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On the Multiplicity of Discriminants of Relative Quadratic Extensions of Quintic Fields

Schehrazad Selmane

Abstract**—**According to Hermite there exists only a finite number of number fields having a given degree, and a given value of the discriminant, nevertheless this number is not known generally. The determination of a maximum number of number fields of degree *10* having a given discriminant that contain a subfield of degree *5* having a fixed class number, narrow class number and Galois group is the purpose of this work. The constructed lists of the first coincidences of *52 (resp. 50, 40, 48, 22, 6)* nonisomorphic number fields with same discriminant of degree *10* of signature *(6,2) (resp. (4,3), (8,1), (2,4), (0,5), (10,0))* containing a quintic field. For each field in the lists, we indicate its discriminant, the discriminant of its subfield, a relative polynomial generating the field over its quintic field and its relative discriminant, the corresponding polynomial over *Q* and its Galois closure are presented with concluding remarks.

Keywords**—**Discriminant, nonisomorphic fields, quintic fields, relative quadratic extensions.

I. INTRODUCTION

ARTIN had conjectured in *1925* that the nonisomorphic fields never have coinciding discriminants, and he asserted that this could be proved using class field theory. Indeed this is true for quadratic fields. Five years later, A. Scholz and O. Taussky have provided the first counter examples for Artin's conjecture with the aid of class field theory and they gave families of *4* nonisomorphic complex cubic fields with same discriminant d $(d = -3299, -3896, -4027, -5703)$ [7].

The number of nonisomorphic fields with a fixed signature and Galois group sharing a common discriminant is called the multiplicity of the discriminant. Theory of multiplicities began with two fundamental methods of deriving exact formulas for the multiplicity m of cubic discriminants. The first method based on class field theory of the associated normal fields is due to H. Hasse [3], and the second method based on Kummer theory of the associated radical fields is due to H. Reichart [6]. In *1990* D. C. Mayer have generalized the idea of Hasse for dihedral fields of degree $2p$ with a prime number $p > 2$, and in *1991* for pure metacyclic fields of degree $p(p-1)$ [4], [5].

According to Hermite, there exist only a finite number of number fields having a given degree and a given value of the discriminant. In all known cases, the coincidences of discriminants between nonisomorphic fields with same signature do not involve small discriminants. For instance, in degree *3*, the first coincidence occurs for $d = -972$ for $r = 1$ and $d = 3969$ for $r = 3$; there is *120* fields with $-972 < d < 0$ and *133* fields with $0 < d < 3969$. In degree 4, for $r = 0, 2,$ and 4 there are respectively *18, 30,* and *96* fields before the first coincidence. The results known for $n \geq 5$ confirm the trend observed in degree $n \leq 4$ [11].

In previous papers [8]-[10], we have examined the existence of several nonisomorphic fields with same discriminant in the limit of constructed lists of quadratic extensions of quintic fields and we have derived some interesting properties of these fields. The findings have enabled us to undertake the issue of determining a maximum number of nonisomorphic fields for a class of number fields of degree *10* having a given discriminant that contain a subfield of degree *5* having a fixed class number, narrow class number and Galois group. We show the existence of *52 (resp. 50, 40, 48, 22, 6)* nonisomorphic number fields with same discriminants of signature *(6,2) (resp. (4,3), (8,1), (2,4), (0,5), (10,0))* containing a quintic field.

The paper is organized as follows. A summary, in the shape of table, of some known results regarding coincidences of discriminants of number fields of degree smaller than *6* is given in Section II. The main stages that allow us to find the targeted fields made the subject of the third section. The obtained results, brief discussion, and lists of polynomials defining the tenth degree number fields are reported in Section IV.

II. KNOWN RESULTS

In Table I, for each degree n $(n \leq 6)$ and signature (r,s), known results regarding coincidences of discriminants within the limits of available tables of number fields are presented [11]. The value d_1 of the discriminant of the first coincidence of discriminant of *2* nonisomorphic fields and the value d_2 of the maximum number β $(\beta > 2)$ of nonisomorphic number fields with same discriminant of discriminant smaller than the bound $\mathcal{B}$ are given. We also indicate the number η (resp. η_1, η_2) of fields of discriminant smaller than $\mathcal{B}$ (resp. d_1, d_2). Finally, ✓ means there are no coincidences of discriminants in such cases.

