Optimal Selling Prices for Small Sized Poultry Farmers

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Abstract— In Japan, meat-type chickens are mainly classified into three categories: (1) Broilers, (2) Branded chickens, and (3) *Jidori* (Free-range local traditional pedigree chickens). The Jidori chickens are certified by the Japanese Ministry of Agriculture, whilst, for the Branded chickens, there is no regulation with respect to their breed (genotype) or methods for rearing them. It is, therefore, relatively easy for poultry farmers to introduce Branded than Jidori chickens. The Branded chickens are normally fed a low-calorie diet with ingredients such as herbs, which lengthens their breeding period (compared with that of the Broilers) and increases their market value. In the field of inventory management, fast-growing animals such as broilers are categorised as ameliorating items. To the best of our knowledge, there are no previous studies that have explicitly considered smaller sized poultry farmers with limited breeding areas. This study develops an inventory model for a small sized poultry farmer that produces both the Broilers (Product 1) and the Branded chickens (Product 2) with different amelioration rates. The poultry farmer's total profit per unit of time is formulated as a function of selling prices by using a price-dependent demand function. The existence of a unique optimal selling price for each product, which maximises the total profit, established. It has also been confirmed through numerical examples that, when the breeding area is fixed, the total profit could increase if the poultry farmer reduced the product quantity of Product 1 to introduce Product 2.

Keywords—Amelioration, deterioration, small sized poultry farmers, optimal price.

I. Introduction

THERE are many small and middle sized poultry farmers in Japan, which results from its smaller size of land and geographical features of a mountainous country. Each small sized poultry farmer would normally sign a certain exclusive contract with a dominant buyer such as large wholesalers or meat processors, which signifies that their financial conditions are completely dependent upon the order quantities of the dominant buyer. There is a possibility that poultry farmers could stabilise and secure profit by producing their own branded chicken.

The chickens reared for slaughter are classified into mainly three categories in Japan [1]: (1) *Wakadori* (Broiler), (2) *Meigaradori* (Branded chicken) and (3) *Jidori* (Free-range local traditional pedigree chicken). Broilers are the most popular young and fast-growing chicken, and it takes about 50 days for broiler chickens to reach a commercial weight of from 2.5 kg to 3.0 kg. The Branded chickens are the same kind as that of broilers, but characterised by being fed with a low-calorie diet including substances such as herbs, medical herbs, tea, enzymes, and/or seaweed. Feeding a low-calorie diet makes their breeding period longer than that of broilers [1], [2]. There is no regulation in regard to how to breed the Branded chickens in Japan. In contrast, the *Jidori* (the Free-range local traditional pedigree chicken) has been certified governmentally in Japanese Agriculture Standards (JAS) by the Ministry of Agriculture in Japan. JAS 844 defines the meaning of *Jidori*, which refers to its genetic characteristics and breeding period as well as lower breeding density [3]. If the poultry farmer enters the market of the *Jidori*, its production cost considerably increases due to the longer breeding period, usually longer than that of the Branded chickens, and the lower livestock density [1]. For this reason, it is relatively easy for the small sized poultry farmers with the constraints such as smaller breeding areas and limited budget to introduce the Branded chickens.

In the field of inventory management, livestock such as chickens, fish and ducklings are referred to as ameliorating items. Several models have been studied for items with the Weibull amelioration rate, since Hwang [4], [5] first developed EOQ models for ameliorating items. Mondal et al. [6] conducted an inventory model for ameliorating items with price dependent demand rate for a prescribed time period. Hwang [7] dealt with a stochastic set-covering location problem for both ameliorating and deteriorating items. Law et al. [8] developed an integrated, production-inventory model for ameliorating and deteriorating, taking into account the time discount. Wee et al. [9] developed an inventory model with consideration of the time value of money in the case where both amelioration and deterioration rates were assumed to follow a Weibull distribution. Vandana and Srivastava [10] proposed an inventory model for ameliorating/deteriorating items with trapezoidal-type demand rate under the condition of inflation and time discounting rate. Chou et al. [11] and Tuan et al. [12] established analytical frameworks to obtain an optimal solution for the EOQ model proposed by Hwang [4].

This study proposes an inventory model for products with Weibull amelioration as follows.

This study considers the situation where a small sized poultry farmer not only produces Broilers (Product 1) to meet the demand of the dominant buyer, but also introduces their own branded chickens (Product 2) for other customers. The breeding area is limited and divided into two regions; one is for Product 1 and the other for Product 2. Each product has a

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different breeding period. The product with longer breeding period has higher selling prices. The poultry farmer's total profit per unit of time is formulated as a function of selling prices to obtain an optimal selling price for each product, which maximises the total profit. Some numerical examples are also presented.

