

# Optimal Controller with Backstepping and BELBIC for Single-Link Flexible Manipulator

Ali Reza Sahab and Amir Gholami Pastaki

**Abstract**—In this paper, backstepping method (BM) is proposed for a single-link flexible mechanical manipulator. In each step of this method a positive value is obtained. Selections of the gain factor values are very important because controller will have different behavior for each different set of values. Improper selection of these gains can lead to instability of the system. In order to choose proper values for gains BELBIC method has been used in this work. Finally, to prove the efficiency of this method, the obtained results of proposed model are compared with robust controller one. Results show that the combination of backstepping and BELBIC that is presented here, can stabilize the system with higher speed, shorter settling time and lower overshoot in than robust controller.

**Keywords**—single-link flexible manipulator, backstepping, BELBIC

## I. INTRODUCTION

**M**ECHANICAL manipulators have lots of applications in industries. Technological developments, controlling methods have been sophisticated and new system control with conventional methods is not feasible. Human capability of dealing with these kinds of systems has resulted in researchers' tendency towards human based controlling methods. Lots of controllers and methods have been proposed in order to control manipulators.

Vicente Feliu et al. [1], have presented a new feed forward method to amend flexible manipulator dynamics in order to have an optimal design for maximum load. M.S. Alam et al. [2], have design a feed forward command shapers with multi-objective genetic optimization for vibration control of a single-link flexible manipulator. A combination of fuzzy logic with genetic algorithm optimization for a single-link flexible manipulator can be seen in [3]. Kerem Gurses et al. [4] have used an array of fiber optic curvature sensors and PZT actuators to control the vibration of a single-link flexible manipulator. A PZT actuator control of a single-link flexible manipulator based on linear velocity feedback and actuator placement can be seen in [5] that used intelligent equipments specifically piezoelectric material in order to obtain Lyapunov function using PD feedback and voltage signals to drive PZT actuator. Barun Pratiher et al. have investigated non-linear vibration and behavior of a single link viscoelastic Cartesian

manipulator [6]. In [7] a combination of sensor based active controller and piezoelectric actuators is utilized in order to control end position a two-link flexible manipulator. Ismael Payo et al. [8], utilized a modified PID to control force of a very lightweight single-link flexible arm based on coupling torque feedback in two states of no-load and tracking. In [9] a simplified and reduced model for a single-link flexible manipulator is presented and system identification has been investigated. S.B. et al. have designed a vibration controller for a single-link flexible arm considering its disturbances [10]. Furthermore, a strain gauge based controller for single-link flexible arms in very lightweight robots is designed that is robust for maximum payload [11]. A robust distributed controller has been designed for a single-link Cartesian smart materials robot (SCARA) that is tolerant of distortions [12].

Y. Cao et al designed a new manipulator that was utilized for a trajectory input tracking experiment [13]. Proposed controller is based on PID and FLT design. In [14] and [15] backstepping method of a single-link flexible robotic manipulator is utilized for trajectory input tracking.

Nonlinearity of unknown parameters is always the main problem of manipulators. Real time implementation of NARMA-L2 controller has designed in [16] for controlling a DC motor of single link manipulator considering unknown dynamics. Positive position feedback (PPF) method is one of the most efficient methods for intelligent actuators.

Slewing and vibration control of a single-link flexible manipulator has done by Jinjun shan et al, [17], using PPF controller for a PZT actuator. A new modeling to single-link flexible manipulator using singular perturbation method is presented in [18] which were controlled by  $H_{\infty}$  method. In [19], a combination of nonlinear backstepping control and adaptive control has been proposed to design a controller for a flexible joint manipulator. A hybrid of adaptive and sliding control method has utilized to control a single-link flexible-joint robot with mismatched uncertainties [20]. PID-based control of a single-link flexible manipulator that was optimized by genetic algorithm was proposed in [21] in order to control vertical motions. B.A. Md Zain, et al [22], utilizing parametric techniques of RLS with Genetic Algorithms proposed a dynamic modeling for a Single-link flexible manipulator.