Among the *121 (resp. 162, 201, 154)* number fields of degree *7* having one *(resp. 3, 5, 7)* real place of discriminant smaller than, in absolute value, $6{\cdot}10^5$ (resp. $18{\cdot}10^5$, $12{\cdot}10^6$, $15{\cdot}10^7$) there exists no coincidences of discriminant.

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TABLE I
KNOWN RESULTS

(n,r)	B	η	d_1	η_1	β	d_2	η_2
(3,1)	10^6	182417	-972	120	9	274347	48356
(3,3)	$2\cdot10^6$	112444	3969	133	4	32009	1400
(4,0)	10^6	81322	576	18	19	705600	56296
(4,2)	10^6	90671	-1472	30	10	-	9517
(4,4)	10^6	13073	16448	96	7	132800	8801
						705600	
(5,1)	10^6	28993	16757	137	5		19630
(5,3)	10^6	10800	-289761	63	3	721872	3618
(5,5)	$2\cdot10^7$	22740	1810969	928	✓	-	✓
						428976	
(6,0)	$2\cdot10^5$	442	-33856	36	3	✓	98
(6,2)	$4\cdot10^5$	1179	111269	161	3		505
(6,4)	10^6	405	✓	✓	✓	-64387	✓
(6,6)	10^7	398	3195392	72	✓	237521	✓
						✓	
						✓	

III. METHOD

If $\mathcal{L}$ is a number field of degree n and of signature (r; s), we denote by $`_{\mathcal{L}}$ its ring of integers, by $d_{\mathcal{L}}$ its discriminant, by $h_{\mathcal{L}}$ the class number of $\mathcal{L}$, $h_{\mathcal{L}}^{+}$ the narrow class number of $\mathcal{L}$, and by $J(\mathcal{L})$ the set of distinct Q isomorphisms of $\mathcal{L}$ into C. For $\beta \in \mathcal{L}$, we denote the corresponding conjugates by $\beta^{(1)}, \ldots, \beta^{(n)}$ and we set $T_2(\beta) = \sum_{i=1}^{n} |\beta^{(i)}|^2$.

In previous papers [8]-[10], we developed a method for the explicit construction of all relative quadratic extensions of quintic fields with discriminant less than some given bounds. This method is applied in the determination of a maximum number of tenth degree nonisomorphic fields with same discriminant of signature *(r,s)* that contain a quintic field. With the chosen bounds $\mathcal{B}$ = 10^{11} (resp: $4\cdot10^{11}$, 10^{13}) in the enumeration of quadratic extensions of quintic fields of signature *(1,2) (resp: (3,1), (5,0))* [8]-[10], we obtained up to *12* nonisomorphic fields having same discriminant for the signatures *(2,4)* and *(0,5) (resp. (4,3)* and *(2,4), (6,2)* and *(4,3))*. The obtained *12* nonisomorphic fields have a value of discriminant of the form:

$$|d_K| = 2^{10}\, p^{f_i}\, d_{\mathcal{F}}^2 \tag{1}$$

where p is a prime number such that $p\vartheta_F = \prod_{i=1}^{t} \wp_i^{e_i}$ with $N(\wp_i) = p^{f_i}$ and where *2* remains inert in the quintic field F. This led us to look for a maximum number of nonisomorphic fields with same discriminant among the fields K containing a quintic field F where *2* remains inert and where p denotes a prime number which splits completely in the field F, and having $2^{10}\, p^{f_i}\, d_{\mathcal{F}}^2$ as discriminant.

Let us fix the quintic field F of signature (,,0≃,,0) and of discriminant d_{F} such that *2* remains inert, and let p be the first prime number which splits completely in the field F. Let K be a quadratic extension of F of absolute discriminant d_{K} smaller than the real constant $\mathcal{B}$. We fix $\mathcal{B} = 5.2^{10}\, p\, d_{\mathcal{F}}^2$ and denote by (,,≃,,) the signature of K.

