International Journal of Information, Control and Computer Sciences
ISSN: 2517-9942
Vol:4, No:1, 2010

Optimal Combination for Modal Pushover Analysis by Using Genetic Algorithm

K. Shakeri and M. Mohebbi

***Abstract*—**In order to consider the effects of the higher modes in the pushover analysis, during the recent years several multi-modal pushover procedures have been presented. In these methods the response of the considered modes are combined by the square-root-of-sum-of-squares (SRSS) rule while application of the elastic modal combination rules in the inelastic phases is no longer valid. In this research the feasibility of defining an efficient alternative combination method is investigated. Two steel moment-frame buildings denoted SAC-9 and SAC-20 under ten earthquake records are considered. The nonlinear responses of the structures are estimated by the directed algebraic combination of the weighted responses of the separate modes. The weight of the each mode is defined so that the resulted response of the combination has a minimum error to the nonlinear time history analysis. The genetic algorithm (GA) is used to minimize the error and optimize the weight factors. The obtained optimal factors for each mode in different cases are compared together to find unique appropriate weight factors for each mode in all cases.

***Keywords*—**Genetic Algorithm, Modal Pushover, Optimal weight.

I. Introduction

In recent years, the utilization of the nonlinear static procedure (NSP) for estimating the response of the inelastic structures has been increased and the pushover analysis has played an important role in the development of the performance-based earthquake engineering concepts in the guideline documents and codes (e.g., ATC-40 [1]; FEMA-356 [2]; Erocode-8 [3] and the Japanese structural design code[4]).

The conventional pushover analysis proposed in the guideline documents and codes accurately estimates the seismic demand of the regular and low-rise buildings [5-7] while, this procedure can not appropriately predict the seismic response of the irregular and high-rise buildings [8-11]. The main reason for such unsuccessful performance is that the conventional pushover is developed based on the assumed single fundamental mode shape and it cannot consider the effects of the higher modes.

Therefore in resent years some advanced multi-modal pushover procedures based on the modal decomposition concept have been proposed which can account for the contributions of the higher modes [12-18]. Also in order to consider the effect of the progressive changes in the structural properties during the nonlinear response, some researchers have proposed the adaptive form of the modal procedures (e.g., References [19-25]) where, in each step, the load patterns are updated with respect to the progressive changes in the structural modal properties. Adaptive procedures are more complex than the conventional pushover methods while in the modal procedures with the invariant load pattern the simplicity of the conventional pushover is retained. In fact these methods are an efficient extension of the single-mode conventional pushover to multi modal procedure.

In the well-known modal pushover analysis (MPA) developed by Chopra and Goel [15], the total seismic response of the structure is estimated by combining the responses due to multiple pushover analyses. Each pushover analysis is performed with lateral load pattern corresponding to the inertial force distribution in the considered mode.

K. Shakeri is with the Department of Engineering, University of Mohaghegh Ardabili, Ardabil, Iran. (corresponding author, phone: +98-451-551-2910, fax: +98-451-551-2904, e-mail: Shakeri@ uma.ac.ir).
M. Mohebbi is with the Department of Engineering, University of Mohaghegh Ardabili, Ardabil, Iran. (e-mail: Mohebbi@ uma.ac.ir).

II. Optimal Combination Rule for Modal Pushover Analysis

In the multi modal pushover procedures whether in adaptive or non-adaptive form the parameters resulting from different considered modes are combined by the square-root-of-sum-of-squares (SRSS) or the complete quadratic combination (CQC) rules. While application of these elastic modal combination rules in the inelastic phases is no longer valid and may using an effective alternative modal combination rule could improve the results.

In this regard, here an efficient optimal weight (OW) combination rule for using in MPA procedure has been proposed tentatively. The weighted responses of every considered mode are combined directly by the algebraic sum. The weight of the each mode is defined so that the resulted response from the optimal weight combination has a minimum error to the nonlinear time history (NTH) analysis response. The optimal weights are obtained by using the genetic algorithm (GA) optimization method. These optimal weight factors are obtained for each particular case study. So they will be different for different cases. However may through a statistical study be able to find convergence between the optimal factors of each mode in different cases. The optimal weight factors for two 9-story and 20-story steel moment frames under ten ground motions are obtained and compared with together to propose unique appropriate weight factors for each mode in all cases.

