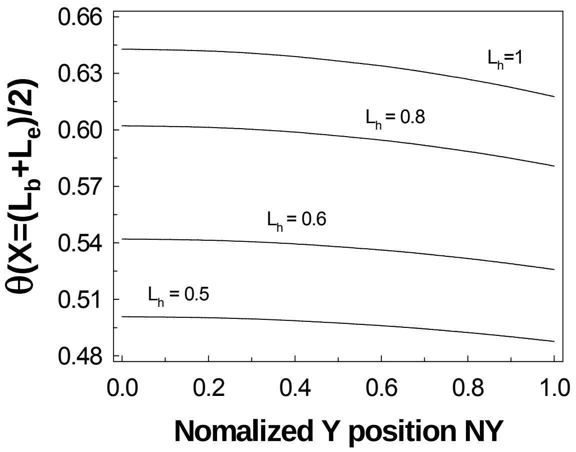

Fig. 2 Dimensionless temperature along the normalized position of Y (L_b=0.1, L_e=1.6, M=0.5)

TABLE I
Dimensionless fin temperature with the variations of L_B and L_H (M=0.1, L_E-L_B=2)

L_b	$\theta(X=L_b+0.1, Y=0)$		
	L_h=0.1	L_h=0.3	L_h=0.5
0.01	0.9018	0.9529	0.9686
0.05	0.8689	0.9358	0.9571
0.1	0.8310	0.9152	0.9428
0.2	0.7642	0.8763	0.9151

Table I lists the dimensionless temperature at the arbitrary fin position (at $X=L_b+0.1$, Y=0) with the variation of the fin base thickness and fin base height for M=0.1 and L_e-L_b=2. As expected, this table illustrates that the temperature decreases as the fin base thickness increases due to the increase of the thermal resistance between the fin base and the inside wall. It also can be noted that the temperature decreases as the fin base height decreases.

The dimensionless heat loss as a function of fin tip length for different values of convection characteristic number is presented in Fig. 3. It is observed that the heat loss increases rapidly when the fin tip length approaches fin base thickness (i.e. very short fin). It is because that the fin base height increases as the fin tip length decreases for the fixed fin volume. Obviously, the design in this case is impractical, although the heat dissipation is large. Another important phenomenon shown in Fig. 3 is that the maximum heat loss does not exist when the convection characteristic number is beyond certain value. For example, the maximum heat loss exists for M=0.2 and M=0.3 whereas it does not exist for M=0.4. The maximum heat loss will be referred to the optimum heat loss and the fin tip length at which the heat loss becomes the maximum is referred to the optimum fin tip length in this study.

Fig. 4 presents the fin effectiveness as a function of fin tip length under the same conditions as given in Fig. 3. It shows that the fin effectiveness decreases whereas the heat loss increases as the fin tip length decreases from 0.8 to 0.4 because of the fixed fin volume. As already mentioned, this phenomenon explains why the fin design is impractical as the fin tip length approaches fin base thickness. Even though the maximum heat loss exists for M=0.2 and M=0.3 as shown in Fig. 3, the fin effectiveness increases continuously as the fin tip length increases for all given values of M. It can also be noted that the effectiveness increases as the convection characteristic number decreases for the same value of fin tip length.

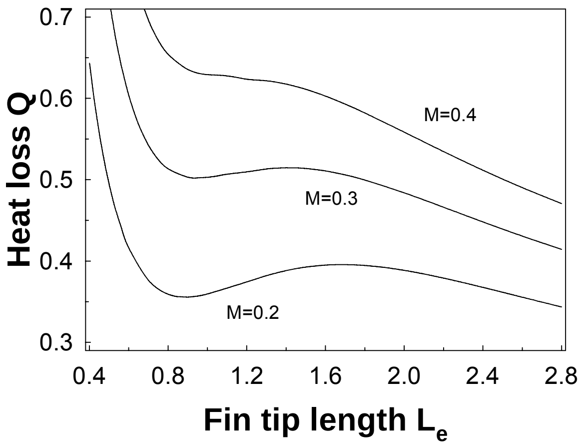

Fig. 3 Heat loss as a function of fin tip length (V=0.5, L_b=0.1)