In [23, 24] new controlling methods based on a hybrid strategy and integral resonant control (IRC) were proposed respectively in order to control vibration damping and precise tip-positioning of manipulator. Utilizing  $H_{\infty}$ -based PID

Ali Reza Sahab is a staff member of Islamic Azad University Lahijan Branch, Lahijan, Guilan, Iran corresponding author of this paper (phone: 0098-9111832163; e-mail: sahab@iaiu.ac.ir).

Amir Gholami Pastaki is a staff member of Electrical Group, Department of Engineering, Islamic Azad University, Lahijan Branch, Lahijan, Guilan, Iran. (e-mail: amgholami@iaiu.lahijan.ac.ir).

controller position of single-link manipulator has been controlled [25]. In [26], a bond graph modeling technique was used to track the trajectory path of flexible space robot. A combination of backstepping method and high-order differential neural network was used to control flexible-joint manipulator [27]. In [28, 29] two BELBIC robust adaptive controller for stable uncertain nonlinear systems and aerospace launch vehicle control were introduced respectively. Also a PID and BELBIC controller designs in path tracking problem was introduced in [30] as an intelligent auto pilot.

In this paper utilizing backstepping method (BM) a controller is designed to stabilize the system states of a single-link flexible mechanical manipulator. Then applying presented BELBIC model introduced in [30], [31] the controller is converted to an intelligent one in order to optimize BM. This proposed model is simulated and obtained results show that in comparison with robust controllers proposed system is faster and more proper.

## II. MODELING AND PROBLEM FORMULATION

The proposed scheme above will be applied to a single-link flexible-joint manipulator. Giving a priori the motor inertia, it identifies the rest parameters of the system without requiring acceleration signals. Details are given in the following. The dynamics of a single-link flexible-joint manipulator, in state-space representation, can be described by [32].

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{mgl}{J_l} \sin x_1 - \frac{k_e}{J_l} (x_1 - x_3) \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= \frac{k_e}{J_r} (x_1 - x_3) - \frac{\mu}{J_r} x_4 + \frac{1}{J_r} u(t)\end{aligned}\quad (1)$$

Where  $x_1$  is the link position,  $x_2$  is the link angular velocity,  $x_3$  is the motor rotor position,  $x_4$  is the motor rotor angular velocity,  $J_l$  is the link inertia,  $J_r$  is the motor rotor inertia,  $k_e$  is the joint elastic constant,  $m$  is the link mass,  $l$  is the link length,  $g$  is the gravity constant,  $\mu$  is the viscosity, and  $u(t)$  is the control input. The system parameters are chosen from [33] that list in table 1.

## III. BACKSTEPPING METHOD

Considering the strictly feedback nonlinear system as (2) where  $x = [x_1, \dots, x_n]^T$ ,  $f_i(0)$  and  $g_i(0)$  are smooth functions with  $f_i(0) = 0$  and  $g_i(0) \neq 0$ .

$$\begin{aligned}\dot{x}_1 &= f_1(x_1, \dots, x_i) + g_1(x_1, \dots, x_i)x_{i+1} \\ \dot{x}_n &= f_n(x) + g_n(x)u \quad ; \quad 1 \leq i \leq n-1\end{aligned}\quad (2)$$

**Step 1.**  $x_2$  is taken as a virtual control input and choose:

$$\dot{x}_2 = \frac{1}{g_1(x_1)}[u_1 - f_1(x_1)]\quad (3)$$

TABLE I  
PARAMETERS OF SYSTEM

Symbols	Values	Units
$mgl$	5	$Nm$
$J_l$	1	$Kg.m^2$
$J_r$	0.3	$Kg.m^2$
$\mu$	0.1	$Kg.m^2/S$
$k_e$	100	$Nm$

The first subsystem is changed to be  $\dot{x}_1 = u_1$ . Choosing  $u_1 = -k_1 x_1$  with  $k_1 > 0$ , the origin of the first subsystem  $x_1 = 0$  is asymptotically stable, and Lyapunov function is  $V_1(x_1) = x_1^2/2$ , (3) is changed to:

**Step 2.**  $x_3$  is taken as a virtual control input and choose:

$$\dot{x}_3 = \frac{1}{g_2(x_1, x_2)}[u_2 - f_2(x_1, x_2)]\quad (5)$$

$$\dot{x}_1 = f_1(x_1) + g_1(x_1)x_2\quad (6)$$

$$\dot{x}_2 = u_2$$

Which is in the form of BM, so  $u_2$  is as follow:

$$u_2 = -\frac{\partial V_1}{\partial x_1} g_1(x_1) - k_2[x_2 - \varphi_1(x_1)] + \frac{\partial \varphi_1}{\partial x_1} [f_1(x_1) + g_1(x_1)x_2]\quad (7)$$

Where  $k_2 > 0$ . This equation asymptotically stabilizes  $(x_1, x_2) = (0, 0)$  and Lyapunov function is as (8).

$$V_2(x_1, x_2) = V_1(x_1) + \frac{1}{2}[x_2 - \varphi_1(x_1)]^2\quad (8)$$

$$\begin{aligned}x_3 = \varphi_2(x_1, x_2) &= \frac{1}{g_2} \left[ -\frac{\partial V_1}{\partial x_1} g_1 - k_2(x_2 - \varphi_1) \right. \\ &\quad \left. + \frac{\partial \varphi_1}{\partial x_1} (f_1 + g_1 x_2) - f_2 \right]\end{aligned}\quad (9)$$

The remaining step can be deduced by analogy. Until step  $n$ , the actual control law  $u = \varphi_n(x)$  shall be determined, which can asymptotically stabilize (2). BM is expanded for class of nonlinear MIMO systems [34].

## IV. CONTROL MANIPULATOR SYSTEM

BM is used to control the states  $x_1, x_2, x_3, x_4$  to origin point  $(0, 0, 0, 0)$ , via the torque  $u$  calculated with four steps.

**Step 1.** The first subsystem of (1) is

$$\dot{x}_1 = x_2\quad (10)$$

Lyapunov function and  $x_2$  are obtained as (11) and (12).

$$V_0(x_1) = \frac{1}{2}x_1^2\quad (11)$$

$$x_2 = \varphi_0(x_1) = -k_1 x_1\quad (12)$$

**Step 2.** Considering  $(x_1, x_2)$  of (1)

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{mgl}{J_l} \sin x_1 - \frac{k_e}{J_l} (x_1 - x_3) \end{aligned} \quad (13)$$

$x_3$  and Lyapunov function are taken as (14) and (15).

$$\begin{aligned} x_3 = \varphi_1(x_1, x_2) &= \frac{J_l}{k_e} \left[ \frac{\partial \varphi_0}{\partial x_1} \dot{x}_1 - \frac{\partial V_0}{\partial x_1} - k_2 (x_2 - \varphi_0) \right. \\ &\quad \left. + \frac{mgl}{J_l} \sin x_1 + \frac{k_e}{J_l} x_1 \right] \end{aligned} \quad (14)$$

$$V_1(x_1, x_2) = V_0(x_1) + \frac{1}{2} (x_2 - \varphi_0)^2 \quad (15)$$

**Step 3.** Considering  $x_1, x_2, x_3$  of the (1)

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{mgl}{J_l} \sin x_1 - \frac{k_e}{J_l} (x_1 - x_3) \end{aligned} \quad (16)$$

$x_4$  and Lyapunov function are taken as (17) and (18).

$$x_4 = \varphi_2(x_1, x_2, x_3) = \frac{\partial \varphi_1}{\partial x_1} \dot{x}_1 + \frac{\partial \varphi_1}{\partial x_2} \dot{x}_2 - \frac{k_e}{J_l} \frac{\partial V_1}{\partial x_2} - k_3 (x_3 - \varphi_1) \quad (17)$$

$$V_2(x_1, x_2, x_3) = V_1(x_1, x_2) + \frac{1}{2} (x_3 - \varphi_1)^2 \quad (18)$$