According to [8-10] there exists an integer $\theta \in K$, $\theta \notin F$ such that:

$$\begin{aligned} &\mathsf{K} = \mathsf{F}(\theta) \\ &Min(\theta, \mathsf{F}) = P(x) = x^2 + ax + b \in \vartheta_{\mathsf{F}}[x] \\ &\sum_{i=1}^{10} \left|\theta^{(i)}\right|^2 \le \frac{1}{2}\sum_{i=1}^{5} \left|\sigma_i(a)\right|^2 + \left(\frac{\mathcal{B}}{4|d_{\mathsf{F}}|}\right)^{\frac{1}{5}} \end{aligned} \tag{2}$$

where σ_i *(1≤ i ≤5)* are the distinct *Q*-isomorphisms of F into *C*. Moreover the signature of K must be compatible with that of F, namely, $s \geq 2s'$.

To construct all polynomials P of which one of the roots generates one of the searched fields K over F, we work in the field F. The computation were done for the first field F such that $p.\, d_{\mathcal{F}}^2$ is as small as possible and this for each type of Galois closure, class number and narrow class number of quintic fields of signature *(5,0) (resp. (1,2), (3,1))* of discriminant smaller than $2\cdot10^7$ *(resp.* 10^6, 10^6*)*. All used quintic fields were taken from the available tables of number fields [11]. We assume that the discriminant d_{F} and an integral basis $W = \{w_1 = 1, w_2, \ldots, w_5\}$ of F are already known and we denote by g the minimal polynomial of ρ, where F = $Q(\rho)$.

A. Choice of A

As inequality (2) remains valid by translation by an element of ϑ_{F} , then we only have to make a run through a system of representants of ϑ_{F} modulo $2\vartheta_{\mathsf{F}}$, and therefore only *32* values can be considered for a :

$$a = \sum_{i=1}^{5} a_i w_i \;\; with \;\; a_i \in \{0,1\} \, for \;\; i = 1, \ldots, 5. \tag{3}$$

The running time for the computation of the possible b's, for a fixed value of a, strongly depends upon the size of the real constant $\kappa = \frac{1}{2}\sum_{i=1}^{5} \left|\sigma_i(a)\right|^2 + \left(\frac{\mathcal{B}}{4|d_{\mathsf{F}}|}\right)^{\frac{1}{5}}$. Each one of the *32* possible values of a have been chosen so that κ is minimum.

B. Choice of B

Once a convenient value of a is determined, we compute the set of suitable values of $b = \sum_{i=1}^{5} b_i w_i$ from the second relative symmetric function $s_2 = \theta^2 + \theta'^2$ where θ' denotes the other root of P. The values of s_2 are computed from the inequality $\sum_{i=1}^{5}\left|s_2^{(i)}\right|^2 \leq \kappa^2$ which is derived from (2) and the following inequality $\sum_{i=1}^{5}\left|s_2^{(i)}\right| \leq \kappa$.

We find ourselves in the presence of a long list of polynomials *P*. The main simplifications used to reduce, as much as possible, the number of polynomials to be considered to construct the complete lists of the desired fields are described below.

First of all, we solved the problem of the signature of the field K by ensuring that the polynomial discriminant $\Delta= a^2 - 4b$ of the real conjugate polynomial has the sign $(-1)^s$. However, for the signature *(0,5),* we needed also to verify that the roots of the other conjugate polynomials are not real for quintic fields having discriminant square. Then, we proceeded with the elimination of polynomials having too large values of $T_2(\theta)$ by checking whether the inequality:

$$\sum_{i=1}^{5}\left|\Delta^{(i)}\right| \leq 2\left(\frac{\mathcal{B}}{4\left|d_{\mathrm{F}}\right|}\right)^{\frac{1}{5}} \tag{4}$$

is fulfilled. Indeed, (2) and (4) are equivalent since

$$\sum_{i=1}^{10}\left|\theta^{(i)}\right|^2 = \frac{1}{2}\sum_{i=1}^{5}\left|a^{(i)}\right|^2 + \sum_{i=1}^{5}\left|\Delta^{(i)}\right|. \tag{5}$$

Finally, we checked the irreducibility of the polynomial *P*.

