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Optimal Space Vector Control for Permanent Magnet Synchronous Motor based on Nonrecursive Riccati Equation

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***Abstract*—**In this paper the optimal control strategy for Permanent Magnet Synchronous Motor (PMSM) based drive system is presented. The designed full optimal control is available for speed operating range up to base speed. The optimal voltage space-vector assures input energy reduction and stator loss minimization, maintaining the output energy in the same limits with the conventional PMSM electrical drive. The optimal control with three components is based on the energetically criteria and it is applicable in numerical version, being a nonrecursive solution. The simulation results confirm the increased efficiency of the optimal PMSM drive. The properties of the optimal voltage space vector are shown.

***Keywords*—**Matlab/Simulink, optimal control, permanent magnet synchronous motor, Riccati equation, space vector PWM

I. Introduction

It is well known that the PMSM is a high efficient electric motor, but in the transient behavior, as starting, stopping or reversing, the efficiency of the energy conversion is diminished down to a values smaller than 60 percent, while in the stationary state it is greater. In order to improve the performances in industry applications, with often dynamic regimes, an optimal control is proposed by using linear quadratic criteria. In the motion control area (metallurgical rolling mills, robotics, manipulators, elevators, escalators) the obtained solution of the optimal control proves to be very adequate, increasing the working period of the electrical drives by decreasing the copper losses in the stator windings of the PMSM.

The optimal control of the three phase surface permanent magnet synchronous machine (PMSM)-as it is presented in this paper-uses a different strategy besides those already offered by the technical literature [1]-[6]. The energetically optimization techniques are classified in accordance with the control principles that are used: single state variable control (the control variable is chosen such as it leads either to the input power minimization by using power factor control [1], or to the motor losses minimization through the slip control [2], model based control [3], [4], and search control [5], [6]. From the point of view of the optimal control theory there are presented both the stationary optimization methods and the dynamic optimization ones. The proposed optimal vector control assures energy minimization by using the feedback component, zero steady state by its forcing component and fast compensation of the load through its feedforward compensating component. The proposed optimal control objectives are: smooth response without overshoot; the load torque compensation, and the input energy minimization.

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As it is nonrecursive, determined on the basis of variational methods, the optimal solution is calculated on-line (to the current time), without memorizing this from the final time to the initial time, as in off-line case where the optimal control solution is based on the recursive method. In this way, the nonrecursive solution can be implemented to any type of electric motor, to usual electrical drive application, for any operating dynamic regimes, with any variation form of the load torque.

II. Problem Formulation

The formulation of the optimal control problem supposes: 1) the mathematical model of the three-phase PMSM; 2) defining the performance functional criteria such that the objectives of the optimal control to be attained.

A. Mathematical model of the PMSM

The PMSM mathematical model is derived from the classical d, q mathematical model of the Synchronous motor with Round Rotor, taking into account that the excitation is provided by the permanent magnet, and the damper windings not exist [7]. By introducing the decoupling terms between d and q axes to the stator voltage u_{Sd} and u_{Sq}, their inter-influence can be compensated. In this manner, the direct-axis stator current i_{sd}, and the quadrature-axis stator current i_{sq} can be independently controlled.

Taking into consideration the rotor surface mounted permanent magnet synchronous motor (SMPMSM) ($L_{sd}=L_{sq}$), and considering $i^*_{sd}(t)=0$, the mathematical model of the PMSM in d,q reference frame is presented:

$$\begin{bmatrix} \overset{\circ}{\omega}_m(t) \\ \overset{o}{i}_{sq}(t) \end{bmatrix} = \begin{bmatrix} -\frac{F}{J} & \frac{3p\Psi_m}{2J} \\ -\frac{\Psi_m}{L_s} & -\frac{R_s}{L_s} \end{bmatrix} \cdot \begin{bmatrix} \omega_m(t) \\ i_{sq}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L_s} \end{bmatrix} \cdot u_{sq}(t) + \begin{bmatrix} -\frac{1}{J} \\ 0 \end{bmatrix} \cdot m_l(t) \tag{1}$$

or in compact form:

$$\overset{\circ}{\mathbf{x}}(\mathbf{t}) = \mathbf{Ax(t)} + \mathbf{Bu(t)} + \mathbf{Gw(t)} \tag{2}$$

where:

i_{sq} – Transversal stator current component in synchronously d,q coordinate system

R_s - Stator phase resistance

$L_{sd}= L_{sq}=L_s$ - Stator phase inductance d, q axes are equal

$u_{sd,q}$ - Stator voltages in d,q coordinate system

ω - Angular speed

J - Moment of inertia

θ - Rotor position angle
Ψ_m - Permanent magnet flux
p - Number of poles pairs,
m_l-load torque
F - Viscous friction force

$\mathbf{x(t)}$, $\mathbf{u}(t)$, and $\mathbf{w}(t)$ are the state, the control and the perturbation vectors.