II. Notation and Assumption

The main notation used in this paper listed below:

i	index of the product ($i = 1, 2$)
p_i	selling price of the Product i (decision variable)
$Q_i(p_i)$	order quantity per cycle of the Product i, a function of p_i
$D_i(p_i)$	demand quantity of the Product i, a function of p_i
D_U	maximum capacity of breeding area
T_i	time interval between successive orders (length of production period) of the Product i ($T_1 < T_2$)
$I_i(t)$	inventory level of the Product i at time t ($0 \le t < T_i$)
h_i	inventory holding cost of the Product i, including the cost of diet, per unit of weight and time ($h_1 < h_2$)
c_i	purchasing price of newly-hatched chick of the Product i per unit of weight
θ_i	deterioration rate of the Product i
s	ordering cost per order

The assumptions in this study are as follows:

1) The quantity of the product is expressed as its weight and treated as continuous for simplicity.
2) No shortages are allowed.
3) The breeding area is divided into two areas; one is for the Product 1 (Broiler chicken) and the other for Product 2 (Branded chicken).
4) No product is shipped during the production period, and all stock on hand of the Product i is shipped as a whole at the end of each production period ($t = NT_i$ ($N = 1,2,\cdots$)).
5) The inventory level of the Product i continuously increases due to growth during the production period ($[(N-1)T_i, NT_i)$ ($N = 1,2,\cdots$)), but at the same time, is depleted due to its deterioration.
6) The instantaneous rate of amelioration of the Product i at time t, denoted by $r_i(t)$, is expressed by [4]-[12]:

$$r_i(t) = \frac{f_i(t)}{1-F_i(t)} = \alpha_i\beta_i t^{\beta_i-1} \ (\alpha_i > 0, \beta_i > 0), \tag{1}$$

where $f_i(t)$ and $F_i(t)$, respectively, represent the probability density function and the distribution function of Weibull distribution, i.e.,

$$f_i(t) = \alpha_i\beta_i t^{\beta_i-1} e^{-\alpha_i t^{\beta_i}}, \tag{2}$$

$$F(t) = 1 - e^{-\alpha_i t^{\beta_i}}. \tag{3}$$

7) The demand quantity of the Product i, denoted by $D_i(p_i)$, is assumed to be deterministic and decreasing in p_i. This study focuses on the case where the demand can be expressed as the following constant price elasticity demand function [13], [14]:

$$D_i(p_i) = a_i p_i^{-b_i}, \tag{4}$$

where $a_i (> 0)$ denotes a scale parameter and $b_i\ (> 1)$ represents the index of price elasticity.

8) The poultry farmer has a maximum capacity, denoted by D_U, of the breeding area, i.e., $\sum_{i=1}^{2} D_i(p_i) \le D_U$, where $D_i(p_i)$ agrees with the maximum inventory level of the Product i during the cycle, for, by Assumption 4), the inventory level of the Product i at the end of each production period NT_i is equal to $D_i(p_i)$ as shown in Fig. 1.

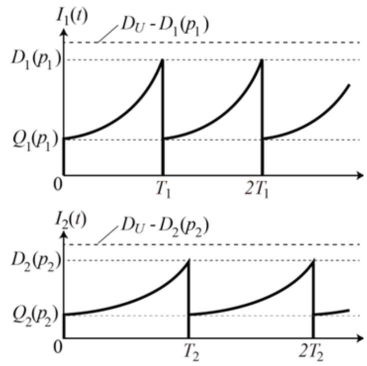

Fig. 1 Transition of inventory level

III. Total Profit

By Assumption 5) and Assumption 6), the inventory level of the Product i, denoted by $I_i(t)$, at time t can be expressed by:

$$I_i'(t) = [r_i(t) - \theta_i] I_i(t), \tag{5}$$

where $r_i(t)$ denotes the amelioration rate given by (1), and θ_i represents the deterioration rate of the Product i. By solving the differential equation of (5) with a boundary condition $I_i(t) = D_i(p_i)$, the inventory level of the Product i at time t is given by

$$I_i(t) = D_i(p_i) e^{-\alpha_i\left(T_i^{\beta_i} - t^{\beta_i}\right) + (T_i - t)\theta_i}, \tag{6}$$

where $D_i(p_i)$ is given by (4). Fig. 1 shows the transition of the inventory level for each product.