For the polynomials surviving these tests, we started by computing the relative discriminant δ using a theorem on ramification in Kummer extensions. Only polynomials for which $N(\delta) \leq \mathcal{B}\, d_{\mathcal{F}}^{-2}$ were kept, which allowed us to obtain the value d_K directly. As we obtained several polynomials for a given discriminant, we used the function OrderIsSubfield in KANT [2] to decide whether or not such polynomials define the same field up to isomorphism. Then, we computed the polynomial

$$f(x) = \prod_{i=1}^{5}\left(x^2 + \sigma_i(a) + \sigma_i(b)\right) = \sum_{i=1}^{10} t_i x^{10-i} \quad (t_0 = 1) \tag{6}$$

and the Galois group of the Galois closure for each field in the lists using KANT [2].

IV. Results and Comments

In this section, some information derived from the obtained results, gathered in Table II, are provided.

For a fixed quintic field F of signature (r', s'), the maximum number η of nonisomorphic fields with same discriminant $2^{10}\, p\, d_{\mathcal{F}}^2$ is obtained with the signature *(r,s)* of K such that $|r - r'| = 1$. For the corresponding signature, namely, $(2r' - r, \frac{1}{2}r + 2s')$, we obtain β $(\beta \leq \eta)$ nonisomorphic fields. Moreover, it is not necessary to do the computation for the determination of polynomials defining the β nonisomorphic fields with same discriminant for the corresponding signature $(2r' - r, \frac{1}{2}r + 2s')$. Indeed, if we denote by

$$f_i(x) = x^{10} + a_1x^8 + a_2x^6 + a_3x^4 + a_4x^2 + a_5 \quad (i = 1, \dots, \eta) \tag{7}$$

The η polynomials defining the nonisomorphic fields with same discriminant of discriminant j○Kj = $2^{10}\, p\, d_{\mathcal{F}}^2$ then among the η polynomials g_i $(i \leq 1 \leq \eta)$

$$g_i(x) = x^{10} - a_1x^8 + a_2x^6 - a_3x^4 + a_4x^2 - a_5 \tag{8}$$

β polynomials define nonisomorphic number fields with same discriminant of signature $(2r' - r, \frac{1}{2}r + 2s')$ and discriminant

j○Kj = $2^{10}\, p\, d_{\mathcal{F}}^2$, and *(η-β)* polynomials define nonisomorphic fields with same discriminant of signature $(2r' - r, \frac{1}{2}r + 2s')$ and discriminant $d_K = p\, d_{\mathcal{F}}^2$. Moreover, the polynomials $f_i\left(x^{1/2}\right)$ and $g_i\left(x^{1/2}\right)$ are just polynomials defining the quintic field F.

The notations for the Galois group Gal of F are similar to those of G. Butler and J. McKay [1], *h* (resp. h^+) corresponds to class number (resp. narrow class number) of F, and *nb* is the number of fields for all signatures.

We list for each fixed signature of the field F, the polynomials defining the constructed tenth degree nonisomorphic fields of signature *(r,s)* and those defining the tenth degree nonisomorphic fields for the corresponding signature $(2r' - r, \frac{1}{2}r + 2s')$.