The rotor field oriented PMSM is controlled at constant flux, the optimal control generates the voltage control of the Space Vector Pulse Width Modulator.

B. The performance functional quadratic criteria

The performance functional quadratic criteria [8] was chosen in order to assure a good behavior in transient state without oscillations of the angle q, and for reaching the stationary state without overshoot

$$J = \frac{1}{2}[\mathbf{x}(t_1) - \mathbf{x}_1]^T \mathbf{S}[\mathbf{x}(t_1) - \mathbf{x}_1] + \frac{1}{2}\int_0^{t_1} [\mathbf{x}^T(t)\mathbf{Q}\mathbf{x}(t) + \mathbf{u}^T(t)\mathbf{R}\mathbf{u}(t)]dt \qquad (3)$$

in which the final free state is $\mathbf{x(t_1)}$ and the required final state is $\mathbf{x}_1 = \begin{bmatrix} \omega_m^* \\ 0 \end{bmatrix}$.

The matrix $\mathbf{S}$ has in view the minimizing of the square error between the reached state and the desired state $\mathbf{x}_1$ in the fixed time t_1

$$\mathbf{S} = \begin{bmatrix} s & 0 \\ 0 & 0 \end{bmatrix}. \qquad (4)$$

the first term of the criteria (3), often called the terminal cost, is given by:

$$\lambda(t_1) = \frac{1}{2}[\mathbf{x}(t_1) - \mathbf{x}_1]^T \mathbf{S}[\mathbf{x}(t_1) - \mathbf{x}_1] = \frac{1}{2}s[\omega(t_1) - \omega_1]^2 \qquad (5)$$

In the same way, by setting

$$\mathbf{Q} = \begin{bmatrix} q_1 & 0 \\ 0 & q_2 \end{bmatrix}, \qquad (6)$$

the second term of (3) is related by equation

$$\frac{1}{2}\int_0^{t_1}[\mathbf{x}^T(t)\mathbf{Q}\mathbf{x}(t)]dt = \frac{1}{2}q_1\int_0^{t_1}[\omega_m^2(t)]dt + \frac{1}{2}q_2\int_0^{t_1}[i_{sq}^2(t)]dt \qquad (7)$$

and minimizes the accumulated energy of the system inertia and the stator copper losses [9], [10].

The third term results as

$$\frac{1}{2}\int_0^{t_1}[\mathbf{u}^T(t)\mathbf{R}\mathbf{u}(t)]dt = \frac{1}{2}r\int_0^{t_1}[u_{sq}^2(t)]dt \qquad (8)$$

where the matrix $\mathbf{R}$=[r] degenerated in a scalar value, determines the transfer from the initial state into the final state with minimal control effort; this has as its consequence the minimizing of the input energy.

By choosing weighting matrices

$$\mathbf{S},\mathbf{Q} \geq 0,\ \mathbf{R} > 0 \qquad (9)$$

and taking into account that rank of the system is equal with the number of variable states:

$$\text{rank} \quad \mathbf{C}_{trb} = 2, \qquad (10)$$

Where $\mathbf{C}_{trb}$ is the controllable matrix of the dynamic system (1), that means the dynamic system (4) is controllable [9].

According to (10) and (11) the optimal control exists and is unique [1], [9], [10].

Therefore, the optimal control problem is with free-end point, fixed time and without restrictions. The restrictions of the magnitude for the control and state could be resolved by the adequate choice of the weighting matrices.