III. Obtaining the Optimal Weight Factors

Since the structural damages are mainly controlled by the story drift, the weight factors are obtained so that the story drifts resulted from the MPA using optimal weight combination have the minimum errors with those resulted from the NTH analysis. All the established steps of the MPA except the SRSS combination step are included in the proposed procedure. The response of the structure are estimated by (1)

$$R = aR_1 + bR_2 + cR_3 + dR_4 \quad (1)$$

Where, R is the final (total) response. R1, R2, R3 and R4 are respectively absolute responses due to the first four considered modes in MPA procedure. a, b, c and d are the optimal weights of the considered modes.

In order to define the amount of the optimal factors, an error vector is defined as:

$$\vec{\Delta}_{error} = |\vec{\Delta}_{NTH}| - \left(a|\vec{\Delta}_1| + b|\vec{\Delta}_2| + c|\vec{\Delta}_3| + d|\vec{\Delta}_4|\right) \quad (2)$$

Where, $|\vec{\Delta}_{NTH}|$ is the vector of peak inter-story drift profile resulted from the NTH analysis and $|\vec{\Delta}_1|, |\vec{\Delta}_2|, |\vec{\Delta}_3|$ and $|\vec{\Delta}_4|$ are the vectors of the absolute inter-story drifts profiles resulted from the pushover analyses according to each considered mode. Whenever the error vector is close to zero vector, the response of the pushover analysis approaches the NTH analysis response.

To minimize the error vector, all its components and sum of them must be minimized, so the objective function is defined as:

$$norm(\vec{\Delta}_{error}) = \sqrt{d_{error1}^2 + d_{error2}^2 + \ldots.. + d_{error\,n}^2} \quad (3)$$

Where, $d_{error\,i}$ is i[th] component of the error vector ($\vec{\Delta}_{error}$). To minimize the objective function and obtain the optimal quantities of a, b, c and d parameters, the genetic algorithm (GA) optimization method is used.

IV. Genetic Algorithms (GAs)

In the traditional optimization the domain is searched using the gradient of the objective function and the limitation of this method arises when the functions of objective function and the constraints of the optimization problem are not continuous and it is not possible to calculate the gradient of the functions. Genetic algorithm (GA) which has been developed by Holland [26] is a computational method. In the application of GA for solving the optimization problems, a design vector can be considered as a chromosome, its design components as the genes, and its value of the objective function as a measure of the fitness. GA starts with a discrete set of design vectors (chromosomes) and changes the current set towards generating a fitter generation of design points, through three genetic algorithm operators including selection, cross over and mutation [27, 28]. In each generation, a set of chromosomes is selected for mating based on their relative fitness. The fitter chromosomes are given more chance of passing their genes into the next generation. This process is operated by selection. In this paper the stochastic universal sampling method [29] has been used for selecting a number of chromosomes for mating, based on their fitness values in the current population. The selected chromosomes are then chosen randomly through cross over to produce offspring. In the present study discrete recombination has been used for cross over operator. In order to maintain the variability of the population, mutation at a specified low rate should be performed in certain chromosomes. The mutation helps GA to provide a guarantee that the probability of searching any given chromosomes will never be zero and helps the GA escape local minima. At the final generation the chromosome which has the best fitness is chosen as the optimum point. Though in the early stages of string coding development, design variables were represented in their binary format but they have some drawbacks in taking continuous problems and it has been shown that for real-valued numerical optimization problems, real- valued coding representations offer certain advantages such as simple programming, less memory required and greater freedom to use different genetic operators over binary versions [30]. Hence in this paper the real-valued coding has been used to represent the chromosomes. Also in this paper the elitist strategy has been used which allows some of the best chromosomes in the current population to go to the next generation without modification.