The order quantity per cycle $Q_i(p_i)$ is obtained by

$$Q_i(p_i) = I_i(0) = D_i(p_i) e^{-\left(\alpha_i T_i^{\beta_i} - \theta_i T_i\right)}. \tag{7}$$

The total inventory of the Product i held during $[0, T_i)$ is given by

$$H_i(T_i) = \int_0^{T_i} I_i(t)dt = Q_i(p_i) \int_0^{T_i} e^{\alpha_i t^{\beta_i} - \theta_i t} dt. \tag{8}$$

Therefore, the total profit of the Product i per unit of time can be represented by

$$P_i(p_i) = \frac{p_i D_i(p_i) - c_i Q_i(p_i) - h_i H_i(T_i) - s}{T_i} = \frac{1}{T_i}\left\{a_i p_i^{-b_i}\left[p_i - \right.\right.$$

$$e^{-\left(\alpha_i T_i^{\beta_i}-\theta_i T_i\right)} \times \left(c_i + h_i \int_0^{T_i} e^{\alpha_i t^{\beta_i}-\theta_i t} dt\right)\Big] - s\Big\}. \quad (9)$$

Let C_i be the positive multipliers, independent of p_i, in (9), i.e.,

$$C_i = e^{-\left(\alpha_i T_i^{\beta_i}-\theta_i T_i\right)} \times \left(c_i + h_i \int_0^{T_i} e^{\alpha_i t^{\beta_i}-\theta_i t} dt\right). \quad (10)$$

Henceforth, (9) is expressed as:

$$P_i(p_i) = \frac{a_i p_i^{-b_i}(p_i - C_i) - s}{T_i}. \quad (11)$$

The poultry farmer's total profit per unit of time is given by

$$P(p_i, p_2) = \sum_{i=1}^{2} \frac{a_i p_i^{-b_i}(p_i - C_i) - s}{T_i}. \quad (12)$$

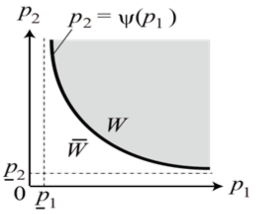

Fig. 2 Feasible region

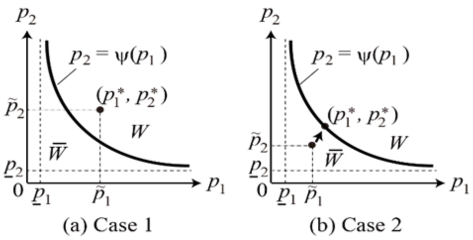

Fig. 3 Optimal prices

IV. Feasible Region

By Assumption 8), the lower bound of p_i $(i = 1, 2)$ is given by

$$\underline{p_i} = (a_i/D_U)^{1/b_i} \ (> 0). \quad (13)$$

For given a value of p_1, $\sum_{i=1}^{2} D_i(p_i) \le D_U$ agrees with

$$p_2 \ge \left(\frac{a_2}{D_U - a_1 p_1^{-b_1}}\right)^{1/b_2} \left(> \underline{p_2}\right). \quad (14)$$

Let $\psi(p_1)$ be the right-hand side of (14), i.e.,

$$\psi(p_1) = \left(\frac{a_2}{D_U - a_1 p_1^{-b_1}}\right)^{1/b_2} \left(> \underline{p_2}\right). \quad (15)$$

It can be shown that $\psi'(p_1) < 0$, $\psi''(p_1) > 0$, $\lim_{p_1 \to \underline{p_1}+0} \psi(p_1) = +\infty$, $\lim_{p_1 \to +\infty} \psi(p_1) = \underline{p_2}$. The behaviour of $\psi(p_1)$ can therefore be characterised as shown in Fig. 2. It also depicts the feasible region W on the (p_1, p_2) plane, which can be expressed as:

$$W = \{(p_1, p_2) |\, p_2 \ge \psi(p_1)\}. \quad (16)$$

V. Optimal Selling Prices

By differentiating $P(p_i, p_2)$ in (12) with respect to p_i $(i = 1, 2)$, we have

$$\frac{\partial P(p_1, p_2)}{\partial p_i} = a_i p_i^{-b_i} \frac{1 - b_i \frac{p_i - C_i}{p_i}}{T_i} \quad (17)$$

Let $\tilde{p}_i$ be a unique solution of $\partial P(p_1, p_2)/\partial p_i = 0$ as:

$$\tilde{p}_i = \frac{bi}{b_i - 1} C_i. \quad (18)$$

It can be proven using the Hessian matrix of $P(p_1, p_2)$ in (12) that $P(p_i, p_2)$ is a strictly concave function of p_i $(i = 1, 2)$ and attains a unique maximum value at the point of $(p_1, p_2) = (\tilde{p}_1, \tilde{p}_2)$.

