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Optimal Location of the I/O Point in the Parking System

Jing Zhang, Jie Chen

***Abstract*—**In this paper, we deal with the optimal I/O point location in an automated parking system. In this system, the S/R machine (storage and retrieve machine) travels independently in vertical and horizontal directions. Based on the characteristics of the parking system and the basic principle of AS/RS system (Automated Storage and Retrieval System), we obtain the continuous model in units of time. For the single command cycle using the randomized storage policy, we calculate the probability density function for the system travel time and thus we develop the travel time model. And we confirm that the travel time model shows a good performance by comparing with discrete case. Finally in this part, we establish the optimal model by minimizing the expected travel time model and it is shown that the optimal location of the I/O point is located at the middle of the left-hand above corner.

***Keywords*—**Parking system, optimal location, response time, S/R machine.

I. Introduction

WITH the development of the society, the number of cars increases greatly. According to the data from the National Bureau of Statistics of China, we know that the number of cars in 2014 is 19 times that of 2010 [1]. The car brings some benefits and also brings some problems. One of these problems is parking problem. We find that AS/RS provides an efficient way to solve this problem; much research work has been done about this topic these years. We find that AS/RS system provides an efficient way to solve this problem; much research work has been done about this topic these years. In literature [2], [3], [5], the authors often develop the continuous model in units of time and establish the optimal model by minimizing the expected travel time model. It is a common and effective way to study on this topic and we also use the similar method in this paper.

In our parking system, the shape of the system is rectangular. The I/O point is on the left-hand of the system. And the S/R machine travels independently in vertical and horizontal directions while carrying a pallet. Each pallet holds only one car. With these characteristics, every parking location in our system is accessed to visit by the S/R machine. We try to find the optimal location of the I/O point in order to obtain the minimal travel time of the crane, so the waiting time to get the car is minimal.

Some assumptions and notations are made in Section II. In Section III, the mathematical model for single command cycles under the randomized storage assignment rule is derived. In Section IV, we compare inferred result with discrete case to emphasize the reality of the derivation and get more conclusions. In Section V, in order to get the optimal I/O point location, we optimize the model by minimizing the expected response time. Finally, in Section VI, we make some conclusions and give some advices on the future research.

Jing Zhang and Jie Chen are with Department of Mathematics and Physics, Qing Dao University of Science & Technology, China (e-mail: zhangjing@qust.edu.cn, 771964371@qq.com)..

II. Assumptions and Notions

In our parking system, the shape of the system is rectangular. The I/O point is on the left-hand of the system. And the S/R machine can travel independently in the vertical and horizontal directions. Fig. 1 shows the general shape of the parking system.

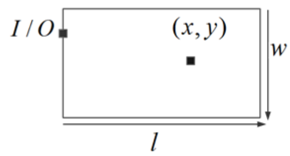

Fig. 1 Continuous representation of the storage area

In our parking system, the shape of the system is rectangular. The I/O point is on the left-hand of the system and the S/R machine can travel independently in the vertical and horizontal directions. Fig. 1 shows the general shape of the parking system.

All assumptions are similar as in [2]-[5] we only emphasize two factors as below:

1. The parking area should be considered to be a continuous pick face and the length of the system is always longer than the width of the system.
2. All parking spaces are the same size and each different car is regarded as a unit load. Therefore all parking spaces are candidates for storing any car.

The following notations are used in this paper:

L : Length of the parking system.

W : Width of the parking system.

S_h : Horizontal speed of the S/R machine.

S_w : Vertical speed of the S/R machine.

t_h : Horizontal response time to the farthest column from the I/O station, $t_h = L/S_h$.

t_w : Vertical response time to the farthest row from the I/O station, $t_w = W/S_w$.

T : $Max\{t_h, t_w\}$.

b : $\min\{t_h/T, t_w/T\}, \quad 0 < b \le 1$.