V. Appling the Proposed Procedure for Different Cases

The proposed algorithm is applied to two nine-story and twenty-story steel frame buildings as medium- and high-rise steel structures where the structural responses are affected by the higher modes effects. Each structure is subjected to different ground motions and the optimal factors of modes are obtained in each case.

VI. Structural Models

A nine-story and a twenty-story perimeter steel moment resistant frame (SMRF) structures denoted SAC-9 and SAC-20 are considered in this study. SAC-9 and SAC-20 are designed for the Phase II of the SAC project as benchmark structures [31]. These structures conform to the requirement of the UBC (1994) provision for the Los Angeles, California region. The two-dimension models of the structures are modeled in the DRAIN-2DX computer program [32]. One of the perimeter SMRFs in the N-S direction with half of the building mass is modeled. The gravity loads are not included in the analysis. For further details about these buildings could refer to Reference [33].

VII. Ground Motions

Ten ground motions are used in this study, as listed in Table I. These ground motions are same as those used in FEMA-440

[11] to investigate the multi-degree-of-freedom effects in pushover analysis. They were recorded at site class C, as defined by the NEHRP Provisions (BSSC, 2000) and originated from the earthquake having magnitudes (Ms) between 6.6 and 7.6. Unlike FEMA-440 in this study all records are taken from the Pacific Earthquake Engineering Research (PEER) site, *(http://peer.berkeley.edu/smcat).* Since the forth record in Table I (E4*)* is not available in the PEER site, it is not included in this study and only the other ten records are considered. The considered records are iteratively scaled so that the peak displacement of the roof to be equal to 4% of the building height for the frames. The scale factors of the records for each frame are also presented in Table I.

VIII. Investigation on the Obtained Optimal Factors

Each structure is subjected to the considered ground motions and the optimal weight factors of modes are obtained in each case. To determine the participation of each mode in the optimal weight (OW) combination for modal pushover analysis, the GA procedure is employed. In the SAC-9 model only three first modes are considered while in the SAC-20 model four first modes are considered. The factor of each mode (a, b, c and d) are chosen as variable parameters while their upper and lower bound values are [-10, 10]. The parameters of GA for using in this study are taken as follows: population size = 50, number of generation = 500 and mutation rate = 0.05. After performing the GA procedure the optimal values of the factors are found and shown in Table II for each building under different records.

TABLE I
Ground motion properties

No.	Earthquake	Date	Ms	Station Location	Component	PGA (g)	PGV (cm/s)	Scale for SAC9	Scale for SAC20
E1	Superstitn	11/24/1987	6.6	El Centro Imp. Co. Cent (01335)	0	0.358	46.4	6.2	7.32
E2	Northridge	1/17/1994	6.7	Canyon Country - W Lost Cany (90057)	0	0.41	43	4.7	20.65
E3	Loma Prieta	10/18/1989	7.1	Gilroy Array #2 (47380)	90	0.322	39.1	8.89	21.57
E4	Chi Chi	9/20/1999	7.6	(TCU122)	N	0.261	34	#	#
E5	Loma Prieta	10/18/1989	7.1	Gilroy Array #3 (47381)	90	0.367	44.7	6.35	18.44
E6	Northridge	1/17/1994	6.7	Canoga Park - Topanga Can (90053)	196	0.42	60.8	6.1	12.25
E7	Chi Chi	9/20/1999	7.6	(CHY101)	W	0.353	70.6	2.24	3.58
E8	Superstitn	11/24/1987	6.6	El Centro Imp. Co. Cent (01335)	90	0.258	40.9	6.85	12.07
E9	Northridge	1/17/1994	6.7	Canoga Park - Topanga Can (90053)	106	0.356	32.1	10.7	24.3
E10	Imperial Valley	10/15/1979	6.9	El Centro Array #2 (5115)	140	0.315	31.5	8.35	15.05
E11	Imperial Valley	10/15/1979	6.9	El Centro Array #11 (5058)	230	0.38	42.1	6.77	8.62

TABLE II
Optimal factors and error indices of different combination methods for different records.