By considering the feasible region W in (16), the maximum total profit per unit of time can be represented as follows:

$$P^* = \max_{p_2 \ge \psi(p_1)} P(p_1, p_2). \quad (19)$$

The optimal selling price $(p_1, p_2) = (p_1^*, p_2^*)$ which attains P^* in (19) can be summarised as follows:

A. In the Case of $\tilde{p}_2 \ge \psi(\tilde{p}_1)$ (Case 1)

In this case, the optimal selling price (p_1^*, p_2^*) is given by

$$(p_1^*, p_2^*) = (\tilde{p}_1, \tilde{p}_2), \quad (20)$$

as shown in Fig. 3 (a).

The maximum total profit P^* is then expressed by

$$P^* = \sum_{i=1}^{2} \frac{\frac{a_i}{b_i} \left(\frac{b_i}{b_i - 1} C_i\right)^{-(b_i - 1)} - s}{T_i}. \quad (21)$$

TABLE I (A)
NUMERICAL EXAMPLES: $h_1 = 0.85$

h_2	p_1^*	p_2^*	$\tilde{p}_1$	$\tilde{p}_2$	P^*	Case
1.0	256.20	370.31	176.05	261.86	1849.62	2
1.5	221.16	453.75	176.05	392.71	1818.74	2
2.0	199.71	555.57	176.05	523.56	1794.10	2
2.5	186.48	668.53	176.05	654.41	1774.21	2
3.142	176.05	822.42	176.05	822.42	1753.64	1

TABLE I (B)
NUMERICAL EXAMPLES: $h_1 = 1.00$

h_2	p_1^*	p_2^*	$\tilde{p}_1$	$\tilde{p}_2$	P^*	Case
1.0	271.16	348.55	207.10	261.86	1837.64	2
1.5	230.65	424.58	207.10	392.71	1804.50	2
1.959	207.10	512.80	207.10	512.80	1779.98	1
2.5	207.10	654.41	207.10	654.41	1757.27	1
3.0	207.10	785.26	207.10	785.26	1740.65	1

TABLE I (C)
NUMERICAL EXAMPLES: $h_1 = 1.15$

h_2	p_1^*	p_2^*	$\tilde{p}_1$	$\tilde{p}_2$	P^*	Case
1.0	288.04	329.38	238.14	261.86	1826.43	2
1.550	238.14	405.66	238.14	405.66	1787.84	1
2.0	238.14	523.56	238.14	523.56	1763.53	1
2.5	238.14	654.41	238.14	654.41	1742.77	1
3.0	238.14	785.26	238.14	785.26	1726.15	1

B. In the Case of $\tilde{p}_2 < \psi(\tilde{p}_1)$ (Case 2)

In this case, the poultry farmer can attain P^* in (19) on the line of $p_2 = \psi(p_1)$ as shown in Fig. 3 (b), in that $P(p_1, p_2)$ in (12) is a strictly concave function of p_i as mentioned above in this section. By letting of $p_2 = \psi(p_1)$ in (12), the total profit on $p_2 = \psi(p_1)$ becomes

$$P_\psi(p_1) = P(p_1, \psi(p_2)) = \frac{a_1 p_1^{-b_1}(p_1 - C_1) - s}{T_1} + \frac{(D_U - a_1 p_1^{-b_1})[\psi(p_1) - C_2] - s}{T_2}. \quad (22)$$

It can be proven that there exists a unique finite $p_1 = p_1^*$ ($> \tilde{p}_1$), which maximises $P(p_1)$ in (22), and $\psi(p_1^*) > \tilde{p}_2$ is satisfied.

Therefore, the optimal selling price (p_1^*, p_2^*) is obtained by

$$(p_1^*, p_2^*) = (p_1^*, \psi(p_1^*)). \quad (23)$$

VI. NUMERICAL EXAMPLES

Table I shows numerical examples in reference to $p_1^*, p_2^*, \tilde{p}_1, \tilde{p}_2$ and P^* for $(a_1, a_2, b_1, b_2, c_1, c_2, \alpha_1, \alpha_2, \beta_1, \beta_2, \theta_1, \theta_2, T_1, T_2, D_U) = (100000, 120000, 1.12, 1.1, 1, 1, 0.8755, 0.6740, 0.4, 0.45, 0.0008, 0.0006, 54, 62, 380)$ when h_2 varies from 1.5 to 3.5. Tables I (A)-(C) summarise the results for h_1 =0.85, 1.0, and 1.25, respectively. Fig. 4 corresponds to Table I. In Table I, the number 1 and number 2 in the column of "Case" represent the Case 1 and Case 2, respectively, which were described in Section V.A. and Section B.