III. Derivation of the Expected Response Time Model

As in [2]-[7], we analyze our system for the single command cycle, therefore the response time equals two times of the response time between I/O point and the random parking space.

We assume the parking area to be rectangle (in the special case of $b=1$, is square) in time and in general. Letting $T=t_h$, then $b=t_w/t_h$. We get the continuous standardized system shown in Fig. 2.

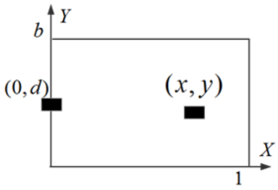

Fig. 2 The standardized system

In Fig. 2, X-axis and Y-axis are in units of time. The point (x,y) represents the storage or retrieval parking space, where $0\le x\le 1$ and $0\le y\le b$. We assume that the I/O point located at the d time units above the left-hand corner is represented by $(0,d)$, where $d\le b$. Now, let t_d represent the response time $(0,d)$ to (x,y). Using the characteristics of the S/R machine, we obtain the response time from the I/O point to location (x,y) as:

$$t_d = x+|y-d| \tag{1}$$

Now, let $G_d(t)$ denote the probability that the response time t_d is less than or equal to t. Thus,

$$G_d(t)=\Pr(t_d\le t)=\Pr(x+|y-d|\le t) \tag{2}$$

Furthermore, we can easily find that it is uniformly distributed for the random storage assignment rule. According to the relationship between b and d, there are some differences in $G_d(t)$. When $d\le b/2$, $G_d(t)$ is as:

$$G_d(t)=\Pr(t_d\le t)=\begin{cases}\dfrac{t^2}{b} & 0\le t\le d\\ \dfrac{t^2+2td-d^2}{2b} & d\le t\le b-d\\ \dfrac{2tb+2db-b^2-2d^2}{2b} & b-d\le t\le 1\\ \dfrac{-2t^2+4t+2bt-2-b^2-2d^2+2bd}{2b} & 1\le t\le 1+d\\ \dfrac{-t^2+2bt-2dt+2t-b^2-d^2+2bd+2d-1}{2b} & 1+d\le t\le 1+b-d\end{cases} \tag{3}$$

When $d>b/2$, $G_d(t)$ is as:

$$G_d(t)=\Pr(t_d\le t)=\begin{cases}\dfrac{t^2}{b} & 0\le t\le b-d\\ \dfrac{t^2+2bt-2dt-b^2-d^2+2bd}{2b} & b-d\le t\le d\\ \dfrac{2tb+2db-b^2-2d^2}{2b} & d\le t\le 1\\ \dfrac{-2t^2+4t+2bt-2-b^2-2d^2+2bd}{2b} & 1\le t\le 1+b-d\\ \dfrac{-t^2+2dt+2t-d^2-2d+2b-1}{2b} & 1+b-d\le t\le 1+d\end{cases} \tag{4}$$

The dashed area in Fig. 3 represents the area of all parking space with the condition of $t_d\le t$.

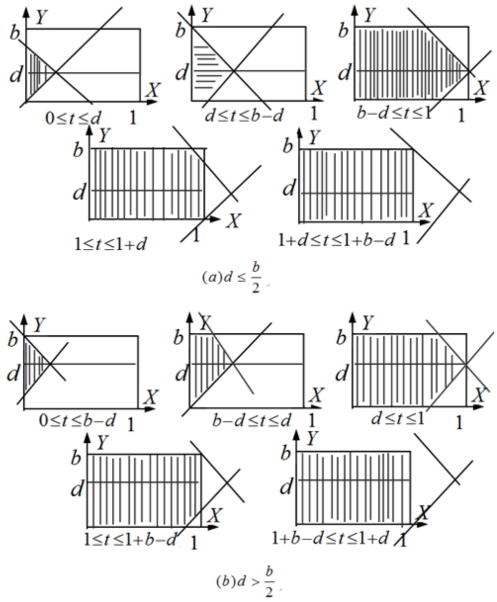

Fig. 3 Area for $t_d\le t$

From (3), (4), the probability density function $g_d(t)$ will be as:

In case of $d\le b/2$:

$$g_d(t)=\begin{cases}\dfrac{2t}{b} & 0\le t\le d\\ \dfrac{t+d}{b} & d\le t\le b-d\\ 1 & b-d\le t\le 1\\ \dfrac{-2t+2+b}{b} & 1\le t\le 1+d\\ \dfrac{-t+b-d+1}{b} & 1+d\le t\le 1+b-d\end{cases} \tag{5}$$

In case of $d>b/2$:

$$g_d(t)=\begin{cases}\frac{2t}{b} & 0\le t\le b-d\\ \frac{t+b-d}{b} & b-d\le t\le d\\ 1 & d\le t\le 1\\ \frac{-2t+2+b}{b} & 1\le t\le 1+b-d\\ \frac{-t+d+1}{b} & 1+b-d\le t\le 1+d\end{cases} \quad (6)$$

Letting $E_d(SC)$ denote the expected travel time from $(0,d)$ under single command cycle, the following result is obtained:

$$E_d(SC)=\begin{cases}2\int_0^{1+b-d} tg_d(t)=\frac{2d^2}{b}-2d+b+1 & d\le\frac{b}{2}\\ 2\int_0^{1+d} tg_d(t)=\frac{2d^2}{b}-2d+b+1 & d>\frac{b}{2}\end{cases} \quad (7)$$

From (7), $E_d(SC)$ can be represented by one equality as:

$$E_d(SC)=\frac{2d^2}{b}-2d+b+1 \quad (8)$$

IV. Comparisons with Discrete Case

In Section III, we derive the response time model for parking system. To emphasize the reality of the above results, we compare the results of the continuous model with the ones from a corresponding discrete parking space. In the discrete system, we assumed that every paring space is same size which is 16.5 ft*9.9 ft and $S_h=356$ fpm, $S_w=100$ fpm. We present some results in Tables I and II.

TABLE I
RESULTS OF FIXED d

System Size	No. of parking spaces	Shape factor b	Discrete case	Continuous model	% Deviation
*300*98*	162	0.8599	1.2842	1.4013	8.3626
*319*98*	171	0.9135	1.3305	1.4280	6.8294
336*98	180	0.9688	1.3769	1.4519	5.1693
348*98	189	0.9975	1.4232	1.4687	3.1019
*356*100*	210	1	1.4683	1.5000	2.1123

TABLE II
RESULTS OF FIXED SYSTEM SIZE

Value of m in d=m*b	Discrete case	Continuous model	% Deviation
0.2	1.5585	1.6447	5.2399
0.35	1.4376	1.5128	4.9648
0.5	1.5921	1.4232	3.1019
0.65	1.4970	1.5128	1.0426
0.8	1.6785	1.6447	1.8940

In Table I, we assumed that the discrete system had a fixed number $d=0.5*b$. We can get the expected response time in the continuous model and discrete case respectively for the different system size. For example, if the system size is 319 ft*98 ft, the expected response time in continuous model equals to 1.4280 min and in discrete case equals to 1.3305 min. From Table I, we can find that the continuous model shows a good performance with the largest percentage deviation being 8.3626% and as the system size becomes large, continuous model performs better.

In Table II, we assumed that the discrete system had a fixed system size which is 348 ft*98 ft. It can be observed that when the value of d equals to $0.5*b$, the response time is the least. It means when the I/O point is located at the middle of the left-hand above the corner, the performance of the parking system is the best.