**Step 4.** Considering all states of (1),  $u$  as an actual control input and Lyapunov function are taken as (19) and (20).

$$\begin{aligned} u = J_r \left[ \frac{\partial \varphi_2}{\partial x_1} \dot{x}_1 + \frac{\partial \varphi_2}{\partial x_2} \dot{x}_2 + \frac{\partial \varphi_2}{\partial x_3} \dot{x}_3 - \frac{\partial V_2}{\partial x_3} - k_4 (x_4 - \varphi_2) \right. \\ \left. - \frac{k_e}{J_r} (x_1 - x_3) + \frac{\mu}{J_r} x_4 \right] \end{aligned} \quad (19)$$

$$V_3(x_1, x_2, x_3, x_4) = V_2(x_1, x_2, x_3) + \frac{1}{2} (x_4 - \varphi_2)^2 \quad (20)$$

#### V. BRAIN EMOTIONAL LEARNING BASED INTELLIGENT CONTROLLER (BELBIC)

In this method base of learning are emotional factors like excitement and anxiety. Here the roots of distress are assumed as stimulants and the control system should react in the way that reduces the system anxiety caused by these stimulants. BELBIC has some input sensors that can be chosen by designer. Each input sensor has two different states that can be described as:

$$\begin{aligned} A_i &= s_i v_i \\ O_i &= s_i w_i \end{aligned} \quad (21)$$

In which  $s_i$  is  $i^{\text{th}}$  input sensor and  $v$  and  $w$  are two states that are depended on input sensor and can be obtained from:

$$\Delta v_i = \alpha s_i \max(0, rew - \sum A_i) \quad (22)$$

$$\Delta w_i = \beta s_i (rew - \sum A_i - \sum O_i - \max(s_i))$$

In which  $\alpha$  and  $\beta$  are teaching coefficients and  $rew$  is reward signal that in this work is chosen as a linear function of system error. Controlling vector  $u$  is:

$$u = \sum A_i - \sum O_i \quad (23)$$

Reward signal is supposed as follow:

$$rew = e_1^2 + e_2^2 + e_3^2 + e_4^2 \quad (24)$$

#### VI. NUMERICAL SIMULATIONS

This section presents numerical simulations flexible link manipulator. The Optimal Backstepping Method (OBM) is used as an approach to control manipulator system and eventually the results of this would be compared with the control result of Robust Control Method (RCM) [35].

Fig. 1, 2, 3 and 4 show that  $x_1, x_2, x_3, x_4$  of Manipulator system can be stabilized with the control law  $u$  that is obtained as (19) to the origin point  $(0,0,0,0)$ .

