Optical Switching Based On Bragg Solitons in A Nonuniform Fiber Bragg Grating
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Abstract—In this paper, we consider the nonlinear pulse propagation through a nonuniform birefringent fiber Bragg grating (FBG) whose index modulation depth varies along the propagation direction. Here, the pulse propagation is governed by the nonlinear birefringent coupled mode (NLBCM) equations. To form the Bragg soliton outside the photonic band gap (PBG), the NLBCM equations are reduced to the well known NLS type equation by multiple scale analysis. As we consider the pulse propagation in a nonuniform FBG, the pulse propagation outside the PBG is governed by inhomogeneous NLS (INLS) rather than NLS. We then discuss the formation of soliton in the FBG known as Bragg soliton whose central frequency lies outside but close to the PBG of the grating structure. Further, we discuss Bragg soliton compression due to a delicate balance between the SPM and the varying grating induced dispersion. In addition, Bragg soliton collision, Bragg soliton switching and possible logic gates have also been discussed.

Keywords—Bragg grating, Nonuniform fiber, Nonlinear pulse.

I. INTRODUCTION

As in conventional optical fibers, the fiber birefringence plays an indispensable role and affects the nonlinear phenomena considerably in FBG also. In FBG, birefringence induces bifurcation or a split of the peak at the Bragg reflection wavelength [1]. Therefore, in general, the birefringence effect requires additional investigations. The birefringence effect should be included if Bragg gratings are made inside the core of polarization-maintaining fibers. From a physical standpoint, the two orthogonally polarized components have slightly different mode indices. Since the Bragg wavelength depends on the mode index, the stop bands of the two modes have the same widths but are shifted by a small amount with respect to each other. As a result, despite both the polarization components having the same wavelength (frequency), one of them may fall within the stop band while the other remains outside the PBG. Further, as the two stop bands shift due to nonlinear index changes, the shift can be different for the two orthogonally polarized components because of the combination of the XPM and birefringence effects. More recently [2]-[4] have extensively investigated the birefringence effect in Bragg grating structure.

Recently, with the help of perturbation theory, [5] investigated the role of inhomogeneity in the chirped Bragg gratings. That is, they demonstrated that the inhomogeneity affects the amplitude and the width, as well as phase and the velocity of the soliton. Further, by using the Inverse Scattering transform (IST) they analyzed the dynamics of a two soliton pulse of the perturbed NLS equation which governs pulse propagation outside but close to PBG. Finally, they also discussed the dynamics of a multisoliton pulse in the chirped grating. However, in this sub-section, by considering the effect of birefringence, we analytically show that the inhomogeneity present in the chirped grating affects the amplitude as well as width of the pulse. Hence, this process leads to the compression of Bragg (two) soliton pulse outside the PBG structure. Under the elastic collision condition, we discuss the scenario of these Bragg solitons’ interaction with and without (inhomogeneity) compression. On the other hand, under the inelastic collision condition, we analyze the occurrence of Bragg soliton switching in the chirped Bragg grating.

II. THEORETICAL MODEL

Note that the coupled mode theory can easily be extended to account for fiber birefringence in FBG. Therefore, the nonlinear pulse propagation in FBG in the presence of birefringence is described by the nonlinear birefringent coupled mode (NLBCM) equations [6].

\[
\frac{\partial E_{1f}}{\partial z} + \frac{\partial E_{2f}}{\partial \tau} + \alpha k_{\xi} E_{1f} + \gamma \beta E_{1f} E_{2f} E_{2f} + \left| E_{1f} \right|^2 E_{1f} = 0
\]

\[
\frac{\partial E_{1b}}{\partial z} + \frac{\partial E_{2b}}{\partial \tau} + \alpha k_{\xi} E_{1b} + \gamma \beta E_{1b} E_{2b} E_{2b} + \left| E_{1b} \right|^2 E_{1b} = 0
\]

Here the field envelopes \( E_{1f}, E_{1b}, E_{2f} \) and \( E_{2b} \) represent the forward and backward propagating fields in FBG in the presence of birefringence. The physical parameter namely nonlinear coefficient \( \alpha _i \) is defined as

\[
\alpha _i = \frac{n_i k_i}{A_{ni}}
\]
where \((j = 1, 2)\); \(n_2\) is the nonlinear index of refraction; \(k_0\) is the Bragg wavevector; \(A_{ef}\) is the effective cross-sectional area. The other two coefficients can be determined from \(c_0\) since for a weakly birefringent optical fiber, they are in the ratio
\[
\left\{a_j : b_j : c_j \right\} = \left\{3 : 2 : 1 \right\} \tag{3}
\]

