

# Optimal Controllers with Actuator Saturation for Nonlinear Structures

M. Mohebbi and K. Shakeri

**Abstract**—Since the actuator capacity is limited, in the real application of active control systems under severe earthquakes it is conceivable that the actuators saturate, hence the actuator saturation should be considered as a constraint in design of optimal controllers. In this paper optimal design of active controllers for nonlinear structures by considering actuator saturation, has been studied. The proposed method for designing optimal controllers is based on defining an optimization problem which the objective has been to minimize the maximum displacement of structure when a limited capacity for actuator has been used. To this end a single degree of freedom (SDF) structure with a bilinear hysteretic behavior has been simulated under a white noise ground acceleration of different amplitudes. Active tendon control mechanism, comprised of prestressed tendons and an actuator, and extended nonlinear Newmark method based instantaneous optimal control algorithm have been used. To achieve the best results, the weights corresponding to displacement, velocity, acceleration and control force in the performance index have been optimized by the Distributed Genetic Algorithm (DGA). Results show the effectiveness of the proposed method in considering actuator saturation. Also based on the numerical simulations it can be concluded that the actuator capacity and the average value of required control force are two important factors in designing nonlinear controllers which consider the actuator saturation.

**Keywords**—Active control, Actuator Saturation, Distributed-genetic algorithms, Nonlinear.

## I. INTRODUCTION

IN recent years intensive research efforts have been made to improve the reliability and safety of structures under earthquake and strong winds. To this end the use of protective systems such as passive, active, semi-active or hybrid control systems have received considerable attention.

In the area of passive control systems much progress has been accomplished in base isolation and different types of mechanical energy dissipater and in some cases these systems have been installed in actual buildings [1]. While passive control systems are effective in some cases they also suffer from a number limitation such as dependency to nature of earthquake.

Active control systems such as active mass dampers, active

tendon systems and active tuned liquid column dampers have been developed and tested in the laboratory and in a few cases installed in pro-type full scale buildings[2]-[3].

There are many active control algorithms proposed in the literature, most of which have been developed for linear systems. Some examples are the classical optimal control, pole assignment, bounded state control and predictive control methods [4] as well as intelligent control methods such as neural network and fuzzy logic based control [5]-[6].

In reality many buildings undergo large deformations or yielding when subjected to earthquake ground motions, hence exhibit nonlinear elastic or inelastic behavior, also in the most hybrid control systems, passive devices such as sliding isolation system and lead-core rubber bearing isolation systems behave nonlinearly or hysterically. Consequently active control systems should be capable of dealing with nonlinear structures.

On the other hand in the most previous researches in the field of active control of linear and nonlinear structures it has been assumed that the actuator can provide any desired control force which is determined according to control law, while in practical applications of active control systems it is conceivable that the required control force be larger than the actuator capacity, consequently the actuators saturate. So in this paper, it has been decided to study the effect of actuator saturation on the performance of control systems and designing optimal controllers.

There are some methods which have been developed for active control of nonlinear systems [7]-[8] such as active pulse control [8], optimal control of nonlinear structures [9] and hybrid control of nonlinear and hysteretic structures [10]. Chang and Yang [11] have developed an algorithm based on the Newmark integration algorithm and the instantaneous optimal control method in which the performance index includes displacement and velocity feedback. Bahar et al. [12] have improved the algorithm proposed by Chang and Yang [11] by using Wilson's- $\theta$  instead of Newmark integration algorithm. They have proposed a control algorithm for the linear systems that weighting parameters in performance index are determined by trial and error or some simplified assumptions. Joghataie and Mohebbi [13] have proposed an algorithm for active control of nonlinear frames which uses full feedback of response in performance index and applies genetic algorithm to determine the parameters of weighting matrices for optimal design of controllers. In this paper

M. Mohebbi is with the Faculty of Engineering, University of Mohaghegh Ardabili, Ardabil, Iran (corresponding author to provide by phone: +98-451-551-2910, fax: +98-451-551-2904, e-mail: mohebbi@uma.ac.ir).

K. Shakeri is with the Faculty of Engineering, University of Mohaghegh Ardabili, Ardabil, Iran (e-mail: shakeri@uma.ac.ir).

following the method proposed by Joghataie and Mohebbi[13] for determining the weighting matrices, nonlinear Newmark based instantaneous optimal control method has been developed and used for optimal design of controllers for nonlinear frames considering actuator saturation.

In the following sections, first nonlinear Newmark based instantaneous optimal control algorithm extended for nonlinear structures will be briefly reviewed. An explanation of the Distributed GA and designing optimal controller including actuator saturation will be presented followed by an SDOF nonlinear frame example and conclusions.