C. The solution of the optimal control problem

By using the variational method, the Hamiltonian of the optimal control problem is

$$H(y, \mathbf{x}, \mathbf{u}, t) = \frac{1}{2}\left[x^T(t)\mathbf{Q}\mathbf{x}(t) + \mathbf{u}^T(t)\mathbf{R}\mathbf{u}(t) + \mathbf{y}(t)\cdot\overset{\circ}{\mathbf{x}}(t)\right] \qquad (11)$$

in which $\mathbf{y}(t) \in \Re^2$ is the associated costate vector. The costate vector is the solution of the differential matrix equation

$$\overset{\circ}{\mathbf{y}}(t) = -\nabla_x H \qquad (12)$$

or in other form

$$\overset{\circ}{\mathbf{y}}(t) = -[\mathbf{Q}\mathbf{x}(t) + \mathbf{A}^T\mathbf{y}(t)] \qquad (13)$$

The necessary condition to obtain the optimal control is

$$\nabla_u H = 0 \qquad (14)$$

This implies:

$$\mathbf{u}^*(t) = -\mathbf{R}^{-1}\cdot\mathbf{B}^T\cdot\mathbf{y}(t) \qquad (15)$$

By substituting $\mathbf{u}^*(t)$ in the dynamic system (4), and by using (14) we get the canonical system associated to the linear quadratic problem.

$$\begin{cases} \overset{\circ}{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) - \mathbf{B}\mathbf{R}^{-1}\cdot\mathbf{B}^T\cdot\mathbf{y}(t) + \mathbf{G}\mathbf{w}(t) \\ \overset{\circ}{\mathbf{y}}(t) = -\mathbf{Q}\mathbf{x}(t) - \mathbf{A}^T\mathbf{y}(t), \end{cases} \qquad (16)$$

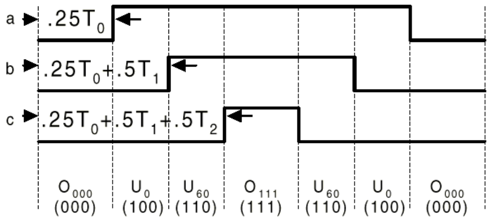

Fig. 2 Switching pattern for the first sector

Taking into consideration a starting of the PMSM, the integration conditions of the canonical system are:

- the initial condition of the system

$$\mathbf{x}(t_0)=\begin{bmatrix}x_1(t_0)\\x_2(t_0)\end{bmatrix}=\begin{bmatrix}\omega(t_0)\\q(t_0)\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}\quad t_0=0\text{s} \tag{17}$$

- the final costate vector condition

$$\mathbf{y}(t_1)=\nabla_x\Lambda(t_1), \tag{18}$$

i.e.,

$$\mathbf{y}(t_1)=\mathbf{S}[\mathbf{x}(t_1)-\mathbf{x}_1]\qquad t_1=0.4\text{s} \tag{19}$$

the transversality condition of the costate vector.

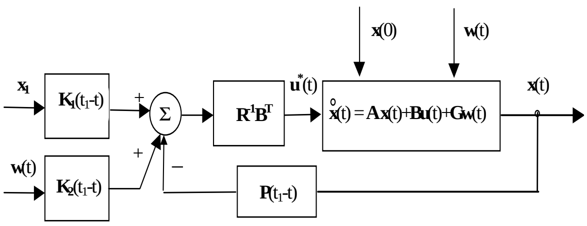

Fig. 1 The structure of the optimal control $u^*(t)$

By solving the canonical system (16) the optimal state trajectory and the costate vector are obtained such that the optimal control $\mathbf{u}^*(t)=u^*_{sq}(t)$ is provided by eq. (15). In order to avoid the classic recursive solution of the matrix Riccati differential equation, with the well known disadvantages and the positive eigenvalues of the system, the current time ' t ' goes to ' t_1-t ', time remaining until the end of the optimal process through the adequate conversion of the state coordinates [8].

The optimal control, at any moment ' t ', is [9], [10]:

$$\begin{aligned}\mathbf{u}^*(t)=&-\mathbf{R}^{-1}\mathbf{B}^T\mathbf{P}(t_1-t)\mathbf{x}(t)+\\&+\mathbf{R}^{-1}\mathbf{B}^T\mathbf{K}_1(t_1-t)\mathbf{x}_1+\mathbf{R}^{-1}\mathbf{B}^T\mathbf{K}_2(t_1-t)\mathbf{w}(t)\end{aligned} \tag{20}$$

in which $\mathbf{P}(t_1 - t)$ is the solution of the matrix Riccati differential equation, and the matrices $\mathbf{K}_1$ and $\mathbf{K}_2$ are calculated via $\mathbf{P}(t_1$-$t)$.

The optimal control law has three components: the state feedback, the forcing component to achieve the desired state $\mathbf{x}_1$ and the compensating feed forward of the perturbation $\mathbf{w}(t)$.

Obviously, the analytical solution supposes the knowledge of the perturbation $\mathbf{w}(t)=m_l(t)$, which could be available by using a torque estimator.