The birefringence coefficient is given by,
\[
\delta = 2ck_0 \left( \frac{1}{n_{01}} - \frac{1}{n_{02}} \right) \tag{4}
\]

The grating coefficient is defined as
\[
\kappa_{i,2} = \frac{\delta n_i \pi}{2n_{0i} a} \tag{5}
\]

It should be noted that the inclusion of birefringence in (1) results in a dispersion relation that is different for the 1 and 2 polarization components. The corresponding dispersion curves are shown in Fig. 1 where the stop bands 1 and 2 are centered about their respective Bragg frequencies as,
\[
\omega_{\theta i,j} = \frac{c}{n_{0i,j}} \frac{\pi}{d} \tag{6}
\]

which cause them to be separated. This separation is important for the dynamics of the optical AND gate to be discussed in the following section.

As we consider the pulse propagation outside PBG, we now turn our attention to the nonlinear Schrödinger equations for a birefringent medium. In the absence of birefringence, it has already been shown that, for pulse widths not too narrow and center frequencies not too deep in the gap, the NLCM equations scale into a NLS type equation [7], [8], [10]. The resulting NLS equation is known to describe the dynamics of fields at carrier frequencies far from the band gap [7]. The same arguments hold in the presence of birefringence except that the resulting nonlinear birefringent Schrödinger (NLBS) equation involves two coupled equations

\[

\begin{align*}
\left( \frac{\partial}{\partial t} + a_j \frac{\partial}{\partial z} \right) E_j + \frac{1}{2} \omega_j \frac{\partial^2 E_j}{\partial z^2} + \left\{ \alpha_{\text{sym}} E_j \right\}^n + \alpha_{\text{sym}} \alpha_j E_j E_j E_j e^{2i\omega t} &= 0 \\
\left( \frac{\partial}{\partial t} + b_j \frac{\partial}{\partial z} \right) E_j + \frac{1}{2} \omega_j \frac{\partial^2 E_j}{\partial z^2} + \left\{ \alpha_{\text{sym}} E_j \right\}^n + \alpha_{\text{sym}} \alpha_j E_j E_j E_j e^{2i\omega t} &= 0
\end{align*}
\]

Fig. 1 Dispersion relation for a birefringent medium with \(n_c < n_e\). The \(Y\) band is shifted up relative to the \(X\) band so that an input frequency as shown will be more deeply in the \(Y\) gap than in the \(X\) gap.

The field variables \(E_1^i\) and \(E_2^j\) describe the amplitudes of envelope functions that modulate carrier Bloch functions at a specified \(k\), belonging to either the upper or lower branch. The birefringence parameter is defined as, \(\Delta \beta = \beta_{01} - \beta_{02}\) which is related to the beat length \(L_B\) as \(\Delta \beta = 2\pi / L_B\). The nonlinear coefficients close to the bandgap, in the presence of a weak birefringence are given by
\[
\alpha_{\text{sym}} = \frac{n_2}{A_{ef}} \frac{k_0}{n_{01}^2} \frac{3 - \rho_j^2}{2 \rho_j^2} \tag{7}
\]

where the velocity coefficients \(\rho_j = d\omega_j / dk\); the velocity fraction \(\rho_2 = \rho_1 / \rho_j\); and the other quantities are defined as in (2). It is worthy to note that (6) is valid only when the grating induced dispersion is significantly larger than the underlying neglected material dispersion.