It is observed from Table I that $p_i^* > \tilde{p}_i$ in the case of Case 2, as mentioned in Section V.B. This implies that the total profit could increase if the poultry farmer reduced the product quantity of the Product 1 to make room for introducing the Product 2 for the following reason.

By Assumption 7), the order quantity $Q_i(p_i)$ is decreasing in p_i, which signifies that the production quantity of the Product i decreases with increasing p_i . Since $\tilde{p}_1$ is determined independent of $\tilde{p}_2$, $\tilde{p}_1$ coincides with an optimal price in the case where the poultry farmer produces only the Product 1. The maximum profit in this case can be calculated by using $P_1(\tilde{p}_1)$ in (11). The values of $P_1(\tilde{p}_1)$, though not shown in Table I, are obtained as follows: (a) $P_1(\tilde{p}_1) = 870.50$ for h_1= 0.85, (b) $P_1(\tilde{p}_1) = 853.34$ for h_1= 1.0, and (c) $P_1(\tilde{p}_1) = 838.85$ for h_1= 1.15. By comparing these values of $P_1(\tilde{p}_1)$ with those of P^* in Table I, it is observed that $P_1(\tilde{p}_1) < P^*$ is satisfied in the case of $\tilde{p}_2 < \psi(\tilde{p}_1)$. In this case, it is difficult to prove analytically that $P_1(\tilde{p}_1) < P^*$ is satisfied, but this magnitude relationship can be confirmed by the results for other realistic parameters.

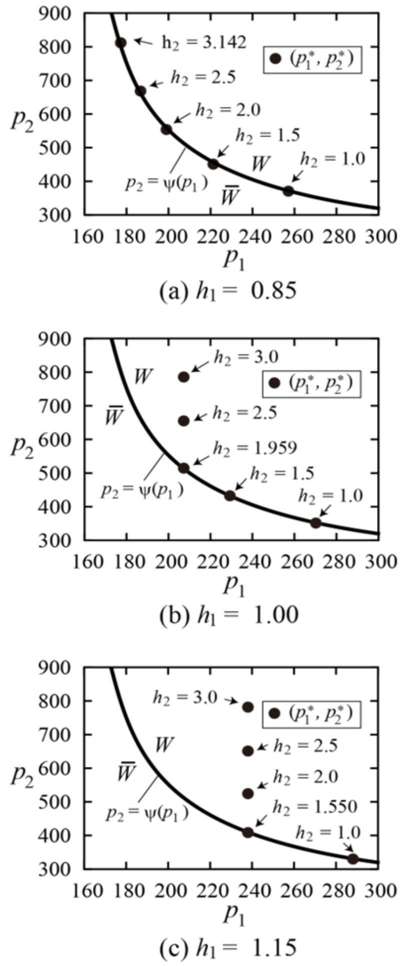

Fig. 4 Numerical Examples

Table I and Fig. 4 demonstrate that p_i^* increases with h_i $(i = 1, 2)$. Especially, as h_2 increases, the optimal price $(p_1^*, p_2^*) = (p_1^*, \psi(p_1^*))$ is replaced by $(p_1^*, p_2^*) = (\tilde{p}_1, \tilde{p}_2)$ at the points of $h_2 = 2.498$ and 1.694 when for $h_1 = 1.0$ and 1.25, respectively. This implies that the livestock density can be decreased as the cost of feed for the Product 2 becomes much higher than that for the Product 1.

VII. CONCLUSION

In this study, an inventory model for the items with Weibull amelioration rate has been proposed under the following circumstances: The small sized poultry farmer not only produces Broilers (Product 1) to meet the demand of the dominant buyer, but also introduces their own branded chickens (Product 2) for other customers. The breeding area is limited and divided into two regions; one is for Product 1 and the other for Product 2. Each product has a different breeding period. The product with longer breeding period has higher selling prices.

The poultry farmer's total profit per unit of time is formulated as a function of selling prices, and is has been proven that there exists a unique positive finite optimal selling price for each product, which maximises the total profit. Results from some numerical examples confirm that the total profit could increase if the poultry farmer reduced the product quantity of the Product 1 to make room for introducing the Product 2.

In real situations, most of the small sized poultry farmers have little freedom but to accept the order quantity of the broilers as well as their selling prices, offered by the dominant buyer and bounded by the contract. On the other hand, the demand of the branded chickens for other customers might vary with uncertainty. These factors will be considered as an extension of this model in the future.

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