## II. NEWMARK BASED NONLINEAR INSTANTANEOUS OPTIMAL CONTROL ALGORITHM

In this paper for active control of nonlinear n-DOF structure, following the DGA based nonlinear optimal control [13] the Newmark based nonlinear instantaneous optimal control has been developed and used. The equation of motion of a controlled nonlinear n-DOF structure with  $m$  actuators at times  $(k-1)\Delta t$  and  $(k)\Delta t$  can be written as:

$$\mathbf{M}\ddot{\mathbf{X}}_{k-1} + \mathbf{F}_{D_{k-1}} + \mathbf{F}_{S_{k-1}} = \mathbf{M}\mathbf{e}\ddot{\mathbf{X}}_{g_{k-1}} + \mathbf{D}\mathbf{u}_{k-1} \quad (1)$$

$$\mathbf{M}\ddot{\mathbf{X}}_k + \mathbf{F}_{D_k} + \mathbf{F}_{S_k} = \mathbf{M}\mathbf{e}\ddot{\mathbf{X}}_{g_k} + \mathbf{D}\mathbf{u}_k \quad (2)$$

where  $t=\text{time}$ ,  $\ddot{\mathbf{X}}_g=\text{ground acceleration}$ ,  $\mathbf{X}$ ,  $\dot{\mathbf{X}}$  and  $\ddot{\mathbf{X}}$  are displacement, velocity and acceleration vectors respectively,  $\mathbf{M}=n \times n$  mass matrix,  $\mathbf{F}_D=\text{vector of damping forces}$  which is a function of velocity,  $\mathbf{F}_S=\text{vector of restoring forces}$  which is a function of displacement,  $\mathbf{D}=n \times m$  location matrix of actuators,  $\mathbf{e}=[-1, -1, \dots, -1]^T=n\text{-dimensional ground acceleration transformation vector}$ ,  $\mathbf{u}(t)=m\text{-dimensional control force vector}$ ,  $k=\text{integration time step}$ .

Subtracting (1) from (2) gives:

$$\mathbf{M}\Delta\ddot{\mathbf{X}}(t) + \mathbf{C}^*\Delta\dot{\mathbf{X}}(t) + \mathbf{K}^*\Delta\mathbf{X}(t) = \Delta\mathbf{P}(t) \quad (3a)$$

where

$$\Delta\ddot{\mathbf{X}}(t) = \ddot{\mathbf{X}}_k - \ddot{\mathbf{X}}_{k-1} \quad (3b)$$

$$\Delta\dot{\mathbf{X}}(t) = \dot{\mathbf{X}}_k - \dot{\mathbf{X}}_{k-1} \quad (3c)$$

$$\Delta\mathbf{X}(t) = \mathbf{X}_k - \mathbf{X}_{k-1} \quad (3d)$$

$$\Delta\mathbf{P}(t) = \mathbf{P}_k - \mathbf{P}_{k-1} \quad (3e)$$

$$\mathbf{P}_k = \mathbf{M}\mathbf{e}\ddot{\mathbf{X}}_{g_k} + \mathbf{D}\mathbf{u}_k \quad (3f)$$

$$\mathbf{P}_{k-1} = \mathbf{M}\mathbf{e}\ddot{\mathbf{X}}_{g_{k-1}} + \mathbf{D}\mathbf{u}_{k-1} \quad (3g)$$

Also  $\mathbf{C}^*$  and  $\mathbf{K}^*$  are tangential damping and stiffness matrices respectively.

Based on Newmark method [14], by solving the set of (3a) to (3g) the response of a nonlinear structure can be obtained as follows:

$$\mathbf{X}_k = \mathbf{X}_{k-1} + \Delta\mathbf{X}_k \quad (4a)$$

$$\dot{\mathbf{X}}_k = (1 - a_5)\dot{\mathbf{X}}_{k-1} - a_6\ddot{\mathbf{X}}_{k-1} + a_4\Delta\mathbf{X}_k \quad (4b)$$

$$\ddot{\mathbf{X}}_k = (1 - a_3)\ddot{\mathbf{X}}_{k-1} - a_2\dot{\mathbf{X}}_{k-1} + a_1\Delta\mathbf{X}_k \quad (4c)$$

$$\Delta\mathbf{X}_k = \mathbf{K}_{n_k}^{*-1}\Delta\mathbf{F}_k \quad (4d)$$

$$\mathbf{K}_{n_k}^* = a_1\mathbf{M} + a_4\mathbf{C}_{k-1}^* + \mathbf{K}_{k-1}^* \quad (5)$$

$$\Delta\mathbf{F}_k = (\mathbf{P}_k - \mathbf{P}_{k-1}) + \mathbf{M}(a_2\dot{\mathbf{X}}_{k-1} + a_3\ddot{\mathbf{X}}_{k-1}) + \mathbf{C}_{k-1}^*(a_5\dot{\mathbf{X}}_{k-1} + a_6\ddot{\mathbf{X}}_{k-1}) \quad (6)$$

