

On Solving Single-Period Inventory Model under Hybrid Uncertainty

Madhukar Nagare, Pankaj Dutta

Abstract— Inventory decisional environment of short life-cycle products is full of uncertainties arising from randomness and fuzziness of input parameters like customer demand requiring modeling under hybrid uncertainty. Prior inventory models incorporating fuzzy demand have unfortunately ignored stochastic variation of demand. This paper determines an unambiguous optimal order quantity from a set of n fuzzy observations in a newsvendor inventory setting in presence of fuzzy random variable demand capturing both fuzzy perception and randomness of customer demand. The stress of this paper is in providing solution procedure that attains optimality in two steps with demand information availability in linguistic phrases leading to fuzziness along with stochastic variation. The first step of solution procedure identifies and prefers one best fuzzy opinion out of all expert opinions and the second step determines optimal order quantity from the selected event that maximizes profit. The model and solution procedure is illustrated with a numerical example.

Keywords—Fuzzy expected value, Fuzzy random demand, Hybrid uncertainty, Optimal order quantity, Single-period inventory

I. INTRODUCTION

DECISION making environment of inventory management in retail supply chain is full of uncertainties especially when dealing with demand data pertaining to new innovative products or fashion goods. There are two main sources of uncertainty (i) randomness (ii) fuzziness. Conventional inventory models consider uncertainty only as randomness and incorporate it through probability theory. Implicit assumption in pure probabilistic models is the availability of sufficient amount of information to determine reliable estimates of the various inventory parameters and ignoring or approximately considering inaccurate/imprecise data. Therefore, despite existence of large number of stochastic inventory models, there exists a need for models that handle uncertainty and vagueness in human judgment, lack of evidence and lack of certainty in evidence [1]. Decision making in such business environment poses a great challenge due to difficulty faced in capturing and representing vital yet imprecise information in mathematical models. In such situations, uncertainties arising from vagueness and fuzziness are represented and treated with fuzzy set theory. Pure fuzzy model though consider the imprecision of the available information, the inherent randomness is unfortunately neglected. But in real-life inventory problems, fuzziness and

randomness do not have exclusive existence and very commonly found to co-exist; such situations warrants inventory models developed for mixed fuzzy random environment [2].

The characteristic of long lead times and short selling season for products such as fashion items, seasonal products, sports goods, greeting cards etc provides only one order opportunity and hence inventory problem of such products can be solved suitably in the newsvendor (NV) problem setting. Single period inventory model [SPIM] is frequently discussed in the literature and applied to solve inventory problem of aforementioned goods and many more. Most of the NV model extensions have been developed in the probabilistic framework i.e. the demand uncertainty is described by probability distributions. However, in real world, sometimes the probability distributions of the demands are difficult to obtain due to lack of information and historical data. In such case, the demands are approximately specified based on the experience and subjective judgments of experts and described linguistically such as “demand is nearly d ”. In such cases, the fuzzy set theory is the best form that adapts all the uncertainty set to the model [3]. Therefore; literature provides SPIM with either probabilistic demand [4] or with fuzzy demand [5-8].

A common and major drawback of these studies is that the stochastic variation of demand is not reflected in their models. Stochastic variation poses great difficulty in predicting demand as demand information is available takes form of linguistic phrases like ‘demand around d_1 ’ or ‘demand is nearly d_2 ’ which are subjective and only partially quantifiable. In such cases, fuzzy random variable (FRV) [9-11] has become an appropriate mathematical tool for modeling. However, no attempt is made in the literature to define the demand of single period in *hybrid environment* except for the work of Dutta et al. [12]. They considered the model with FRV demand involving imprecise probabilities and presented a fuzzy optimal solution.

This paper deals with a different model in a SPIM setting that captures both fuzzy perception and random behavior of customer demand and extend their earlier model in determining a crisp optimal solution from a set of n fuzzy observations. As the coexistence of fuzzy perception and random behavior appear into the demand level, in the current study, we first determine one preferred expert’s-opinion as the best among all fuzzy observations for the variable “customer demand” and then determine the optimal order quantity. Instead of taking fuzzy probability of a fuzzy event, here we consider crisp probability as the chance. Fortunately, this consideration does not affect the solution procedure of the

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model. The aim of this study is to provide a more realistic inventory model, especially from a decision-maker's (DM) point of view in determining an unambiguous optimal order quantity.

The paper is organized as follows. In Section 2, a SPIM in presence of FRV demand is formulated and then solution methodology is developed. The graded mean representation of a fuzzy number proposed by Chen and Hsieh [13] is used to optimize the fuzzy expected profit function. A numerical illustration is provided in Section 3 and finally the conclusion is presented in Section 4.