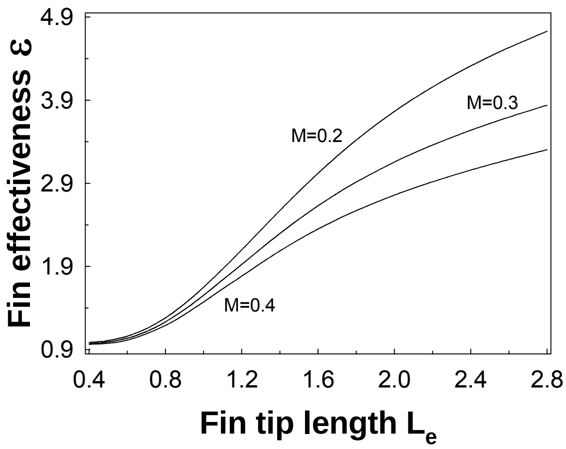

Fig. 4 Fin effectiveness as a function of fin tip length (V=0.5, L_b=0.1)

Fig. 5 depicts the variation of the optimum heat loss and the optimum effectiveness as a function of the fin base thickness for a triangular fin when the dimensionless fin volume is arbitrarily fixed as 0.5. The optimum fin effectiveness means the effectiveness when the heat loss becomes the maximum heat loss for given conditions. It indicates that both the optimum heat loss and the corresponding optimum fin effectiveness decrease as the fin base thickness increases. Note that the optimum heat loss increases whereas the corresponding optimum effectiveness decreases as the convection characteristic number increases for the same value of fin base thickness.

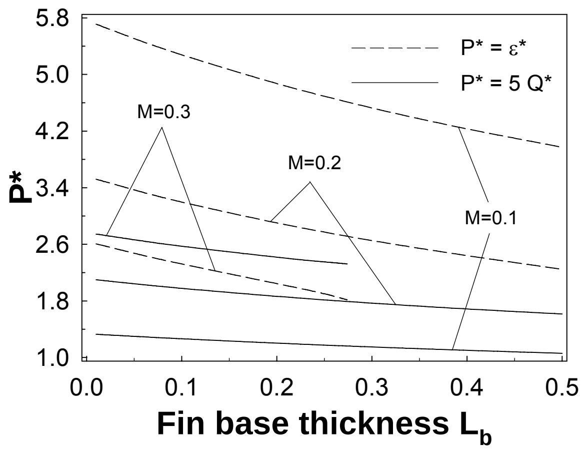

Fig. 5 Optimum heat loss and fin effectiveness versus the fin base thickness (V=0.5)

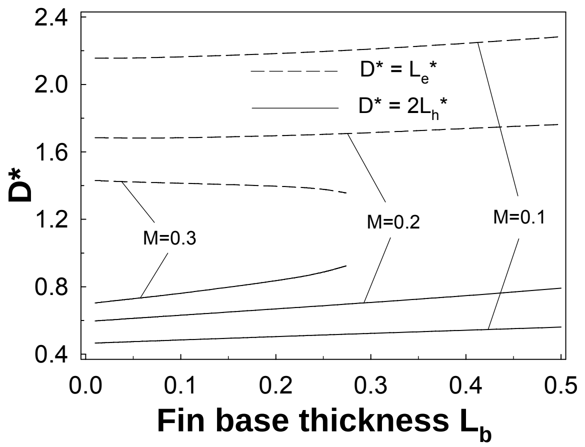

Fig. 6 Optimum fin tip length and fin base height versus the fin base thickness (V=0.5)

Fig. 6 presents the variation of the optimum fin tip length and fin base height under the same condition as given in Fig. 5. The variation of fin tip length is relatively not much whereas the fin base height increases monotonically with the increase of fin base thickness. Physically, it means that the actual fin length becomes shorter and the fin shape is fatter since the fin base thickness increases. It also shows that the fin base height increases while the fin tip length decrease as the convection characteristic number increases for the same value of fin base thickness.