$\mathbf{K}_{n_k}^*$  varies at each time step.

$$a_1 = \frac{1}{\delta(\Delta t)^2}; \quad a_2 = \frac{1}{\delta\Delta t}; \quad a_3 = \frac{1}{2\delta}; \quad (7a, b, c)$$

$$a_4 = \frac{\gamma}{\delta\Delta t}; \quad a_5 = \frac{\gamma}{\delta}; \quad a_6 = \Delta t\left(\frac{\gamma}{2\delta} - 1\right); \quad (7d, e, f)$$

where  $\gamma, \delta$  are Newmark parameters [14].

### A. Performance Index

In the instantaneous optimal control, the performance index at time step  $k$  includes feedback of the system response and control force. To assess the effect of displacement, velocity and acceleration response on the performance of control system it has been decided to use full feedback of the system response and control force in the performance index as:

$$J_k = \frac{1}{2}(\mathbf{X}_k^T\mathbf{Q}_1\mathbf{X}_k + \dot{\mathbf{X}}_k^T\mathbf{Q}_2\dot{\mathbf{X}}_k + \ddot{\mathbf{X}}_k^T\mathbf{Q}_3\ddot{\mathbf{X}}_k + \mathbf{u}_k^T\mathbf{R}\mathbf{u}_k) \quad (8)$$

where  $\mathbf{Q}_1, \mathbf{Q}_2$  and  $\mathbf{Q}_3$  are  $n \times n$  positive semi-definite weighting matrices corresponding to the penalty for large displacements, velocities and accelerations, and  $\mathbf{R}$  is a  $m \times m$  positive definite matrix representing the cost for applying large forces [4].

### B. Determination of Control Force Vector

In the instantaneous optimal control at each time step  $k$ , the control force  $\mathbf{u}_k$  is determined by minimizing the performance index  $J_k$  at that same step which has been defined in (8). To this end the equations of motion, (4a-c), are considered as constraints and the Hamiltonian of the optimization problem is formed according to Chang and Yang [11] as follows:

$$H_k = \frac{1}{2}(\mathbf{X}_k^T\mathbf{Q}_1\mathbf{X}_k + \dot{\mathbf{X}}_k^T\mathbf{Q}_2\dot{\mathbf{X}}_k + \ddot{\mathbf{X}}_k^T\mathbf{Q}_3\ddot{\mathbf{X}}_k + \mathbf{u}_k^T\mathbf{R}\mathbf{u}_k) + \lambda_1^T(\mathbf{X}_k - \mathbf{X}_{k-1} - \Delta\mathbf{X}_k) + \lambda_2^T[\dot{\mathbf{X}}_k - (1 - a_5)\dot{\mathbf{X}}_{k-1} + a_6\ddot{\mathbf{X}}_{k-1} - a_4\Delta\mathbf{X}_k] + \lambda_3^T[\ddot{\mathbf{X}}_k - (1 - a_3)\ddot{\mathbf{X}}_{k-1} + a_2\dot{\mathbf{X}}_{k-1} - a_1\Delta\mathbf{X}_k] \quad (9)$$

Where  $\lambda_i$ s = Lagrangian multipliers. The necessary conditions for minimizing the performance index  $J(t)$  are:

$$\frac{\partial H_k}{\partial \mathbf{X}_k} = \frac{\partial H_k}{\partial \dot{\mathbf{X}}_k} = \frac{\partial H_k}{\partial \ddot{\mathbf{X}}_k} = \frac{\partial H_k}{\partial \mathbf{u}_k} = \frac{\partial H_k}{\partial \lambda_1} = \frac{\partial H_k}{\partial \lambda_2} = \frac{\partial H_k}{\partial \lambda_3} = 0 \quad (10)$$

Substituting (9) into (10) gives:

$$\mathbf{Q}_1\mathbf{X}_k + \lambda_1 = 0 \quad (11)$$

$$\mathbf{Q}_2\dot{\mathbf{X}}_k + \lambda_2 = 0 \quad (12)$$

$$\mathbf{Q}_3 \ddot{\mathbf{X}}_k + \lambda_3 = 0 \quad (13)$$

$$\mathbf{R} \mathbf{u}_k - \mathbf{D}^T \mathbf{K}_{n_k}^{*-T} (\lambda_1 + a_4 \lambda_2 + a_1 \lambda_3) = 0 \quad (14)$$

$$\mathbf{X}_k - \mathbf{X}_{k-1} - \Delta \mathbf{X}_k = 0 \quad (15)$$

$$\dot{\mathbf{X}}_k - (1 - a_5) \dot{\mathbf{X}}_{k-1} + a_6 \ddot{\mathbf{X}}_{k-1} - a_4 \Delta \mathbf{X}_k = 0 \quad (16)$$

$$\ddot{\mathbf{X}}_k - (1 - a_3) \ddot{\mathbf{X}}_{k-1} + a_2 \dot{\mathbf{X}}_{k-1} - a_1 \Delta \mathbf{X}_k = 0 \quad (17)$$