The nonlinear effects in a set of (6) can be classified into phase modulation terms and phase conjugation or energy exchange terms. Phase modulation terms involve one component of the electric field seeing an enhanced index of refraction due to the intensity of the other components of the field. The \(\alpha_{\text{sym}}\) term in the NBSE describes an enhanced index for the field \(1^i\) due to its own intensity (SPM). The \(\alpha_{\text{sym}}\) term enhances the index for the field \(2^j\) due to the intensity of the field \(2^j\) (XPM). However, there are two types of XPM in the NLBCM Equation (1). The first is cross phase modulation between different directions of the same polarization \(i.e., E_{11f}\) being affected by the intensity of \(E_{21j}\). The second is cross phase modulation between different polarizations \(i.e., E_{11f}\) being affected by the intensity of \(E_{22f}\). The energy exchange terms are those that couple into the field conjugates, including the \(\alpha_{\text{ex}}\) terms in the NBS equation and the corresponding terms in the NLBCM equation. If the birefringence is high, then the quickly varying \(\exp(\pm i \delta t)\)
term will cause the effect of these energy exchange terms to vanish.

For the sake of convenience, we now rewrite (6) in terms of a circular basis by letting $Q_1 = (E_1 + iE_2)/\sqrt{2}$ and $Q_2 = (E_1 - iE_2)/\sqrt{2}$. The resulting transformed equations are

\[
\begin{align*}
\frac{i}{\gamma} \frac{\partial Q_1}{\partial z} + \frac{1}{2} \beta_1 \frac{\partial^2 Q_1}{\partial t^2} + i\Gamma \left|Q_1\right|^2 + \frac{1}{2} \left|Q_2\right|^2 = 0 \\
\frac{i}{\gamma} \frac{\partial Q_2}{\partial z} + \frac{1}{2} \beta_1 \frac{\partial^2 Q_2}{\partial t^2} + i\Gamma \left|Q_2\right|^2 + \frac{1}{2} \left|Q_1\right|^2 = 0
\end{align*}
\]

(8)

where the physical parameters $Q_1$ and $Q_2$ refer to right and left circularly polarized fields respectively and $\Gamma = \alpha / 3$. The nonlinear coefficient $\alpha$ is related to $\alpha_{\text{swm}}$ in (7) via $\alpha = \alpha_{\text{swm}}/\omega^2$. Now the energy exchange terms have disappeared, and hence the only nonlinear effects are self and cross phase modulations. These terms can be considered an induced birefringence because, if $\left|Q_1\right| \neq \left|Q_2\right|$, then the nonlinear terms in the two equations are unequal. Therefore one of the polarizations will experience a different nonlinear refractive index. In the case of linearly polarized light this induced birefringence is trivial because one can always rotate to an appropriate basis where the two equations (8) reduce to a single NLS equation. However, for general elliptical polarization, the situation is more complicated.

It is well known fact that each point in the pulse has its own polarization state which can be specified either by using the Stokes polarization parameters or Jones calculus. As the field components $Q_1$ and $Q_2$ have different intensity profiles, the polarization state varies across the pulse. Recently, with the help of Stokes polarization parameters, we have analyzed [9] the polarization evolution of a soliton pulse in a birefringent fiber under the influence of quintic self and cross phase modulation effects. Therefore, it is also possible to study the polarization dynamics of Bragg grating solitons in the transmission regime of FBG in the presence of birefringence effect. Recently, [11], [12] analyzed the polarization dynamics of Bragg grating solitons by NLS type equation in birefringent Bragg grating structure.

Note that (6) has been extensively investigated by [13] and proposed an all-optical switching and an AND gate operation scheme based on the transmission of coupled gap solitons, with the help of NLBS equation, for a birefringent nonlinear Bragg grating structure. Recently, [6] have investigated switching behavior in terms of gap solitons in nonlinear Bragg grating structure in the presence of birefringence effect.

At this juncture, it is worth to note that the physical parameters, in (8), such as quadratic dispersion, $\beta_2$, and nonlinear coefficient, $\Gamma$, are no longer constants but depend upon the propagation direction as we consider the soliton pulse propagation through a nonuniform Bragg grating structure. Therefore, we reduce the NLBS Equation (8) with ‘$z$’ dependent parameters of $\beta_2$, and $\Gamma$ using the following transformation

\[
\xi = \frac{1}{T^2(0)} \int_{0}^{L} \frac{1}{\Gamma(\xi)} d\zeta', \quad q_j = T(0) \sqrt{\beta_j(\xi)} Q_j
\]

After the transformation, NLBS equation becomes

\[
\begin{align*}
q_{ji} + q_{mi} + 2 \left( \sum_{j=1}^{2} \left| q_j \right|^2 \right) q_j + ig(q_j) q_j = 0, \quad j = 1, 2
\end{align*}
\]