The next step consists in the computation of the stator reference frame voltage components $u^*_{s\alpha}(t)$ and $u^*_{s\beta}(t)$ by using the inverse Clark transformation:

$$\begin{bmatrix}u^*_{s\alpha}\\u^*_{s\beta}\end{bmatrix}=\begin{bmatrix}\cos\theta & \sin\theta\\-\sin\theta & \cos\theta\end{bmatrix}\cdot\begin{bmatrix}u^*_{sd}\\u^*_{sq}\end{bmatrix} \tag{21}$$

where

$$u^*_{sd}=0 \tag{22}$$

The optimal stator voltage space-vector is obtained by using

$$\underline{u}^*_s=u^*_{s\alpha}+ju^*_{s\beta} \tag{23}$$

The method used to generate the required voltage for the PMSM feeding is based on pulse with modulation technique via space vector modulation approach. Space vector theory demonstrated certain improvement for output crest voltage and harmonic copper loss. In addition to these advantages, the maximum output voltage based on the space vector theory is $2/\sqrt{3}$ times higher than conventional sinusoidal modulation, and allows a higher efficiency and a higher torque at high speeds.

The purpose of space vector PWM technique is to approximate the reference voltage vector U_{out} by a combination of the eight switching patterns.

If, in a small PWM cycle period T_{pwm}, the average output of the inverter is the same like reference voltage average U_{out}, we get the following equation:

$$\underline{U}_{out}=\frac{T_1}{T_{pwm}}\underline{U}_x+\frac{T_2}{T_{pwm}}\underline{U}_{x\pm60} \tag{24}$$

where T_1 and T_2 are the durations in time for which switching patterns $\underline{U}_x$ and $\underline{U}_{x\pm60}$ are applied within the period T_{pwm}, and $\underline{U}_x$ and $\underline{U}_{x\pm60}$ form the sector containing U_{out}.

Since the sum between T_1 and T_2 is less than or equal to T_{pwm}, the power inverter needs to have a 0 (000 or 111) pattern inserted for the rest of the period. Therefore, the above equation becomes:

$$\underline{U}_{out}=\frac{T_1}{T_{pwm}}\underline{U}_x+\frac{T_2}{T_{pwm}}\underline{U}_{x\pm60}+\frac{T_o}{T_{pwm}}\underline{U}_{000\ or\ 111} \tag{25}$$

Note that the third term on the right side of equation (25) does not affect the vector sum on the left side.

A 3-phase AC induction motor command algorithm based on discussed SV-PWM principle contains the following steps:

- Configure the timers and compare units to generate symmetric PWM outputs;

- Obtain the magnitude of reference voltage vector U_{out} (command voltage) based on the optimal control;
- Obtain the phase of $\underline{U}_{out}$ based on the orthogonal component;
- Determine which sector $\underline{U}_{out}$ is in;
- Decompose $\underline{U}_{out}$ to obtain T_1, T_2 and T_0;
- Determine the switching pattern or sequence to be used and load the calculated compared values into the corresponding compare registers.

A look-up table stores the *acos* function values necessary to get the right value for the phase of reference voltage vector, common *modulo* function technique being avoided in this case. Comparing theta with the sector limits, the sector *s* of theta results.

Decomposition of the reference voltage vector onto the basic space vectors of the sector is done by 2-by-2 matrix multiplication, as shown in equation (26).

$$\begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \mathbf{M}(\mathbf{s}) \begin{bmatrix} U_{out\_\alpha} \\ U_{out\_\beta} \end{bmatrix} \tag{26}$$

Based on the decomposition matrix $\mathbf{M}(\mathbf{s})$ the basic space vectors are fetched and stored in another look-up table.

The switching patterns implementing space vector PWM are such that only one channel toggles at a time, except for the case when the reference voltage vector is one of the basic space vectors. This approach has chosen the switching direction for each sector that results in one channel toggling at a time, as shown in Fig. 2 above for the first sector. Therefore, once the sector of U_{out} has been determined, the channels that toggle first, second and third are determined too. Based on this analysis, two look-up tables are constructed to use the sector *s* as an index for the comparing register. The compare registers are then loaded with the obtained compared values. The correct PWM output pattern is then generated by the compare logic.

III. Numerical Simulation Results

The optimal control PMSM drive system has been numerically simulated by discretization using Z transform and zero order hold for a starting of a 0.75 kW, 3000 rpm PMSM under a rating load of 2.4Nm. The motor parameters are: R_s = 2.83Ω, *d*-axis winding inductance L_d = 10.1e-3H, the rotor magnetic flux Ψ_m = 0.198 Wb.