TABLE II
NUMBER OF FIELDS WITH SAME DISCRIMINANT: $|d_K| = 2^{10}\, p\, d_{\mathcal{F}}^2$

r'	*Gal*	$d_{\mathcal{F}}$	*p*	*(0,5)*	*(2,4)*	*(4,3)*	*(6,2)*	*(8,1)*	*(10,0)*	*nb*
				(h,h⁺)=(1,1)						
5 5 5 5 3 1 1 1 1	T_1 T_2 T_4 T_5 T_5 T_2 T_3 T_4 T_5	14641 160801 15784729 24217 -7367 2209 44217 42849 1649	23 83 14 9 33 29 27 7 83 35 9 91 1 49 9	1 5 5 5 10 19 20 18 19	5 25 23 24 29 20 20 20 20	9 47 50 50 30 - - - -	10 50 48 46 10 - - - -	5 23 25 25 - - - - -	1 5 4 5 - - - - -	31 15 5 15 5 15 5 79 39 40 38 39
				(h,h⁺)=((1,2)						
5 5 5 3	T_3 T_4 T_5 T_5	6725897 11812969 144209 -39231	59 76 9 22 7 17 3	6 6 6 12	22 24 22 28	48 48 48 28	52 48 52 12	20 26 20 -	6 6 6 -	15 4 15 8 15 4 80
				(h,h⁺)=((1,4)						
5	T_5	1476577	10 3	4	36	38	40	34	4	15 6
				(h,h⁺)=((2,2)						
5 3 1	T_5 T_5 T_5	12284977 -550151 64665	89 70 9 26 9	2 12 8	10 36 8	20 36 -	18 12 -	10 - -	2 - -	62 96 16
				(h,h⁺)=((2,4)						
5 3	T_5 T_5	7322417 -936823	19 1 13 7	0 16	16 32	14 32	16 16	14 -	0 -	60 96
				(h,h⁺)=((2,8)						
5	T_5	15216977	41 9	0	48	40	48	40	0	17 6
				(3,3)						
1 1	T_4 T_5	426409 271785	63 1 70 1	19 20	20 18	- -	- -	- -	- -	39 38
				(4,4)						
1	T_5	791825	59	22	24	-	-	-	-	46
				(h,h⁺)=((5,5)						
1 1	T_2 T_5	717409 792425	14 9 93 7	20 20	19 20	- -	- -	- -	- -	39 40

TABLE III
$(R',S')=(1,2)$ AND $D_K=(-1)^s\, 2^{10}\cdot 531\cdot 791825^2$

(0,5)	(2,4)
(1,0,-2,0,9,0,65,0,-119,0,531)	(1,0,2,0,9,0,-65,0,-119,0,-531)
(1,0,2,0,32,0,217,0,392,0,531)	(1,0,-2,0,32,0,-217,0,392,0,-531)
(1,0,6,0,65,0,234,0,968,0,531)	(1,0,-6,0,65,0,-234,0,968,0,-531)
(1,0,8,0,46,0,177,0,304,0,531)	(1,0,-8,0,46,0,-177,0,304,0,-531)
(1,0,8,0,-31,0,334,0,838,0,531)	(1,0,-8,0,-31,0,-334,0,838,0,-531)
(1,0,15,0,69,0,-14,0,-397,0,531)	(1,0,-15,0,69,0,14,0,-397,0,-531)
(1,0,-27,0,266,0,-1113,0,1559,0,531)	(1,0,27,0,266,0,1113,0,1559,0,-531)
(1,0,28,0,251,0,767,0,1019,0,531)	(1,0,-28,0,251,0,-767,0,1019,0,-531)
(1,0,30,0,49,0,6,0,278,0,531)	(1,0,-30,0,49,0,-6,0,278,0,-531)
(1,0,-1,0,-78,0,505,0,-1277,0,1475)	(1,0,1,0,-78,0,-505,0,-1277,0,-1475)
(1,0,4,0,21,0,254,0,736,0,1475)	(1,0,-4,0,21,0,-254,0,736,0,-1475)
(1,0,8,0,-5,0,237,0,-321,0,1475)	(1,0,-8,0,-5,0,-237,0,-321,0,-1475)
(1,0,11,0,70,0,306,0,729,0,1475)	(1,0,-11,0,70,0,-306,0,729,0,-1475)
(1,0,20,0,169,0,568,0,1412,0,1475)	(1,0,-20,0,169,0,-568,0,1412,0,-1475)
(1,0,21,0,201,0,761,0,1116,0,1475)	(1,0,-21,0,201,0,-761,0,1116,0,-1475)
(1,0,22,0,193,0,835,0,1773,0,1475)	(1,0,-22,0,193,0,-835,0,1773,0,-1475)
(1,0,23,0,208,0,895,0,1823,0,1475)	(1,0,-23,0,208,0,-895,0,1823,0,-1475)
(1,0,28,0,273,0,1105,0,1503,0,1475)	(1,0,-28,0,273,0,-1105,0,1503,0,-1475)
(1,0,9,0,28,0,595,0,5425,0,13275)	(1,0,-9,0,28,0,-595,0,5425,0,-13275)
(1,0,-44,0,723,0,-5115,0,11525,0,13275)	(1,0,44,0,723,0,5115,0,11525,0,-13275)
(1,0,-26,0,175,0,700,0,-12150,0,36875)	(1,0,26,0,175,0,-700,0,-12150,0,-36875)
(1,0,49,0,885,0,7250,0,26925,0,36875)	(1,0,-49,0,885,0,-7250,0,26925,0,-36875)
	(1,0,-15,0,74,0,258,0,-283,0,-1475)
	(1,0,-1,0,-17,0,72,0,181,0,-531)