Earthquake	SAC 9			Error			SAC 20				Error		
	a	b	c	OW	Mode1+3	MPA	a	b	c	d	OW	Mode1+3	MPA
E1	1.0196	-0.44753	1.2555	0.0385671	0.1093544	0.188868	1.0708	0.19705	-0.11597	0.39321	0.1284429	0.1750054	0.2786633
E2	0.91331	-0.00614	1.3846	0.0399578	0.0501507	0.0495622	1.1779	0.23919	-0.69169	0.3817	0.5788125	0.7658389	1.4251248
E3	1.196	-0.13889	0.90666	0.0262056	0.11463	0.3157394	1.2401	0.39253	-0.38098	0.16796	0.7307616	1.1637642	1.3847598
E5	1.0465	-0.22825	1.5269	0.0346528	0.0613771	0.1517144	1.1326	0.36837	-0.52328	0.79481	0.5878639	0.9303437	1.2255308
E6	0.93798	-0.1489	1.3613	0.0723429	0.083802	0.1184834	1.0993	0.044465	0.22292	-0.076179	0.1913083	0.2616303	1.253979
E7	0.88387	0.16937	-0.35273	0.0262141	0.1430039	0.105101	1.0679	0.23506	0.81373	-0.60166	0.1080991	0.1274015	0.1499573
E8	0.82211	-0.36102	1.2239	0.035564	0.1472927	0.1022744	0.67763	0.19542	0.10912	-0.21505	0.1508435	0.5474549	0.6531868
E9	1.1807	0.12739	0.76623	0.1590645	0.1915944	0.2611635	1.0577	0.33628	-0.6678	0.57601	0.5243727	0.7807868	1.1009209
E10	0.99463	-0.27595	0.95995	0.0226463	0.0631295	0.077531	0.87994	0.21968	0.06927	0.06175	0.0983073	0.2094565	0.4216832
E11	1.2087	0.21474	0.49207	0.0653009	0.1084208	0.1427034	1.0629	0.47089	0.011385	-0.66773	0.2699116	0.2993564	0.3179182