By substituting (11)-(13) into (14) and after some rearrangement the control force is determined as:

$$\mathbf{u}_k = -\mathbf{R}^{-1} \mathbf{D}^T \mathbf{K}_{n_k}^{*-T} (\mathbf{Q}_1 \mathbf{X}_k + a_4 \mathbf{Q}_2 \dot{\mathbf{X}}_k + a_1 \mathbf{Q}_3 \ddot{\mathbf{X}}_k) \quad (18)$$

where superscript  $(-T)$  means transpose of inverse matrix.

According to (18) it is obvious that the control force is dependent to full feedback of response and weighting matrices.

### III. DISTRIBUTED GENETIC ALGORITHM (DGA)

Genetic algorithm (GA) developed by Holland [15] and has been documented in his pioneering book in this area. GA is a computational method which is inspired by natural Darwinian evolution. In GAs chromosomes evolving under a certain environment are represented by bit strings or real-valued coding. In the early stages of string coding, design variables were represented in their binary format [16]-[17]. Whilst binary-coded GAs appear to be more suitable to complex problems, they have some drawbacks in taking continuous problems and it has been shown that for real-valued numerical optimization problems, real-valued coding representations offer certain advantages such as simple programming, less memory required, no need to convert chromosomes and greater freedom to use different genetic operators over binary versions [16].

There are three genetic algorithm operators including selection, cross over and mutation. In every generation, a set of chromosomes is selected for mating based on their relative fitness. The fitters are given more chance of passing their genes into the next generation. This process of natural selection is operated by selection. The basic operator for producing new individuals in the GA is that of cross over. Cross over produces new individuals that have some parts of both parents genetic material. The role of mutation is often seen as providing a guarantee that the probability of searching any given string will never be zero. In this paper the elitist strategy has been used which allows the best chromosomes of the current generation to go to the next generation without modification.

In Distributed Genetic Algorithms (DGA), a large population is divided into smaller subpopulations, and a traditional GA is executed on each subpopulation separately. Some individuals are selected from each subpopulation and migrated to different subpopulations periodically. For migration of individuals different methods has been proposed such as the ring topology, neighborhood migration and unrestricted migration. In this paper the unrestricted migration which is the most common used method, has been used. In the

literature the use of DGA has shown that smaller number of individuals in DGA leads to quicker convergence and higher searching capability as compared to the conventional GAs [18]-[19].

### IV. OPTIMAL CONTROLLERS CONSIDERING ACTUATOR SATURATION

The control force is defined as a function of the weighting matrices  $\mathbf{R}$ ,  $\mathbf{Q}_1$ ,  $\mathbf{Q}_2$  and  $\mathbf{Q}_3$  in (18) where the weight matrices  $\mathbf{R}$ ,  $\mathbf{Q}_1$ ,  $\mathbf{Q}_2$  and  $\mathbf{Q}_3$  can be determined so that some constraints on the response or control force are satisfied. By assuming unlimited capacity for actuators for any set of weighting matrices the control force is determined according to (18). In practical application it is possible that the required control force be larger than the actuator capacity so it is required to consider the saturation of actuator in designing the controllers. To consider the actuator saturation for a pre-specified actuator capacity,  $u_{sat}$ , two strategies can be used as follows:

#### A. Case (a):

Considering actuator capacity as a constraint in optimization problem and designing optimal controller to minimize the maximum displacement, for this case the optimization problem can be defined as:

$$\text{Find } \mathbf{Q} = (\mathbf{Q}_1, \mathbf{Q}_2, \mathbf{Q}_3) \quad (19a)$$

$$\text{Minimize } X = \text{Max. } (X_k = |\mathbf{X}_k|, k=1, 2, \dots, k_{max}) \quad (19b)$$

$$\text{Subject to } g_1 = u_{max}/(u_{sat}) - 1 \leq 0.0 \quad (19c)$$

where  $u_{max}$  and  $u_{sat}$  are maximum required control force and the capacity of actuator respectively. Also:

$$X_{max} = \max. (|\mathbf{X}_k|, k=1, 2, \dots, k_{max}) \quad (20a)$$

$$k_{max} = \text{total number of time steps} \quad (20b)$$

In this case the maximum required control force is equal with the actuator capacity. In the optimization problem defined in (19a-c), it is desired to find the set of weighting parameter  $\mathbf{Q}^* = (\mathbf{Q}_1^*, \mathbf{Q}_2^*, \mathbf{Q}_3^*)$  so that both the maximum displacement is minimized and also the control force remains in specified limits. In this paper distributed genetic algorithm (DGA) which is an improved version of traditional genetic algorithm (GA), has been used to solve the optimization problem defined in (19a-c).