(9)

where $g(\xi) = - \frac{\Gamma}{2 \beta_2^2} \frac{\partial \Gamma}{\partial \xi} - \frac{1}{2 \beta_2} \frac{\partial \beta_2}{\partial \xi}$ represents an effective gain. As we consider the nonuniform FBG where the dispersion alone varies along the propagation direction of the grating structure, the gain term $g(\xi)$ reduces to

\[
g(\xi) = - \frac{1}{2 \beta_2} \frac{\partial \beta_2}{\partial \xi}.
\]

The resulting (9) describes the nonlinear pulse propagation outside the PBG in nonuniform birefringent FBG.

III. PULSE COMPRESSION BY BRAGG TWO SOLITON

In this sub-section, by employing the birefringence effects, we discuss the Bragg two soliton pulse compression and all optical switching (outside the PBG) in the nonuniform FBG. The two soliton solution of (9) cannot be derived directly. Therefore, by introducing the following variable transformation

\[
q_j(\xi, t) = a Q_j(\xi, \tau) \text{exp}(b), \quad j = 1, 2,
\]

where, $a = \frac{\xi}{(\xi + \xi_0)}$, $b = \frac{\text{i}t^2}{4(\xi + \xi_0)}$, $\xi = \frac{\xi}{(1 + \xi / \xi_0)}$ and $\tau = \frac{t}{(1 + \xi / \xi_0)}$

the inhomogeneous NLBS Equation (9) can be transformed into the well known NLBS Equation (8). The bright two-soliton solutions of the above Manakov system (8) are well known [14].

\[
\begin{align*}
Q_1 &= \frac{\alpha_{11}^{(i)} e^{\eta_1} + \alpha_{12}^{(i)} e^{\eta_2} + e^{\eta_1 + \eta_2 + 2\delta_1} + e^{\eta_1 + \eta_2 + 2\delta_2}}{D} \\
Q_2 &= \frac{\alpha_{21}^{(i)} e^{\eta_1} + \alpha_{22}^{(i)} e^{\eta_2} + e^{\eta_1 + \eta_2 + 2\delta_1} + e^{\eta_1 + \eta_2 + 2\delta_2}}{D}
\end{align*}
\]

where, $\eta_j = k_j (t + ik_j z)$, $e^{\delta_1} = \frac{\kappa_{12}}{k_j + k_j}$, $e^{\delta_2} = \frac{\kappa_{22}}{k_j + k_j}$

\[
\begin{align*}
\end{align*}
\]

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\[ e^{\delta_1} = \frac{k_1 - k_2}{(k_1 + k_1^*)(k_1 + k_2)}(\alpha_1^{(2)}k_{12} - \alpha_2^{(2)}k_{11}) \]
\[ e^{\delta_2} = \frac{k_2 - k_1}{(k_2 + k_2^*)(k_1 + k_2)}(\alpha_2^{(2)}k_{22} - \alpha_1^{(2)}k_{21}) \]
\[ e^{\delta_3} = \frac{k_2 - k_1}{(k_2 + k_2^*)(k_1 + k_1^*)}(\alpha_2^{(2)}k_{12} - \alpha_1^{(2)}k_{22}) \]

\[ s_{ij} = \frac{|k_1 - k_2|^2}{(k_1 + k_1^*)(k_2 + k_2^*)}\left(\kappa_{11} - \kappa_{22} - \kappa_{12} - \kappa_{21}\right) \]
and
\[ \kappa_{ij} = \frac{\mu(\alpha_1^{(1)}r_{11}^{(i)} + \alpha_2^{(1)}r_{12}^{(i)})}{k_i + k_j^*}, \quad i, j = 1, 2 \]

This is the bright two-soliton solutions with six arbitrary complex parameters \( k_1, k_2, \alpha_i^{(1)}, \text{ and } \alpha_i^{(2)}, j = 1, 2 \). Using the above solution and by varying the physical parameters, [14] have discussed the asymptotic behavior of the two soliton at \( \eta_{gl} \to \pm \infty \), elastic and inelastic collision of two soliton interaction, and the effect of the relative phase change on the two soliton interaction properties. The symmetrical or asymmetrical nature of the two soliton interaction plots i.e., elastic and inelastic collision plots can be achieved by suitably choosing the values of \( \alpha_i^{(1)} \) and \( \alpha_i^{(2)}, j = 1, 2 \). For \( k_1 = k_2 \), two-soliton pulses coalesce into a one pulse and by slightly varying either of the values of \( k_1 \) and \( k_2 \), the two-soliton plot is recovered and the distance of the interaction between them is very long i.e. soliton period is very less. If the difference between the \( k_1 \) and \( k_2 \) values increases, we observe an increase in the periodicity of the interaction. Separation between the two-solitons is large for \( k_1 = 1 - i \) and \( k_2 = 1 + i \) before and after interaction.