TABLE IV
$(r',s')=(3,1)$ AND $d_K=(-1)^s\, 2^{10}\cdot 709\cdot(-550151)^2$

(2,4)	(4,3)
(1,0,-10,0,-50,0,187,0,210,0,-709)	(1,0,10,0,-50,0,-187,0,210,0,709)
(1,0,-30,0,305,0,338,0,-686,0,-709)	(1,0,30,0,305,0,-338,0,-686,0,709)
(1,0,17,0,33,0,3,0,-212,0,-709)	(1,0,-17,0,33,0,-3,0,-212,0,709)
(1,0,17,0,9,0,-423,0,-188,0,-709)	(1,0,-17,0,9,0,423,0,-188,0,709)
(1,0,31,0,359,0,1684,0,2563,0,-709)	(1,0,-31,0,359,0,-1684,0,2563,0,709)
(1,0,11,0,-184,0,-917,0,-1405,0,-709)	(1,0,-11,0,-184,0,917,0,-1405,0,709)
(1,0,0,0,41,0,458,0,-1550,0,-709)	(1,0,0,0,41,0,-458,0,-1550,0,709)
(1,0,6,0,-173,0,-1933,0,-5481,0,-709)	(1,0,-6,0,-173,0,1933,0,-5481,0,709)
(1,0,-7,0,56,0,427,0,-17,0,-709)	(1,0,7,0,56,0,-427,0,-17,0,709)
(1,0,6,0,-37,0,-88,0,574,0,-709)	(1,0,-6,0,-37,0,88,0,574,0,709)
(1,0,33,0,145,0,104,0,-465,0,-709)	(1,0,-33,0,145,0,-104,0,-465,0,709)
(1,0,39,0,514,0,2750,0,5181,0,-709)	(1,0,-39,0,514,0,-2750,0,5181,0,709)
(1,0,-18,0,13,0,351,0,-751,0,-709)	(1,0,18,0,13,0,-351,0,-751,0,709)
(1,0,0,0,-136,0,-691,0,-1184,0,-709)	(1,0,0,0,-136,0,691,0,-1184,0,709)
(1,0,17,0,35,0,-254,0,719,0,-709)	(1,0,-17,0,35,0,254,0,719,0,709)
(1,0,17,0,-1213,0,15238,0,-58031,0,-709)	(1,0,-17,0,-1213,0,-15238,0,-58031,0,709)
(1,0,2,0,50,0,199,0,-2746,0,-709)	(1,0,-2,0,50,0,-199,0,-2746,0,709)
(1,0,-25,0,186,0,-207,0,-1205,0,-709)	(1,0,25,0,186,0,207,0,-1205,0,709)
(1,0,27,0,-126,0,-1077,0,4555,0,-6381)	(1,0,-27,0,-126,0,1077,0,4555,0,6381)
(1,0,12,0,-43,0,-1111,0,-4891,0,-6381)	(1,0,-12,0,-43,0,1111,0,-4891,0,6381)
(1,0,65,0,-133,0,-1655,0,6460,0,-6381)	(1,0,-65,0,-133,0,1655,0,6460,0,6381)
(1,0,48,0,893,0,7626,0,24562,0,-6381)	(1,0,-48,0,893,0,-7626,0,24562,0,6381)
(1,0,3,0,-82,0,10,0,977,0,-6381)	(1,0,-3,0,-82,0,-10,0,977,0,6381)
(1,0,37,0,381,0,1351,0,-46,0,-6381)	(1,0,-37,0,381,0,-1351,0,-46,0,6381)
(1,0,-1,0,-279,0,-2482,0,-8017,0,-6381)	(1,0,1,0,-279,0,2482,0,-8017,0,6381)
(1,0,40,0,353,0,644,0,-1844,0,-6381)	(1,0,-40,0,353,0,-644,0,-1844,0,6381)
(1,0,-9,0,-159,0,-800,0,-8743,0,-34741)	(1,0,9,0,-159,0,800,0,-8743,0,34741)