#### B. Case (b):

In this method the control force is determined based on control law and if the required control force is larger than the actuator capacity then the control force is considered equal with the capacity of actuator. In this case to design a controller which minimizes the maximum displacement of structure under the actuator capacity constraint, the optimization problem can be defined as:

$$\text{Find } \mathbf{Q} = (\mathbf{Q}_1, \mathbf{Q}_2, \mathbf{Q}_3) \quad (21a)$$

$$\text{Minimize } X = \text{Max. } (X_k = |\mathbf{X}_k|, k=1, 2, \dots, k_{max}) \quad (21b)$$

$$\text{if } u_{max} \geq u_{sat} \text{ then } u_{max} = u_{sat} \quad (21c)$$

### V. NUMERICAL EXAMPLE

For numerical analysis a single degree of freedom (SDOF) structure has been considered as shown in Fig.1 which its

structural properties have been taken from Yang et al. [9] and modeled according to bilinear hysteretic model shown in Fig.2 and mitigation of its vibrations by active controlling has been studied. The stiffness is bilinear elastic-plastic with an elastic stiffness  $K_1=8.5273 \times 10^4$  kN/m and a post elastic stiffness  $K_2=9.7455 \times 10^3$  kN/m. The floor mass is 345.6 tons and the natural frequency of the structure based on initial stiffness is 2.5 Hz. The linear viscous damping coefficient  $C$  is 54.29 kN.sec/m which corresponds to a damping ratio of 0.5%. Yielding occurs at a lateral relative displacement of  $X_{yielding}=2.4$  cm. In this study, it has been assumed that the actuator-structure interaction effect is not significant.

The uncontrolled structure was subjected to white noise ground accelerations of different intensities, denoting by  $W_1(t)$ , a white noise with  $PGA=100\text{cm/s}^2$  as shown in Fig. 3, the white noise,  $W_\alpha(t) = \alpha W_1(t)$  has a  $PGA=100\alpha \text{ cm/s}^2$ .

The effect of  $\alpha$  on the maximum displacement assuming the system would not fail is represented in Fig.4. For  $\alpha \geq 2$ , the system has experienced nonlinearity beyond  $X_{yielding}=2.4$  cm. Hence to design the controller, it was decided to use the white noise with  $\alpha=4.9$  which could produce large nonlinear uncontrolled displacement, denoted by  $X_u$ , where  $X_u = 3.6 \text{ cm} = 150\% X_{yielding}$ .

The extended nonlinear Newmark method with  $\gamma = 0.25$  and  $\delta = 0.5$  as suggested in literature [14] to stability of numerical analysis has been used for nonlinear analysis of the system where the integration time interval has been 0.002 seconds to achieve the required accuracy.