The dimensionless fin volume, V, was arbitrarily selected to be 0.5 in the previous discussion. The variations of the optimum performance and dimension as a function of V are shown in Figs. 7-8. As expected, the increase of V enhances the optimum heat loss. The corresponding optimum effectiveness decreases remarkably first and then decreases slowly with the increase of the fin volume because the increasing rate of optimum fin height is larger than that of the optimum fin tip

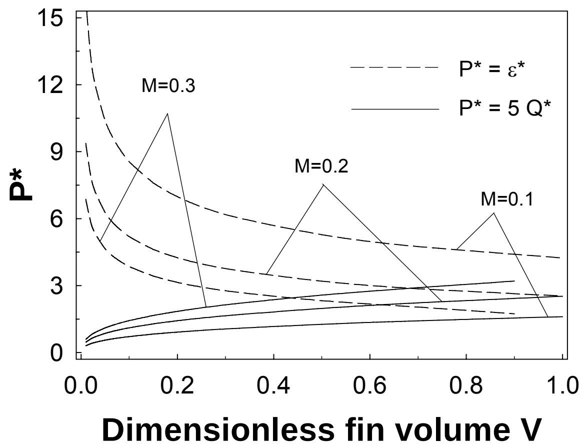

Fig. 7 Optimum heat loss and fin effectiveness versus the fin volume (L_b=0.1)

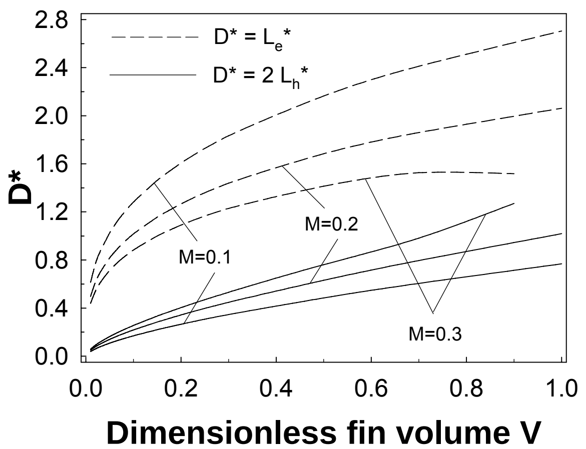

Fig. 8 Optimum fin tip length and fin base height versus the fin volume (L_b=0.1)

length with the increases of the fin volume as shown in Fig. 8. For one example, in the case of M=0.2, fin base height increases from 0.05 to 1.02 (i.e. 20.4 times) whereas the fin tip length increases from 0.5 to 2.06 (i.e. 4.1 times) as the fin volume increases from 0.01 to 1. Fig. 8 also shows the optimum fin tip length increases rapidly first and then levels off whereas the optimum fin base height increases almost linearly as the fin volume increases. Physically, the optimum straight triangular profile fin becomes rather 'fatter' with the increase of the fin volume. It can be noted that the optimum fin tip length increases as the convection characteristic number decreases for the same fixed fin volume.

Fig. 9 represents the variation of the optimum heat loss and the optimum fin effectiveness as a function of the convection characteristic number for several fixed fin volume. It shows that the variation trend of the performance with the variation of the convection characteristic number is somewhat similar to that with the variation of the fin volume.

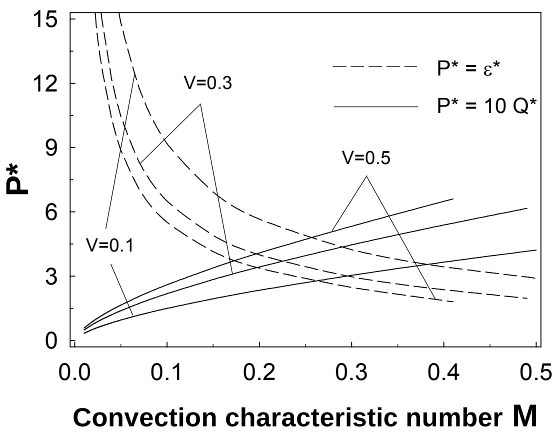

Fig. 9 Optimum heat loss and fin effectiveness vs. convection characteristic number (L_b=0.05)

The optimum fin tip length and fin base height as a function of convection characteristic number for the same condition as given in Fig. 9 is shown in Fig. 10. As shown in this figure, the optimum fin tip length decreases as the convection characteristic number increases and the optimum fin base height increases due to the fixed fin volume. Physically, this means that the shape of the optimum triangular fin becomes shorter and fatter with the increase of the convection characteristic number.