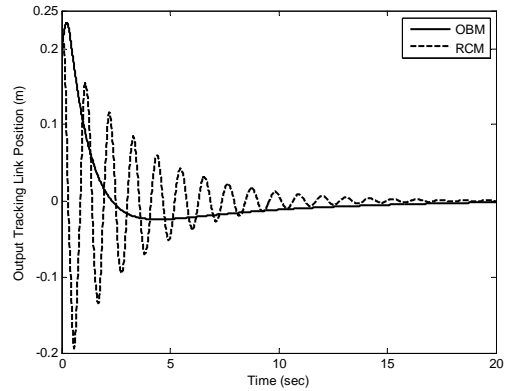


Fig. 1 Link position of manipulator

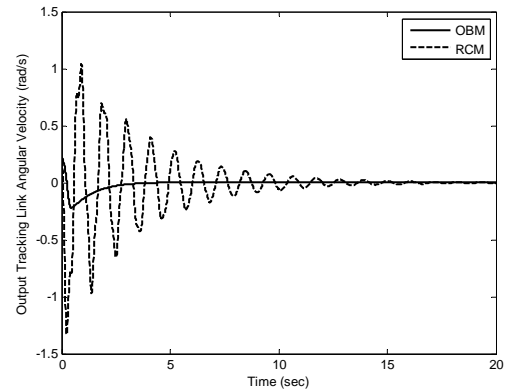


Fig. 2 Link angular velocity of manipulator

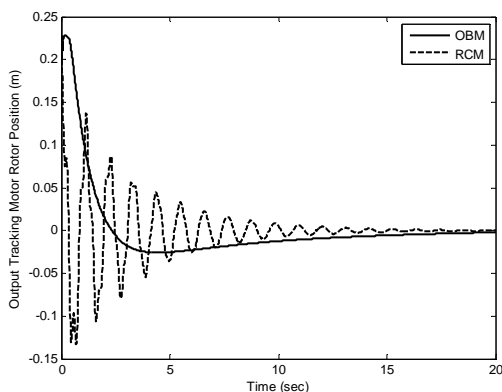


Fig. 3 Motor rotor position of manipulator

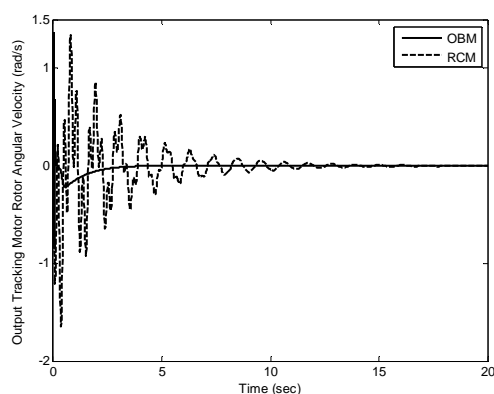


Fig. 4 Motor rotor angular of manipulator

## VII. CONCLUSION

In this paper a single-link flexible manipulator and its stability had been investigated. Then, in order to improve its stability a non-linear backstepping controller had been designed. Whereas, backstepping based controller had positive gain factors, choosing improper values might result instability of the controlled system. Hence, to modify the controller a BELBIC controller was utilized. Comparison between the results of backstepping and BELBIC hybrid controller and conventional robust one proved the efficiency of this work design. Obtained results from simulation illustrated that proposed controller could settle and stabilize mentioned manipulator with higher speed, lower overshoot and shorter settling time.