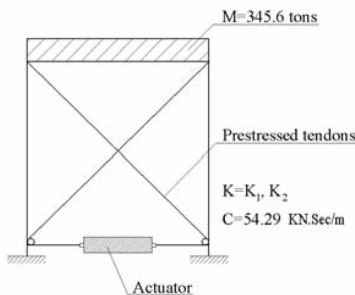


Fig. 1 SDOF Structure-Actuator model with active tendon control system

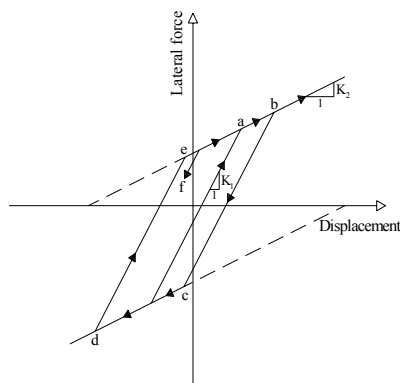


Fig. 2 Nonlinear bilinear Elasto-Plastic stiffness model

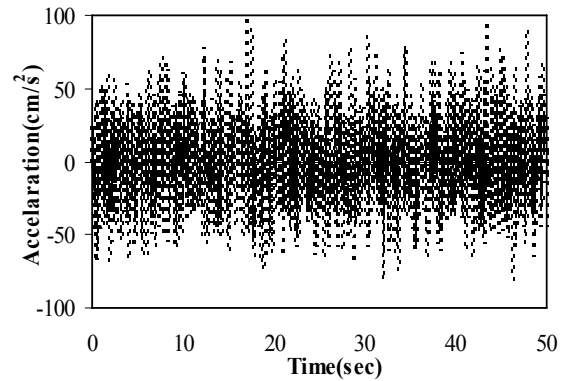


Fig. 3 White noise excitation,  $W_1(t)$ , with  $PGA=100 \text{ cm/s}^2$

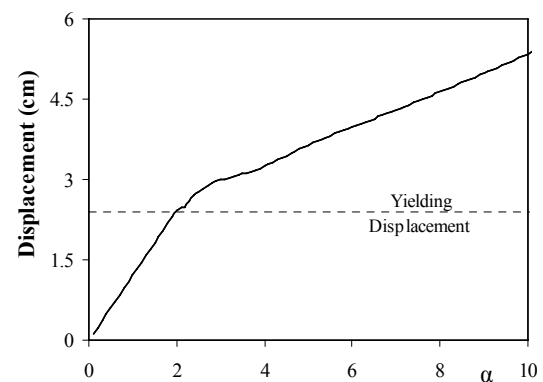


Fig. 4 Maximum displacement of the nonlinear SDOF frame versus different white noise amplitude factor ( $\alpha$ )

#### A. Optimal Design of Controllers According to Case (a):

For different values of actuator capacity,  $u_{sat}$ , optimization problem defined through (19a-c) has been solved. The parameters of the DGA have been as follows:

Number of subpopulations = 2, Number of individuals in each subpopulation = 40, Number of elites = 8, Number of the newborns = 40, mutation rate = 0.04, Migration interval = 20 and Migration rate = 0.20.

##### 1) Finding the Optimum $Q$ by DGA for $u_{sat}=100 \text{ kN}$

It was desired to design the controller to minimize the displacement under a ground white noise acceleration of amplitude  $\alpha=4.9$  while the maximum control force is below the actuator capacity.

Following the DGA procedure, 2 subpopulations each with 40 randomly generated vectors of control parameters  $Q = (Q_1, Q_2, Q_3)$  were generated as the initial population. The response as well as the maximum displacement was recorded and the objective function was calculated for each  $Q$ . The convergence behavior of the DGA towards the optimum answer  $Q^*$  is shown in Fig.5 for three runs, where the optimum objective function value for each generation has been plotted versus the generation number in three runs.

Obviously the convergence is monotonic because the elites in each generation have survived to enter the next generation, taking the best objective function value of any generation to the next one. Also all runs ended approximately with the same objective function value. From solving the optimization problem by DGA the optimum answer has been as follows:

$u_{\max}$  = maximum control force = 99.98 kN;

$X_{\max}$  = maximum displacement = 2.71 cm;

It is clear that the maximum control force is approximately equal with the actuator capacity, as expected in Case (a).

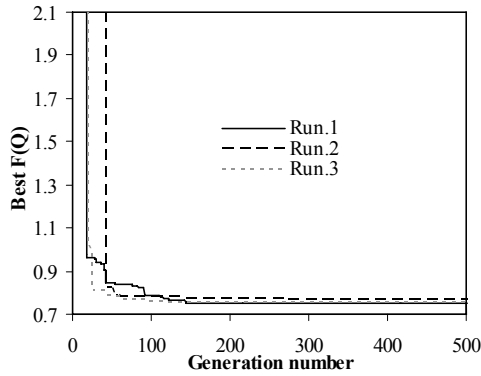


Fig. 5 The best fitness value of chromosomes in three runs of DGA

## 2) Designing of Optimal Controllers for Different Actuator Capacity:

Following the same procedure, new controllers were designed for different values of  $u_{\text{sat}}$  but for the same white noise with  $\alpha = 4.9$ . The maximum normalized response of controlled structure has been shown for different actuator capacity, in Fig.6. The average control force,  $u_{\text{ave}}$ , which can be used as an index to show the value of consuming energy for control system, has been defined in (22).

$$u_{\text{ave}} = \frac{\sum_{k=1}^{k_{\max}} |u_k|}{k_{\max}} \quad (22)$$

For different  $u_{\text{sat}}$  the normalized average control force has been shown in Fig.6, too.

### B. Optimal Design of Controllers According to Case (b):

Following the same procedure explained for Case (a), new controllers have been designed for different actuator capacity while the Case (b) has been used for considering the actuator

saturation constraint according to equations defined in (21a-c). Fig.7 shows the maximum normalized response of controlled structure and average control force. From the results it is clear that in this case average control force is approximately equal with the actuator capacity which shows that in most times the applied control force is equal with the actuator capacity.