(1,0,38,0,-622,0,-683,0,26278,0,-34741)	(1,0,-38,0,-622,0,683,0,26278,0,34741)
(1,0,22,0,-100,0,-5427,0,-35886,0,-34741)	(1,0,-22,0,-100,0,5427,0,-35886,0,34741)
(1,0,16,0,-30,0,-2155,0,-15480,0,-34741)	(1,0,-16,0,-30,0,2155,0,-15480,0,34741)
(1,0,15,0,-95,0,-2929,0,-18114,0,-34741)	(1,0,-15,0,-95,0,2929,0,-18114,0,34741)
(1,0,24,0,187,0,348,0,-5416,0,-34741)	(1,0,-24,0,187,0,-348,0,-5416,0,34741)
(1,0,-1,0,-403,0,-488,0,22563,0,-57429)	(1,0,1,0,-403,0,488,0,22563,0,57429)
(1,0,38,0,-239,0,-4357,0,39559,0,-85789)	(1,0,-38,0,-239,0,4357,0,39559,0,85789)
(1,0,43,0,347,0,-6263,0,-83906,0,-85789)	(1,0,-43,0,347,0,6263,0,-83906,0,85789)
(1,0,52,0,581,0,-6840,0,-133974,0,-516861)	(1,0,-52,0,581,0,6840,0,-133974,0,516861)