## REFERENCES

- [1] Vicente Feliu, Emiliano Pereira, Iv'an M. D'iaz, Pedro Roncero, "Feed forward control of multimode single-link flexible manipulators based on an optimal mechanical design", *Robotics and Autonomous Systems* 54 (2006) 651–666.
- [2] M.S. Alam, M.O. Tokhi, "Designing feed forward command shapers with multi-objective genetic optimization for vibration control of a single-link flexible manipulator", *Engineering Applications of Artificial Intelligence* 21 (2008) 229–246. A. Alpher. *Fröbnication. Journal of Foo*, 12(1):234–778, 2002.
- [3] M.S. Alam, M.O. Tokhi, "Hybrid fuzzy logic control with genetic optimization for a single-link flexible manipulator", *Engineering Applications of Artificial Intelligence* 21 (2008) 858–873.
- [4] Kerem Gurses, Bradley J. Buckham, Edward J. Park, "Vibration control of a single-link flexible manipulator using an array of fiber optic curvature sensors and PZT actuators", *Mechatronics* 19 (2009) 167–177.
- [5] Dong Sun, James K. Mills, Jinjun Shan, S.K. Tso, "A PZT actuator control of a single-link flexible manipulator based on linear velocity feedback and actuator placement", *Mechatronics* 14 (2004) 381–401.
- [6] Barun Pratiher, Santosha K. Dwivedy, "Non-linear vibration of a single link viscoelastic Cartesian manipulator", *International Journal of Non-Linear Mechanics* 43 (2008) 683–696.
- [7] Ho-Cheol Shin, Seung-Bok Choi, "Position control of a two-link flexible manipulator featuring piezoelectric actuators and sensors", *Mechatronics* 11 (2001) 707–729.
- [8] Ismael Payo, Vicente Feliu, Osvaldo Daniel Cortazar, "Force control of a very lightweight single-link flexible arm based on coupling torque feedback", *Mechatronics* 19 (2009) 334–347.
- [9] K. Liu and X.sun, "System Identification and Model Reduction For A Single-link Flexible Manipulator", *Journal of Sound and vibration* (2001) 242(5), 867-891.
- [10] S.B. Choi, J.W. Cheon, "Vibration control of a single-link flexible arm subjected to disturbances", *Journal of Sound and Vibration* 271 (2004) 1147–1156.
- [11] V. Feliu, F. Ramos, "Strain gauge based control of single-link flexible very lightweight robots robust to payload changes", *Mechatronics* 15 (2005) 547–571.
- [12] S.S. Ge, T.H.Lee, J.Q.Gong, "A robust distributed controller of a single-link SCARA/Cartesian smart materials robot", *Mechatronics* 9 (1999) 65-93.
- [13] Y. Cao, V.J. Modi, C.W. de Silva, M. Chu, Y. Chen, A.K. Misra, "Trajectory tracking experiments using a novel manipulator", *Acta Astronautica* 52 (2003) 523 – 540.
- [14] Jhih-Wei Huang, Jung-Shan Lin, "Backstepping Control Design of a Single-Link Flexible Robotic Manipulator", *Proceedings of the 17th World Congress, The International Federation of Automatic Control*, Seoul, Korea, July 6-11, 2008.
- [15] Jong H. Oh, Gin S. Lee, "Control of Flexible Joint Robot system by Backstepping Design Approach", *International Conference on Robotics and Automation*, Albuquerque, New Mexico, April 1997.
- [16] S.S. Mokri, H. Husain, W. Martono and A. Shafie, "Real Time Implementation of NARMA-L2 Control of a Single Link Manipulator", *American Journal of Applied Sciences* 5 (12): 1642-1649, 2008.
- [17] Jinjun Shan, Hong-Tao Liu, Dong Sun, "Slewing and vibration control of a single-link flexible manipulator by positive position feedback (PPF)", *Mechatronics* 15 (2005) 487–503.
- [18] H. R. Karimi, M. J. Yazdanpanah, "A new modeling approach to single-link flexible manipulator using singular perturbation method", *Electrical Engineering* (2006) 88: 375–382.
- [19] Woosoon Yim, "Adaptive Control of a Flexible Joint Manipulator", *Proceedings of the 2001 IEEE, International Conference on Robotics & Automation*, Seoul, Korea. May 21-26, 2001.
- [20] An-Chyau Huang and Yuan-Chih Chen, "Adaptive Sliding Control for Single-Link Flexible-Joint Robot with Mismatched Uncertainties", *IEEE TRANSACTIONS ON CONTROL SYSTEMS TECHNOLOGY*, VOL. 12, NO. 5, SEPTEMBER 2004.
- [21] B.A. Md Zain, M. O. Tokhi, and S.F. Toha, "PID-based Control of a Single-Link Flexible Manipulator in Vertical Motion with Genetic Optimisation", *2009 Third UKSim European Symposium on Computer Modeling and Simulation*.
- [22] B.A. Md Zain, M. O. Tokhi and S. Md Salleh, "Dynamic Modelling of a Single-Link Flexible Manipulator Using Parametric Techniques with Genetic Algorithms", *2009 Third UKSim European Symposium on Computer Modeling and Simulation*.
- [23] Emiliano Pereira, Sumeet S Aphalet, Vicente Feliu and S O Reza Moheimani, "A hybrid control strategy for vibration damping and precise tip-positioning of a single-link flexible manipulator", *Proceedings of the 2009 IEEE International Conference on Mechatronics*. Malaga, Spain, April 2009.
- [24] Emiliano Pereira, Sumeet S. Aphale, Vicente Feliu and S. O. Reza Moheimani, "Integral Resonant Control for Vibration Damping and Precise Tip-Positioning of a Single-Link Flexible Manipulator", *IEEE/ASME TRANSACTIONS ON MECHATRONICS*.