Now we proceed to discuss the Bragg soliton compression and its collision properties under the elastic collision condition, whereas the Bragg soliton switching is discussed under the inelastic collision condition.

**IV. BRAGG SOLITON PULSE COMPRESSION UNDER ELASTIC COLLISION**

In order to study the impact of \( g(\xi) = 1/2(\xi + \xi_0) \) on Bragg soliton in nonuniform FBG, using (10), the two-soliton solutions of the CINLS Equations (9) have been constructed. When the ratios between the parameters are such that, \( \alpha_2 = \alpha_2 \), the collision between the two solitons is observed as elastic collision which is clearly depicted in Fig. 2 and 4. Under this elastic collision condition, the Bragg soliton pulse compression takes place due to enormous grating induced dispersion, outside FBG in the nonuniform FBG and the corresponding surface and contour plot are shown in Figs. 2 (b) and 3 (b), respectively. The physical mechanism for the Bragg soliton pulse in the nonuniform Bragg grating structure has been clearly discussed in the earlier sub-section.

![Fig. 2 Comparison of Bragg soliton interaction 3D plots without (a) and with pulse compression (b)](image-url)
Fig. 3 Contour plots without (a) and with pulse compression (b)
It is well known that the most fascinating feature of solitons is their particle like behavior. It is well known that the solitons tend to survive not only with perturbations but also collisions with other solitons. Survival of two such colliding solitons is even more remarkable if one notes that solitons interact strongly with each other during the collision. It has been well established that for copropagating NLS solitons, the interaction is either attractive or repulsive, depending on the relative phase between two solitons [15]-[18]. Here, we continue to discuss the interaction between two copropagating Bragg solitons in a nonuniform Bragg grating structure. In a uniform FBG, it has already been shown that collision between two counterpropagating Bragg solitons having their central frequencies in the middle of the PBG has been numerically studied by [19]. Recently, [20] have also been shown that the interaction between two copropagating Bragg solitons in a FBG is reminiscent of the NLS solitons except for the new feature that the relative phase of two Bragg solitons depends on their initial separation. Thus, the interaction studies of these Bragg grating solitons have been carried out only in the uniform Bragg grating structure. Therefore, it is of great interest to study such kind of collisions among the Bragg solitons in the real world system i.e., nonuniform FBG. Now, we proceed to discuss the interaction of these solitons in both uniform and nonuniform Bragg grating structure. The Bragg soliton interaction plots, for both uniform and nonuniform FBG, are depicted in Figs. 4 (a) and (b). Fig. 4 (a) shows the interaction of Bragg grating solitons in the uniform grating. From the figure, it is clear that after the interaction, the soliton emerges with no change in its physical parameters except with its phase. On the other hand, Fig. 4 (b) depicts the interaction of Bragg solitons in the nonuniform grating wherein the pulse compression takes place during collision process and hence pulse width gets reduced. This is mainly because of the presence of the inhomogeneity present in the Bragg grating structure. Here also, after the collision process, the soliton emerges as in the case of uniform grating. Further, we note that in the elastic-uniform FBG case, the velocities of the incoming and outgoing pulses are identical and the same is clearly depicted in 2D plots of Fig. 4 (a). However, in the elastic-nonuniform FBG case, the exiting pulse envelopes are closer than the incoming pulse envelopes at the same value of $\xi$. The separation of the centre frequencies of the emerging Bragg solitons is observed which is mainly due to the reduced differential group velocity after the collision process.