V. Optimize the Expected Response Time Model

As in the literature [1], [2], [4]-[6], the actual expected response time $E(SC)$ is as: $E(SC)=TE_d(SC)$. Besides (8), $E(SC)$ will be:

$$E(SC)=TE_d(SC)=T(\frac{2d^2}{b}-2d+b+1) \quad (9)$$

We have known that $T=t_h, b=t_h/t_w$, $d=mb(0\le m\le 1)$, we can get $E(SC)$ as:

$$E(SC)=2m^2t_w-2mt_w+t_w+t_h \quad (10)$$

According to the above result, we can obtain the optimal value of m,t_w,t_h. Let S denotes the area of the system in time units. Following is the process of derivation:

$$\text{Object function: } \min E(SC) \quad (11)$$

Subject to:

$$t_w\le t_h \quad (12)$$

$$t_w*t_h=S \quad (13)$$

$$0\le m,t_w,t_h\le 1 \quad (14)$$

From (10) and (13), we have

$$E(SC)=t_w(2m^2-2m+1)+\frac{S}{t_w} \quad (15)$$

Therefore, we can easily get the optimal $m=0.5$. Substitute $m=0.5$ into (15),

$$E(SC)=0.5t_w+\frac{S}{t_w} \quad (16)$$

When $t_w=t_h=\sqrt{S}$, we get the optimal solution. It means when $m=0.5$ and $t_w=t_h=\sqrt{S}$, the system has the best performance. This conclusion is consistent with the conclusion we get from Section IV. In conclusion, the optimal I/O point location is located at the middle of the left-hand above the corner. We prove our results by MATLAB as listed in Table III.

TABLE III
THE OPTIMAL SOLUTION WITH DIFFERENT S

Value of S	THE OPTIMAL SOLUTION (m, t_w, t_h)	$E(SC)$
10000	(0.5,100,100)	150
15000	(0.5,122.4745,122.4745)	183.7117
20000	(0.5,141.4241,141.4241)	212.1320
25000	(0.5,158.1139,158.1139)	237.1708

VI. CONCLUSIONS

In this paper, we mainly analyze the automated storage/ retrieval parking systems where the S/R machine travels independently in vertical and horizontal directions. We develop the response time model in the continuous system for the single command under the randomized storage assignment rule. And we compare the model with discrete case to emphasize the reality of the model and get more conclusions. Finally, we optimize the response time model and get the optimal I/O point location.

In the future, we can extend the paper to the following cases. Firstly, we can develop our travel time models using different storage assignment rule, for example, class-based policy. Secondly the dual command cycle can be studied. Lastly, we can consider two or more I/O points are used, and how to find the optimal locations of the I/O points.

REFERENCES

[1] National Bureau of statistics. China Statistical Yearbook [J]. Beijing: China Statistics Press, 2000~2014

[2] W. H. Hausman, L. B. Schwarz, and S. C. Graves, “Optimal Storage Assignment in Automatic Warehousing Systems”, *Management Science*, vol. 22, no. 2, pp. 629-638, 1976.

[3] S. C. Graves, W. H. Hausman, and L. B. Schwarz, “Storage-Retrieval Interleaving in Automatic Warehousing Systems”, *Management Science*, vol. 23, no. 9,pp. 935-945, 1977.

[4] L. B. Schwarz, S. C. Graves, and W. H. Hausman, “Scheduling Policies for Automatic Warehousing Systems: Simulation Results”, *IIE Transactions*, vol. 10, no. 3, pp. 260-27, 1978.

[5] Y. A. Bozer and J. A. White, “Travel Time Models for Automated Storage/Retrieval Systems”, *IIE Transactions*, vol. 16, no. 4, pp. 329-338, 1984.

[6] Zaerpour, Nima, Y. Yu and R. B. M. D. Koster, “Small is Beautiful: A Framework for Evaluating and Optimizing Live-Cube Compact Storage Systems”, *Transportation Science*, 2015.

[7] S. G. Koh, B. S. Kim, and B. N. Kim, "Travel time model for the warehousing system with a tower crane S/R machine", Computers & Industrial Engineering, vol. 43, no. 3, pp. 495-507, 2002.