Comparing the results shown in Fig.6 and Fig.7 shows that applying Case (b) for considering the actuator saturation, leads to more reduction in maximum displacement of structure in comparison with Case (a), while it requires larger average control force consequently larger amount of required consuming energy.

For the same average control force, for Cases (a) and (b) the maximum response of uncontrolled and controlled structures has been shown in Table I for different actuator capacity. According to results shown in Table I, it can be concluded that by considering the same value for the average control force, Cases (a) and (b) have approximately the same performance for considering the actuator saturation in designing the optimal controllers.

## VI. CONCLUSION

In this paper two methods have been proposed for considering the actuator saturation in designing the optimal controllers for nonlinear frames. The methods have been based on defining an optimization problem to minimize the maximum response of structure and considering the actuator capacity as constraint. For active control of nonlinear structures by using Newmark integration method for numerical simulation, the extended nonlinear instantaneous optimal control method that considers full feedback of response in performance index, has been used.

For different values of actuator capacity Distributed Genetic Algorithms (DGA) has been used successfully in the design of optimum nonlinear controllers for the objective of minimizing the maximum displacement of a SDF nonlinear structure modeled by bilinear elastic-plastic stiffness, under white noise excitation. It has been shown that the proposed method has been successful in designing the optimal controllers when the actuator saturation has been considered. Also the results of the numerical simulation show that in considering the actuator saturation, the actuator capacity and the average of required control force are two important indexes in designing the controllers.

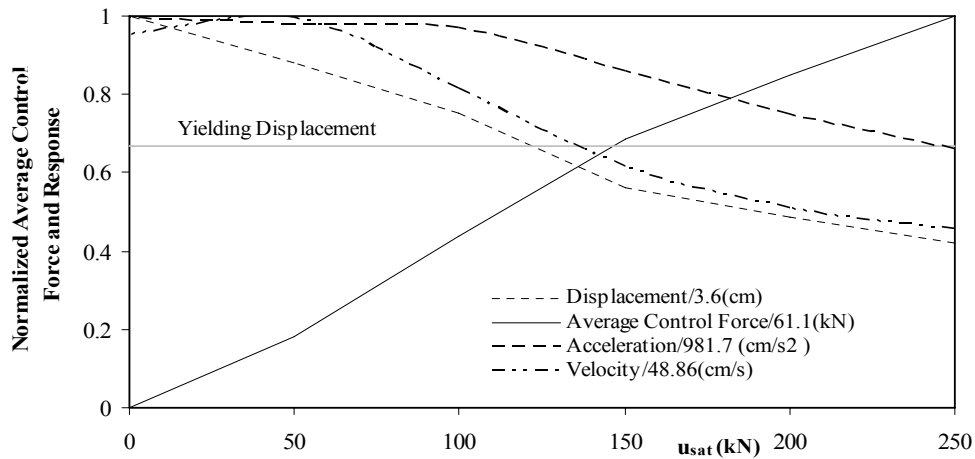


Fig. 6 Normalized maximum response and average control force versus actuator capacity when Case (a) used for considering actuator saturation

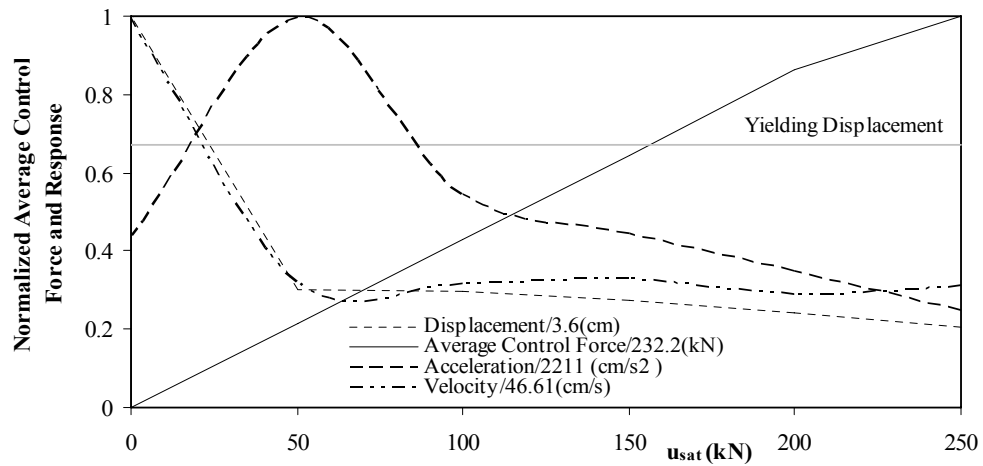


Fig. 7 Normalized maximum response and average control force versus actuator capacity when Case (b) used for considering actuator saturation

TABLE I  
RESPONSE OF UNCONTROLLED AND CONTROLLED STRUCTURES FOR DIFFERENT ACTUATOR CAPACITY WHEN THE SAME AVERAGE CONTROL FORCE USED FOR CASES (A) AND (B).