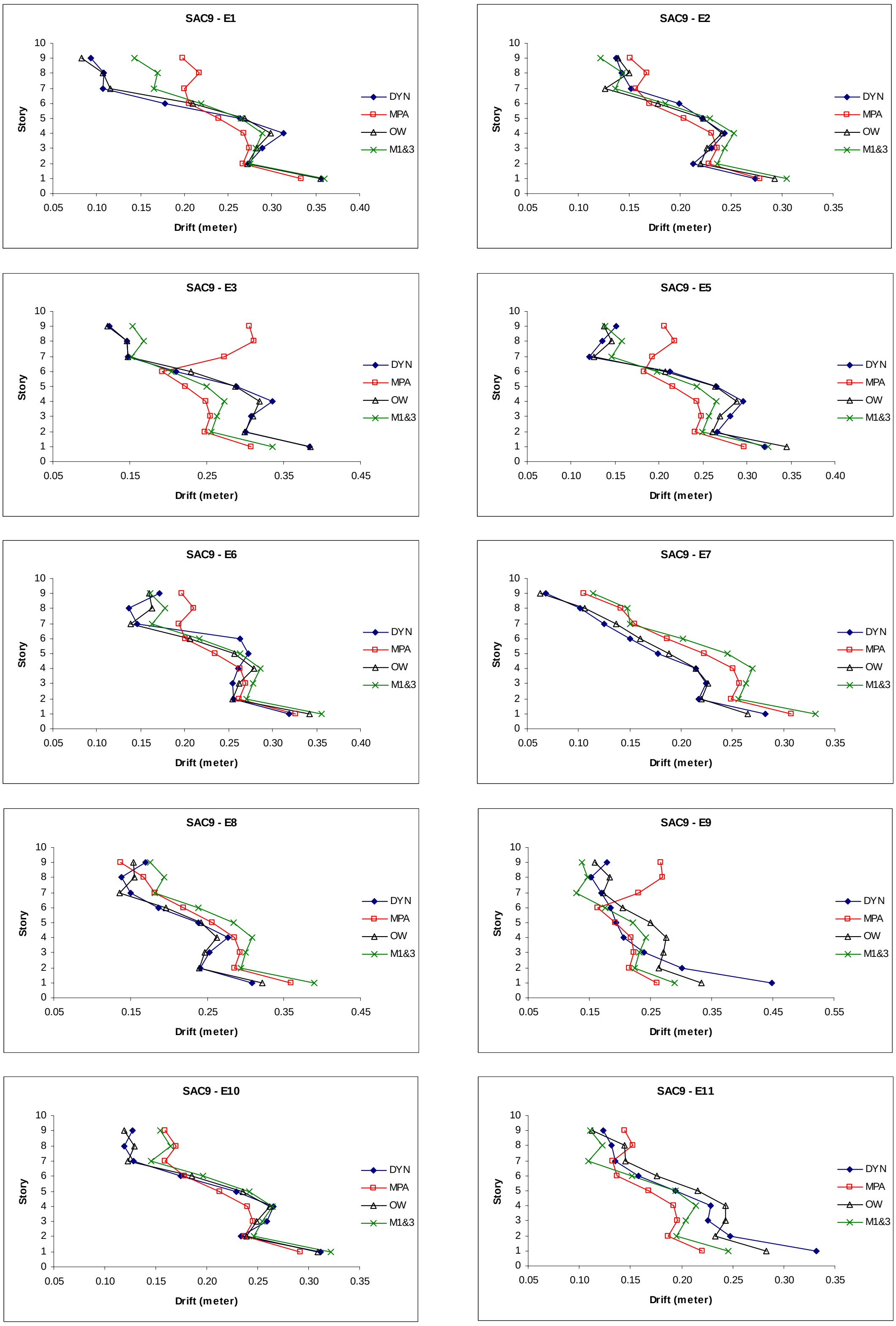

Fig. 1 Peak inter-story drift profiles resulting from the different combination rules used in the multi modal pushover and the NTH analysis for the SAC-9 building under the considered ground motions.