TABLE V
$(r',s')=(5,0)$ AND $d_K=(-1)^s\,2^{10}\cdot 277\cdot 144209^2$

(6,2)	(4,3)
(1,0,-26,0,103,0,132,0,-222,0,-227)	(1,0,26,0,103,0,-132,0,-222,0,227)
(1,0,-12,0,-19,0,111,0,53,0,-227)	(1,0,12,0,-19,0,-111,0,53,0,227)
(1,0,-12,0,-29,0,117,0,69,0,-227)	(1,0,12,0,-29,0,-117,0,69,0,227)
(1,0,24,0,-63,0,-128,0,394,0,-227)	(1,0,-24,0,-63,0,128,0,394,0,227)
(1,0,-22,0,80,0,83,0,-346,0,-227)	(1,0,22,0,80,0,-83,0,-346,0,227)
(1,0,13,0,-15,0,-367,0,772,0,-227)	(1,0,-13,0,-15,0,367,0,772,0,227)
(1,0,1,0,-59,0,44,0,959,0,-2043)	(1,0,-1,0,-59,0,-44,0,959,0,2043)
(1,0,9,0,-103,0,91,0,242,0,-227)	(1,0,-9,0,-103,0,-91,0,242,0,227)
(1,0,2,0,-51,0,-23,0,595,0,-227)	(1,0,-2,0,-51,0,23,0,595,0,227)
(1,0,-39,0,42,0,298,0,-363,0,-227)	(1,0,39,0,42,0,-298,0,-363,0,227)
(1,0,8,0,-117,0,349,0,-215,0,-227)	(1,0,-8,0,-117,0,-349,0,-215,0,227)
(1,0,-33,0,313,0,-759,0,-1002,0,-227)	(1,0,33,0,313,0,759,0,-1002,0,227)
(1,0,11,0,-11,0,-202,0,459,0,-227)	(1,0,-11,0,-11,0,202,0,459,0,227)
(1,0,-47,0,-102,0,299,0,529,0,-227)	(1,0,47,0,-102,0,-299,0,529,0,227)
(1,0,-20,0,103,0,140,0,-1540,0,-227)	(1,0,20,0,103,0,-140,0,-1540,0,227)
(1,0,14,0,-15,0,-172,0,390,0,-227)	(1,0,-14,0,-15,0,172,0,390,0,227)
(1,0,17,0,-2,0,-301,0,513,0,-227)	(1,0,-17,0,-2,0,301,0,513,0,227)
(1,0,14,0,-26,0,-541,0,708,0,-227)	(1,0,-14,0,-26,0,541,0,708,0,227)
(1,0,-8,0,-17,0,144,0,146,0,-227)	(1,0,8,0,-17,0,-144,0,146,0,227)
(1,0,-11,0,23,0,78,0,-173,0,-227)	(1,0,11,0,23,0,-78,0,-173,0,227)
(1,0,20,0,-19,0,-248,0,490,0,-227)	(1,0,-20,0,-19,0,248,0,490,0,227)
(1,0,-1,0,-73,0,329,0,-286,0,-227)	(1,0,1,0,-73,0,-329,0,-286,0,227)
(1,0,5,0,-79,0,-164,0,1145,0,-227)	(1,0,-5,0,-79,0,164,0,1145,0,227)
(1,0,-19,0,35,0,361,0,172,0,-227)	(1,0,19,0,35,0,-361,0,172,0,227)
(1,0,23,0,61,0,-814,0,1323,0,-227)	(1,0,-23,0,61,0,814,0,1323,0,227)
(1,0,29,0,135,0,-629,0,694,0,-227)	(1,0,-29,0,135,0,629,0,694,0,227)
(1,0,-1,0,-58,0,67,0,269,0,-227)	(1,0,1,0,-58,0,-67,0,269,0,227)
(1,0,-3,0,-70,0,145,0,95,0,-227)	(1,0,3,0,-70,0,-145,0,95,0,227)
(1,0,-26,0,151,0,-90,0,-478,0,-227)	(1,0,26,0,151,0,90,0,-478,0,227)
(1,0,3,0,-45,0,-62,0,499,0,-227)	(1,0,-3,0,-45,0,62,0,499,0,227)
(1,0,12,0,-577,0,394,0,326,0,-227)	(1,0,-12,0,-577,0,-394,0,326,0,227)
(1,0,0,0,-31,0,47,0,129,0,-227)	(1,0,0,0,-31,0,-47,0,129,0,227)
(1,0,-1,0,-64,0,-85,0,377,0,-227)	(1,0,1,0,-64,0,85,0,377,0,227)
(1,0,-4,0,-40,0,59,0,230,0,-227)	(1,0,4,0,-40,0,-59,0,230,0,227)
(1,0,1,0,-28,0,17,0,169,0,-227)	(1,0,-1,0,-28,0,-17,0,169,0,227)
(1,0,2,0,-53,0,47,0,333,0,-227)	(1,0,-2,0,-53,0,-47,0,333,0,227)
(1,0,-7,0,-31,0,223,0,-52,0,-227)	(1,0,7,0,-31,0,-223,0,-52,0,227)
(1,0,-7,0,-33,0,109,0,76,0,-227)	(1,0,7,0,-33,0,-109,0,76,0,227)
(1,0,-6,0,-10,0,81,0,8,0,-227)	(1,0,6,0,-10,0,-81,0,8,0,227)
(1,0,-18,0,12,0,237,0,-132,0,-227)	(1,0,18,0,12,0,-237,0,-132,0,227)
(1,0,29,0,159,0,-728,0,757,0,-227)	(1,0,-29,0,159,0,728,0,757,0,227)
(1,0,-6,0,-50,0,267,0,554,0,-2043)	(1,0,6,0,-50,0,-267,0,554,0,2043)
(1,0,-25,0,123,0,538,0,-1433,0,-2043)	(1,0,25,0,123,0,-538,0,-1433,0,2043)
(1,0,-14,0,25,0,289,0,-557,0,-2043)	(1,0,14,0,25,0,-289,0,-557,0,2043)
(1,0,-23,0,24,0,427,0,-287,0,-2043)	(1,0,23,0,24,0,-427,0,-287,0,2043)

(1,0,-7,0,-51,0,426,0,-193,0,-2043)	(1,0,7,0,-51,0,-426,0,-193,0,2043)
(1,0,-32,0,292,0,-365,0,-2630,0,-2043)	(1,0,32,0,292,0,365,0,-2630,0,2043)
(1,0,-37,0,412,0,-1047,0,-3557,0,-2043)	(1,0,37,0,412,0,1047,0,-3557,0,2043)
(1,0,-21,0,99,0,136,0,-911,0,-227)	
(1,0,0,0,-80,0,-99,0,414,0,-227)	
(1,0,-6,0,-111,0,-44,0,424,0,-227)	
(1,0,-28,0,140,0,661,0,434,0,-227)	

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