The PMSM drive response is shown in Fig. 3, in which the speed reaches the desired final value of 3000rpm, and it is also underlined the effect of the load compensating component for a full load perturbation at t=0,2[s]. The impressed three-phase stator voltages based on SVPWM algorithm are shown in the Fig.4 for the same load variation. In Fig. 5 the optimal control, u_{sq}, i.e. the transversal component of the stator voltage is shown. Obviously, the direct voltage component u_{sd}=0. In Fig.6, the voltage loci locus is depicted. The same results are obtained for stopping and reversing periods.

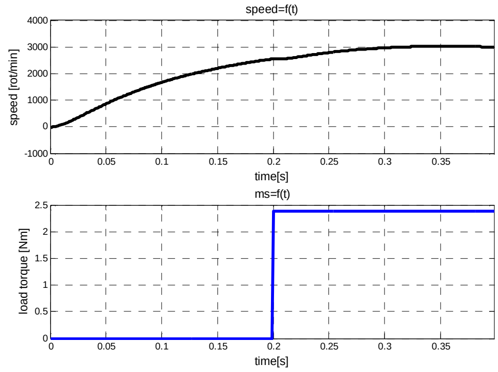

Fig. 3 The PMSM drive system response: speed [rev/min] under the load torque m_l[Nm]

The final time t_1=0,4[s], has been chosen by physical reliability. For these simulations, Matlab/Simulink software has been used.

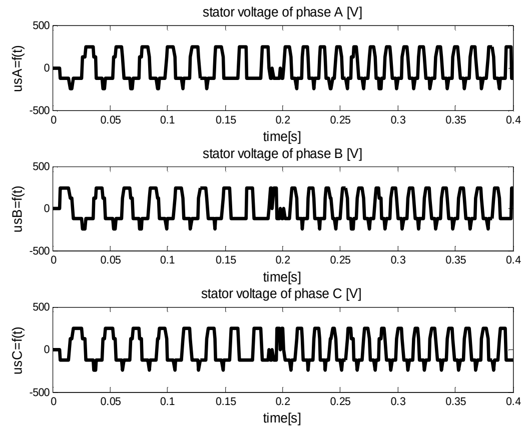

Fig. 4 The optimal three-phase SVMPWM stator voltage of the PMSM drive

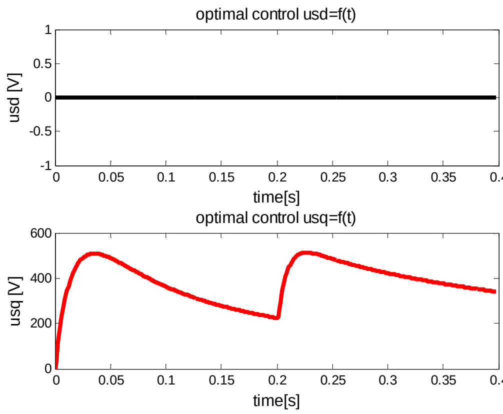

Fig. 5 The optimal control of the PMSM drive system

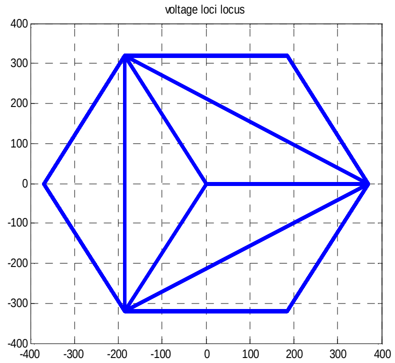

Fig. 6 The optimal voltage loci locus

IV. CONCLUSIONS

A new optimal control for PMSM drive system based on SVPWM is shown. The full optimal control contains three components:

- the state feedback;
- the forcing component to achieve the desired state $\mathbf{x}_1$;
- the compensating feedforward of the perturbation $\mathbf{w}(t)$.

The method used to generate the required voltage for motor feeding is based on pulse with modulation technique via space vector modulation approach. Space vector theory demonstrated a certain improvement for output crest voltage and the harmonic copper loss. In addition to these advantages, the maximum output voltage based on the space vector theory is $2/\sqrt{3}$ times higher than conventional sinusoidal modulation, and allows a higher efficiency and a higher torque at high speeds.

The optimal control is orientated for dynamic regimes and it assures the minimization of both of the PMSM and the inverter losses.

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