- [25] M.-T. Ho and Y.-W. Tu, "Position control of a single-link flexible manipulator using  $H_\infty$ -based PID control", IEE Proc.-Control Theory Appl., Vol. 153, No. 5, September 2006.
- [26] G. Surya Kiran, Amit Kumar, Pushparaj M. Pathak, and N. Sukavanam, "Trajectory Control of Flexible Space Robot", Proceedings of 2008 IEEE International Conference on, Mechatronics and Automation.
- [27] Withit Chatlatanagulchai and Peter H. Meckl, "Backstepping High-Order Differential Neural Network Control of Flexible-Joint Manipulator", 2005 American Control Conference, June 8-10, 2005. Portland, OR, USA.
- [28] Ali Reza Mehrabian, and Caro Lucas, "Emotional Learning based Intelligent Robust Adaptive Controller for Stable Uncertain Nonlinear Systems", INTERNATIONAL JOURNAL OF COMPUTATIONAL INTELLIGENCE VOLUME 2 NUMBER 4 2005 ISSN 1304-4508.
- [29] Ali Reza Mehrabian, Caro Lucas, Jafar Roshanian, "Aerospace launch vehicle control: an intelligent adaptive approach", Aerospace Science and Technology 10 (2006) 149-155.
- [30] Saeed Jafarzadeh, Rooholah Mirheidari, Mohammad Reza Jahed Motlagh, Mojtaba Barkhordari, "Designing PID and BELBIC Controllers in Path Tracking Problem", Int. J. of Computers, Communications & Control, ISSN 1841-9836, E-ISSN 1841-9844, Vol. III (2008), Suppl. issue: Proceedings of ICCCC 2008, pp. 343-348.
- [31] Saeed Jafarzadeh, Rooholah Mirheidari, Mohammad Reza Jahed Motlagh, Mojtaba Barkhordari, "Intelligent Autopilot Control Design for a 2-DOF Helicopter Model", Int. J. of Computers, Communications & Control, ISSN 1841-9836, E-ISSN 1841-9844, Vol. III (2008), Suppl. issue: Proceedings of ICCCC 2008, pp. 337-342.
- [32] Ghorbel, F., J. Hung, and M.W. Spong, "Adaptive control of flexible-joint manipulators". IEEE Control Systems Mag. 9, 1989: p. 9-13.
- [33] Sira-Ramirez, H., S. Ahmad, and M. Zribi, "Dynamical feedback control of robot manipulators with joint flexibility". IEEE Trans. Systems Man Cybernet. 22, 1992: p. 736-747.
- [34] Ali Reza Sahab, Mohammad Haddad Zarif, "Improve backstepping method to GBM". World Applied Sciences Journal 6 (10): 1399-1403, 2009.
- [35] Jeng Tze Huang, "A New Approach to Parametric Identification of a Single-Link Flexible-Joint Manipulator", Journal of Intelligent and Robotic Systems. 37, 3. P.273-284. 2003.



**Ali Reza Sahab** Received the B.S. in control engineering from KNT the university of technology, Tehran, Iran in 2001 and the M.S. and Ph.D. degrees in control engineering from Shahrood university of technology, Shahrood, Iran in 2003 and 2009 respectively.

He is a staff member of Electrical Group, Engineering Faculty of Islamic Azad University, Lahijan Branch. His research interests include nonlinear control and intelligent systems.



**Amir Gholami Pastaki** was born in Rasht, Iran in 1979. He received the B.S. degree in Electronic Engineering from Islamic Azad University, Lahijan branch, Gilan, Iran in 2002 and M.S. degree in Mechatronic Engineering from Islamic Azad University, Qazvin branch, Qazvin, Iran in 2006. He is the staff member of Electronic group of Islamic Azad University,

Lahijan branch, Gilan, Iran. His research interests include robust and nonlinear control.