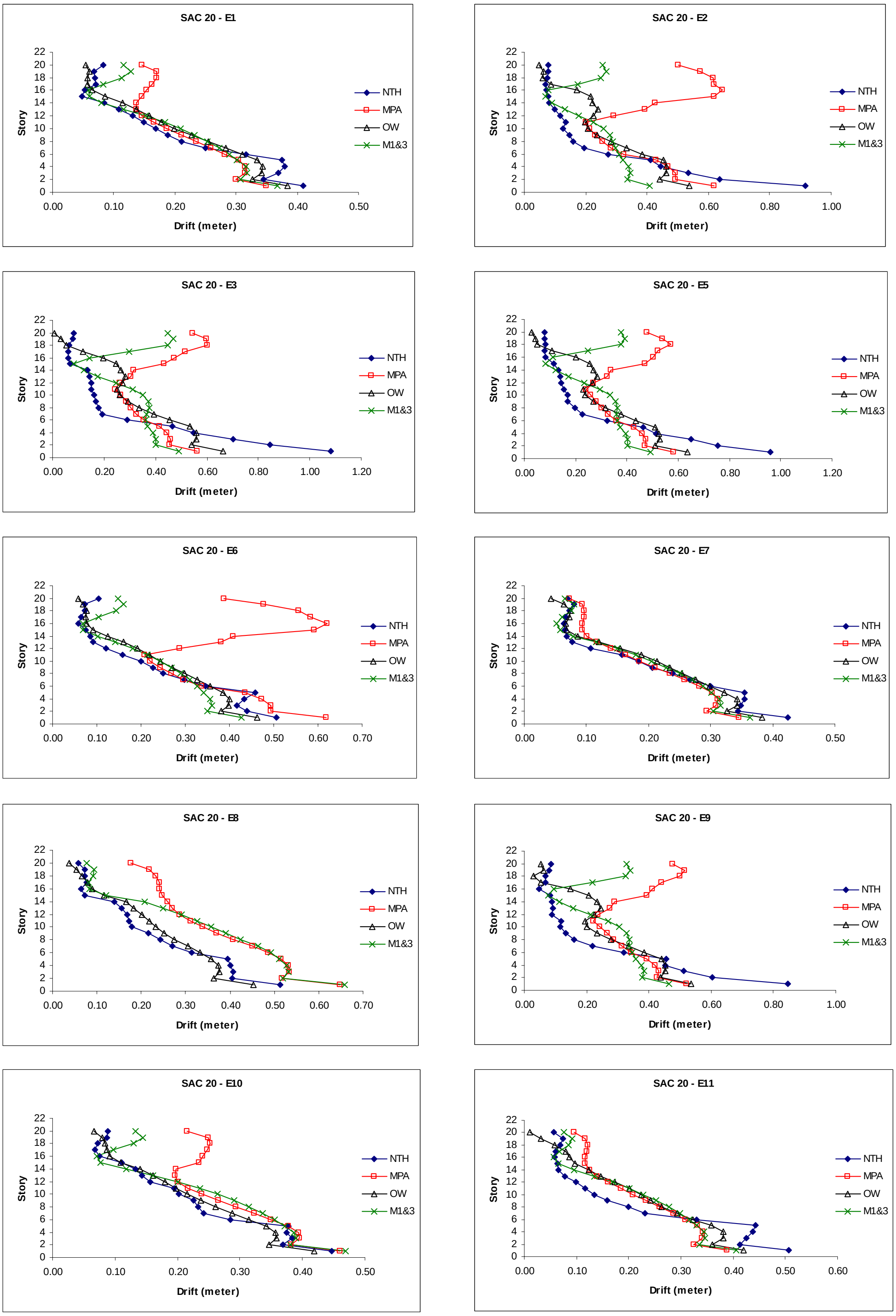

Fig. 2 Peak inter-story drift profiles resulting from the different combination rules used in the multi modal pushover and the NTH analysis for the SAC-20 building under the considered ground motions.

The error of the OW combination rule and other combination rules with respect to the NTH analysis are computed by the error index presented by Lopez-Menjivar and Pinho [34]

$$Error_{\Delta}(\%) = 100 \times \frac{1}{n}\sqrt{\sum_{i=1}^{n}\left(\frac{\Delta_{i-NTHA} - \Delta_{i-Push}}{\Delta_{i-NTHA}}\right)^2} \quad (4)$$

Where, Δ_{i-NTHA} is the peak inter-story drift at a given level i, resulting from the NTH analysis, Δ_{i-Push} is the corresponding inter-story drift of the pushover analysis and n is the number of the stories.

Whenever the error index is close to zero, the responses resulted from the pushover analysis approach the NTH analysis responses. The error index of the different combination rules for both buildings under different record are presented in Table II.

As shown in Table II the error of the OW procedure in all cases is less than the error of the MPA procedure. It means that in any cases the optimal factors could be found so that the OW combination rule has the minimum error with respect to NTH analysis.

The peak inter-story drift profiles resulting from the different combination rules used in the multi modal pushover and the NTH analysis for the SAC-9 and SAC-20 buildings under the considered ground motions are shown resectively in fig. 1 and fig. 2.

If the error of the OW procedure be equal to zero, it means that the response of the NTH analysis exactly can be expressed by the direct algebraic combination of the factored modal responses. So the modal response vector can be interpreted as the ordered basis of the vector space of the NTH analysis response. As shown in the Table II the error of the OW procedure for SAC-9 model in most cases is close to zero and the modal response vectors could be interpreted as the ordered basis vectors. In the SAC-20 building the first four modes are considered however, the error of the OW procedure and other procedures are more than the corresponding errors in SAC-9 model.