IV. Conclusion

From this two-dimensional analysis of a triangular fin, the following conclusions can be drawn:

1. For fixed fin volume, the maximum heat loss in the practical fin length does not exists with the variation of fin tip length when given variables (for example, fin base thickness, convection characteristic number and fin volume) are larger than certain value.

2. Both the optimum heat loss and the corresponding optimum fin effectiveness decrease with the increase of the fin base thickness.

3. Even though the optimum heat loss increases, the corresponding optimum fin effectiveness decreases as the convection characteristic number and the fin volume increase. It is because that the optimum fin tip length decreases whereas the optimum fin base height increases or that the increasing rate of fin tip length is less than that of the fin base height as the convection characteristic number and the fin volume increase.

Nomenclature

h: heat transfer coefficient over the fin [W/m^2℃]
k: thermal conductivity of fin material [W/m℃]
l_b: fin base thickness [m]
L_b: dimensionless fin base thickness, l_b/l_c
l_c: characteristic length [m]
l_e: fin tip length [m]
L_e: dimensionless fin tip length, l_e/l_c

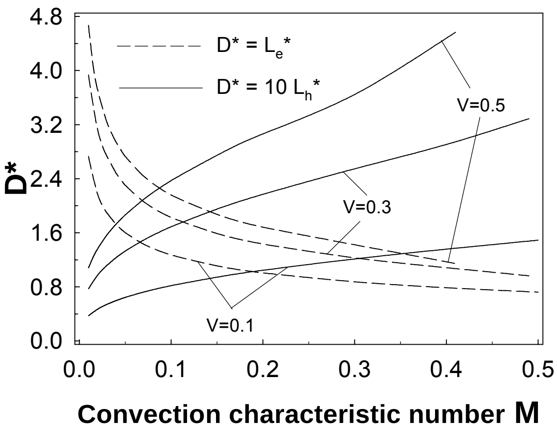

Fig. 10 Optimum fin tip length and fin base height vs. convection characteristic number (L_b=0.05)

l_h: one half fin base height [m]
L_h: dimensionless one half fin base height, l_h/l_c
l_w: fin width [m]
M: convection characteristic number (=hl_c/k)
NY: normalized position of Y (=$2Y/L_h$)
q: heat loss from the fin [W]
Q: dimensionless heat loss from the fin, $q/(kw\Phi_i)$
q_w: heat loss from the bare wall [W]
Q_w: dimensionless heat loss from the bare wall, $q_w/(kw\Phi_i)$
s: fin lateral slope {=$L_h/(L_e-L_b)$}
T: fin temperature [℃]
T_b: fin base temperature [℃]
T_i: temperature of inside wall [℃]
T_∞: ambient temperature [℃]
v: fin volume [m^3]
V: dimensionless fin volume, $v/(l_c^2 \cdot l_w)$
x: length directional variable [m]
X: dimensionless length directional variable, x/l_c
y: height directional variable [m]
Y: dimensionless height directional variable, y/l_c

Greek symbol
ε: fin effectiveness
θ: dimensionless temperature, $(T-T_\infty)/(T_i-T_\infty)$
λ_n: eigenvalues (n = 1, 2, 3, ⋯)

Φ_i: adjusted temperature of inside wall [℃], (T_i-T_∞)

Subscript
b: fin base
c: characteristic
e: fin tip
h: fin base height
i: inside wall
w: outside wall
∞: surrounding

Superscript
* : optimum

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