Response Actuator Capacity (kN)	Case(a)			Case(b)		
	Dis. (cm)	Vel. (cm/s)	Acc. (cm/s <sup>2</sup> )	Dis. (cm)	Vel. (cm/s)	Acc. (cm/s <sup>2</sup> )
0	3.60	46.61	981.7	3.60	46.61	981.7
50	3.17	48.86	964.7	3.40	44.75	977.4
100	2.71	39.98	956.1	2.83	41.52	955.3
150	2.03	30.16	845.2	1.99	29.99	845.8
200	1.75	25.01	736.6	1.75	25.07	736.5
250	1.51	22.39	649.4	1.57	22.97	668.7

Dis. =displacement, Vel. =velocity, Acc. = acceleration

## REFERENCES

- [1] B.F. Spencer and Nagarajaiah S., "State of the art of structural control," *ASCE, J. Struct. Eng.*, vol.129, no.7, pp.845-856, 2003.
- [2] T. Kobori, N. Koshika, K. Yamada and Y. Ikeda, "Seismic-response-controlled structure with active mass driver system .part 1: Design" *Earthquake Eng. Struct. Dynamics*, vol. 20, pp.133-149, 1991.
- [3] T. Kobori, N. Koshika, K. Yamada and Y. Ikeda, "Seismic-response-controlled structure with active mass driver system .part2: Verification," *Earthquake Eng. Struct. Dynamics*, vol.20, pp.151-166, 1991.
- [4] T. T. Soong, *Active structural control: theory and practice*. Longman Scientific & Technical, New York, 1990.
- [5] J. Ghaboussi and A. Joghataie, "Active control of structures using neural networks," *ASCE, J. Eng. Mech.*, 121(4):555-567, 1995.
- [6] A. Joghataie and A. Vahidi, "Designing a general neuro controller for water towers," *ASCE, J.Eng. Mech.*, 126(6): 582-587, 2000.
- [7] A. Reinhorn, M. Manolis G. D. and C. Y. Wen, "Active control of inelastic structures," *ASCE, J. Eng. Mech.*, 113(3):315-333, 1987.
- [8] S.F. Masri, G.A. Bekey and T.K. Caughey, "Optimal pulse control of flexible structures," *ASME, J. App. Mech.*, 48(4):619-626, 1981.
- [9] J. N. Yang, F. X Long and D. Wong, , "Optimal control of nonlinear structures," *Applied Mech.*, vol. 55, pp. 931-938, 1988.
- [10] J. N. Yang, Z. Li, A. Danielians and S. C. Liu, "A seismic hybrid control of nonlinear and hysteretic structures," *ASCE, J. Eng. Mech.*, 118(7):1423-1440, 1992.
- [11] C. C. Chang and H. T., Yang Y., "Instantaneous optimal control of building frames," *ASCE, J. Struct. Eng.*, vol.120, no. 4, pp.1307-1326, 1994.
- [12] O.Bahar, M. R. Banan, M. Mahzoon and Y.Kitagawa, "Instantaneous optimal Wilson- $\theta$  control method," *ASCE, J. Eng. Mech.*, vol.129, no.11, pp.1268-1276, 2003.
- [13] A. Joghataie and M. Mohebbi, "Optimal control of nonlinear frames by Newmark and distributed genetic algorithms," *Struct. Design Tall Spec. Build.*, published online, 2009.
- [14] K. J. Bathe *Finite element procedures*. Prentice-Hall, Inc., New Gersey, 1996.
- [15] J.H. Holland, *Adaptation in natural and artificial systems*. Ann Arbor: The University of Michigan Press, 1975.
- [16] D. E. Goldberg, *Genetic algorithms in search, optimization and machine Learning*. Addison -Wesley Publishing Co., Inc. Reading, Mass, 1989.
- [17] Z. Michalewicz, *Genetic algorithms + data structures=evolution programs*, New York: Springer-Verlag, 1996.
- [18] H. Mühlenbein, M.Schomisch and J. Born, "Parallel genetic algorithms as a function optimizer," *Parallel Computing*, no.17, pp.619-632, 1991.
- [19] T. Starkweather, D. Whitley and K. Mathias, "Optimization using distributed genetic algorithms," *Springer - Verlag Lecture Notes in Computer Science*, no.496, pp.176-185, 1990.