Even though the errors of the OW combination rule in all cases are less than the other procedures, the optimal factors for each building under different record change and the OW combination rule could not be considered as a general rule. However as presented in Table II the optimal factors of the first and third modes in the SAC-9 building subjected to different records are around the one. The mean of the optimal factors for each mode of the SAC-9 model under the different records are 1.02, -0.11 and 0.95 respectively. Therefore the direct combination of the first and third modes (Mode1+3) could be presented as an efficient global combination rule. The efficiency of the Mode1+3 combination rule is investigated through the SAC-9 and SAC-20 models under the considered ground motion records. As presented in Table II the errors of the Mode1+3 rule in all cases are less than the errors of the MPA procedure except the SAC-9 model under the E7 and E8 records. It is to be noted that as shown in fig. 1 the resulted responses from the Mode1+3 rule in these two mentioned records are overestimated.

IX. Conclusion

An efficient optimal weight (OW) combination rule for using in modal pushover analysis has been proposed. The optimal weight of the each mode is defined so that the resulted response of the proposed combination rule has a minimum error to the nonlinear time history analysis. For each considered case study model (SAC-9 and SAC-20) under the different ground motions the optimal weight of the considered modes are defined by using the genetic algorithm.

The responses resulted from the direct algebraic combination of modes factored by optimal weight are very close to the nonlinear time history (NTH) analysis responses. Therefore the modal response vector can be interpreted as the ordered basis of the vector space of the NTH analysis response. However the OW combination rule could not be considered as a general rule. Because the value of the optimal weights depend on the case study and these optimal factors are obtained for a particular case.

By investigating the obtained optimal weight of the considered modes in the different cases an efficient global combination rule denoted Mode1+3 is proposed. In this combination rule the absolute response of the first and third modes are added together directly. The accuracy of the Mode1+3 combination rule is evaluated by applying to two nine and twenty story steel buildings subjected to different ground motions. The error of the Mode1+3 combination rule in most cases is less than the error of the square-root-of-sum-of-squares (SRSS) combination used in the modal pushover analysis.

References

[1] *Seismic evaluation and retrofit of concrete buildings*, ATC, ATC-40. Redwood City, CA: Applied Technology Council, 1996.

[2] *Prestandard and commentary for the seismic rehabilitation of buildings,* FEMA, FEMA 356. Washington, DC: Federal Emergency Management Agency, 2000.

[3] *Eurocode-8: Design of structures for earthquake resistance, Part 1*, Pr-EN 1998-1, Final Draft, Brussels (BEL): European Committee for Standardization; 2003.

[4] S. Otani, H. Hiraishi, M. Midorikawa and M. Teshigawara, "New seismic design provisions in Japan," In: *Proc. Uzumeri symposium, ACI Annual Convention.* Canada, 2000.

[5] M. Saiidi and M.A. Sozen, "Simple non-linear seismic analysis of RC structures," *ASCE J Struct Div* 1981;107(ST5):937–951.

[6] P. Fajfar and P. Gašperšič, "The N2 method for the seismic damage analysis of RC buildings," *Earthquake Eng & Struct Dyn* 1996;25(1):31-46.

[7] A. Gupta and H. Krawinkler, "Estimation of seismic drift demands for frame structures," *Earthquake Eng & Struc Dyn* 2000;29(9):1287-1305.

[8] H. Krawinkler and G.D.P.K. Seneviratna, "Pros and cons of a pushover analysis of seismic performance evaluation," *Eng Struct* 1998;20(4-6):452-464.

[9] S. Kim and E. D'Amore, "Push-over analysis procedures in earthquake engineering," *Earthq Spectra* 1999;15(3):417-434.

[10] A.M. Mwafy and A.S. Elnashai, "Static pushover versus dynamic collapse analysis of RC buildings," *Eng Struct* 2001;23:407-424.