V. OPTICAL SWITCHING BASED ON BRAGG TWO-SOLITON UNDER INELASTIC COLLISION

To achieve the two-soliton inelastic collision in the optical birefringent fiber, we vary the complex parameters $\alpha_{2j}^{(j)}$, $\alpha_{2j}^{(j)}$, $j = 1, 2$, in such a way that the above condition can change the relative phases of the two optical modes. Initially amplitudes of the two optical modes do not change, but a change in the relative phase changes the amplitude distribution after interaction for the set of values $\alpha_1^{(1)} = 1 - 1.0 \alpha_2^{(1)} = 0.02 + 1.64i$, $\alpha_1^{(2)} = 1.0$, $\alpha_2^{(2)} = 1.0$, $Z = i$ and $k_1 = 1 + i$ and $k_2 = 2 - i$. For the uniform FBG case, after the soliton-soliton interaction, the two soliton is switched into a one soliton and consequently the energy is completely transferred from one mode to the other. This phenomenon is clearly seen with the surface plot of Fig. 5 (a) and the corresponding contour plot is provided in Fig. 6 (a). Now by introducing the inhomogeneity parameter, such that $\xi = -3.4$, after the interaction, two-soliton switches into a single soliton with pulse compression and, as a result, an increase in amplitude of the pulse takes place due to the influence of SPM as in Fig. 5 (b). The corresponding contour plot is shown in Fig. 6 (b).

In what follows, we discuss the physical mechanism of the Bragg soliton switching in both uniform and nonuniform Bragg grating structure. Before entering into the discussion, we just review earlier investigations that have been done in this field. Indeed, there are two ways of performing switching in a FBG. The first one is to use an optical pulse, which is...
tuned to lie within the PBG. Here the intensity of the pulse is sufficient to detune itself from the Bragg resonance and thus allowing the propagation through the periodic structure. The second way is to use a high intensity pump beam, tuned far from the Bragg resonance, to alter the propagation constant of a weak signal beam whose wavelength is near or within the grating bandgap. Such a kind of XPM effects were first observed by [21] and [22].

The concept behind AND gate, which is shown in Figs. 5 and 7, can easily be explained as follows. The two orthogonally polarized components $Q_1$ and $Q_2$ (along X and Y direction) of the input light pulse represent hits for the gate. Note that each bit can have a value either 1 or 0 depending on whether the corresponding light pulse is present or absent.

![Fig. 5 Comparison of Bragg soliton switching 3D plots without (a) and with pulse compression (b)](image)

![Fig. 6 Contour plots for Bragg soliton switching without (a) and with pulse compression (b)](image)
As well known, output in the AND gate is obtained only when both components are present simultaneously in the input.

We note that the average refractive index of the two components is defined as \( \bar{n} = \frac{n_1 + n_2}{2} \). Hence, the \( Q_2 \) Bragg frequency is slightly higher than the \( Q_1 \) Bragg frequency. To achieve AND gate operation, we just tune two input pulses inside the PBG but close to the upper branch of the dispersion curve. The combined intensity of these two fields \( Q_1 \) and \( Q_2 \) can increase the refractive index through the XPM phenomenon. As a result, detuning takes place and hence both the components are transmitted. However, if one of the components is absent at the input, instead of transmission, both the components are reflected as the XPM contribution vanishes in the PBG. This kind of inelastic interaction scenario and Bragg soliton switching are clearly shown in Figs. 7 (a) and (b), respectively. Fig. 7 (a) depicts the operation principle of the Bragg soliton switching at the different stages in the uniform FBG. On the other hand, 2D plots of Fig. 7 (b) explain the different scenario of Bragg soliton switching in the nonuniform FBG wherein the pulse compression takes place during the switching process and hence an increase in amplitude is seen. This is mainly owing to the presence of inhomogeneity in the nonuniform FBG.

**VI. CONCLUSION**

To investigate the optical switching based on Bragg soliton, we have considered the nonlinear pulse propagation through the nonuniform birefringent fiber Bragg grating. We have analyzed the high degree of pulse compression based on Bragg two soliton under the elastic collision condition. The occurrence of Bragg soliton switching has also been discussed in both uniform and nonuniform fiber Bragg gratings under the inelastic collision condition. In order to have good understanding about this switching process, 2D plots of the soliton-soliton interactions and 3D surface plots have been clearly depicted. We do envisage that the results presented in this paper would definitely attract the experimentalists who could realize this experimentally.
REFERENCE