[11] *Improvement of nonlinear static seismic analysis procedures*, FEMA, FEMA 440. Washington, DC: Federal Emergency Management Agency, 2005.
[12] T.F. Paret, K.K. Sasaki, D.H. Elibeck and S.A. Freeman, "Approximate inelastic procedures to identify failure mechanism from higher mode effects," in Proc. *the eleventh world conference on earthquake engineering.* 1996.
[13] K.K. Sasaki, S.A. Freeman and T.F. Paret, "Multi-mode pushover procedure (MMP) – a method to identify the effects of higher modes in a pushover analysis," in Proc. *sixth US. national conference on earthquake engineering*. 1998.
[14] A.S. Moghadam, "A pushover procedure for tall buildings," in Proc. *the twelfth European conference on earthquake engineering*. 2002.
[15] A.K. Chopra and R.K. Goel, "A modal pushover analysis procedure for estimating seismic demands for buildings," *Earthquake Eng & Struct Dyn* 2002;31:561-582.
[16] K. Shakeri, M.A. Shayanfar and A.S. Moghadam, "An efficient method for optimum combination of modes required for pushover analysis," in Proc. *the 9th Canadian conference on earthquake engineering*. 2007.
[17] E. Hernandez-Montes, O-S. Kwon and M.A. Aschheim, "An energy-based formulation for first-and multiple-mode nonlinear static (Pushover) analyses," *J Earthquake Eng* 2004;8:69-88.
[18] A.K. Chopra, R.K. Goel and C. Chinatanapakdee, "Evaluation of a modified MPA procedure assuming higher modes as elastic to estimate seismic demands," *Earthq Spectra* 2004;20(3):757-778.
[19] B. Gupta and S.K. Kunnath, "Adaptive spectra-based pushover procedure for seismic evaluation of structures," *Earthq Spectra* 2000;16(2):367-391.
[20] M.N. Aydinoğlu, "An incremental response spectrum analysis procedure based on inelastic spectral displacements for multi-mode seismic performance evaluation," *Bull Earthquake Eng* 2003;1:3-36.
[21] E. Kalkan and S.K. Kunnath, "Adaptive modal combination procedure for nonlinear static analysis of building structures," *J Struct Eng ASCE* 2006;132(11):1721-1731.
[22] A.S. Elnashai, "Advanced inelastic static (pushover) analysis for earthquake applications," *Struct Eng Mech* 2001;12(1):51-69.
[23] S. Antoniou and R. Pinho, "Advantages and limitations of adaptive and non-adaptive force-based pushover procedures," *J Earthquake Eng* 2004;8(4):497-522.
[24] S. Antoniou, R. Pinho, "Development and verification of a displacement-based adaptive pushover procedure," *J Earthquake Eng* 2004;8(5):643-661.
[25] K. Shakeri, M.A. Shayanfar and T. Kabeyasawa, "A story shear-based adaptive pushover procedure for estimating seismic demands of buildings," *Eng. Struct.* 2010:32(1) 174-183.
[26] J.H. Holland, "Adaptation in natural and artificial systems," Ann Arbor: The University of Michigan Press. 1975.
[27] D.E. Goldberg, "Genetic algorithms in search, optimization and machine Learning. Addison -Wesley Publishing Co., Inc. Reading, Mass. 1989.
[28] Z. Michalewicz, "Genetic algorithms + data structures=evolution programs," New York: Springer-Verlag. 1996.
[29] J.E. Baker, "Adaptive selection methods for genetic algorithms," in Proc. *ICGA*, 1985, 1: 101-111.
[30] W.M. Jenkins, "A decimal-coded evolutionary algorithm for constrained optimization," *Comp. and Struct*., 2002, 80:471-480.
[31] *State of the art report on systems performance of steel moment frames subject to earthquake ground shaking*, FEMA 355C. Sacramento, CA: SAC Joint Venture; 2000.
[32] V. Prakash, G.H. Powell and S. Campbell, "DRAIN-2DX base program description and user guide," Version 1.10, Report No. UCB/SEMM-93/17. Berkeley, CA: Department of Civil Engineering, University of California at Berkeley; 1993.
[33] A. Gupta and H. Krawinkler, "Seismic demands for performance evaluation of steel moment resisting frame structures (SAC Task 5.4.3)," Report no. 132. Palo Alto, CA: John A. Blume Earthquake Engineering Center, Stanford University; 1999.
[34] M.A. Lopez-Menjivar and R. Pinho, "A review of existing pushover methods for 2-D reinforced concrete buildings," Pavia (Italy): Rose School; 2004.