II. SINGLE-PERIOD INVENTORY MODEL IN FUZZY-STOCHASTIC ENVIRONMENT

In order to develop the hybrid environment inventory model, the following notation and assumptions are used.

Notation

- Q Order quantity
 \tilde{D} Fuzzy random discrete demand
 c Unit purchase cost of an item
 p Unit selling price of the item
 h Holding cost per item after the end of the period
 s Unit shortage cost
 \tilde{d}_i Fuzzy observation for the variable \tilde{D} for ($i = 1$ to n)
 p_i Probability of occurrence of an event \tilde{d}_i

Assumptions

- The time horizon is precise (single-period) and the DM will order optimal Q for the product at the beginning of the period.
- The demand \tilde{D} is considered as a discrete FRV with the given set of data $\{(\tilde{d}_1, p_1), (\tilde{d}_2, p_2), \dots, (\tilde{d}_n, p_n)\}$, where all the observations of \tilde{D} are triangular fuzzy numbers. Let $\mu_{\tilde{d}_i}$ denote the membership function of \tilde{d}_i where

$$\mu_{\tilde{d}_i}(x) = \begin{cases} L_i(x) = \left(\frac{x - \underline{d}_i}{\underline{d}_i - \underline{d}_i} \right), & \underline{d}_i \leq x \leq \underline{d}_i, \\ R_i(x) = \left(\frac{\bar{d}_i - x}{\bar{d}_i - \underline{d}_i} \right), & \underline{d}_i \leq x \leq \bar{d}_i, \\ 0, & \text{otherwise.} \end{cases}$$

- Since the products have a finite shelf-life (e.g., fresh foods, seasonal items etc.), it is assumed that left over items are salvaged at the end of the period/season. Lost sales penalty cost is assumed to be zero.

As mentioned earlier, demand for the given period is described as fuzzy random where the variable values are

linguistic in nature. Thus if \tilde{d}_k items are procured at the beginning of the period, then the profit function \tilde{P} is given by

$$\tilde{P}(\tilde{d}_k, \tilde{D}) = \begin{cases} p\tilde{d}_i - c\tilde{d}_k - h(\tilde{d}_k - \tilde{d}_i); & \text{for } \tilde{d}_i \preceq \tilde{d}_k \\ (p-c)\tilde{d}_k - s(\tilde{d}_i - \tilde{d}_k); & \text{for } \tilde{d}_i \succeq \tilde{d}_k \end{cases} \quad (1)$$

for some $i = 1$ to n

In the following subsections, the approach for analyzing the fuzzy profit function $\tilde{P}(\tilde{d}_k, \tilde{D})$ is developed.

A. Determining \tilde{d}_k^*

We first determine the expected profit of problem (1) where the demand is treated as a FRV and then suggest an optimal \tilde{d}_k^* from all the observations \tilde{d}_i ; $i = 1$ to n . Let us introduce the following proposition:

Proposition 1. The profit function $\tilde{P}(\tilde{d}_k, \tilde{D})$ defined in (1) itself is a FRV.

Proof: We recall, a FRV associated with a random experiment is an appropriate formulation of a process assessing a fuzzy value to each experimental outcome. Here the profit is a function of uncontrollable variable \tilde{D} . Furthermore, the expert's opinions about \tilde{D} are linguistic, i.e., fuzzy. Therefore, for each \tilde{d}_i of \tilde{D} , there would be a corresponding fuzzy value of $\tilde{P}(\tilde{d}_k, \tilde{D})$ with probability p_i . Consequently, $\tilde{P}(\tilde{d}_k, \tilde{D})$ becomes a fuzzy valued random variable. To restate, the profit function is also a FRV.

In order to determine the optimal order quantity \tilde{d}_k^* , we now find out the expected value of $\tilde{P}(\tilde{d}_k, \tilde{D})$. Corresponding to the crisp expected value of a positive classical random variable, we recall the expectation of FRV is a unique fuzzy number.

In other words, the fuzzy expected value is a summarizing fuzzy value of the central tendency of FRV. Therefore, the expected resultant profit of $\tilde{P}(\tilde{d}_k, \tilde{D})$ becomes a fuzzy quantity on \mathfrak{R} . Let \tilde{E}_p be the fuzzy expected value of $\tilde{P}(\tilde{d}_k, \tilde{D})$.

Then, the total expected profit is determined by

$$\tilde{E}_p = \tilde{E}_p(\tilde{d}_k, \tilde{D}) = \sum_{i=1}^k [p\tilde{d}_i - c\tilde{d}_k - h(\tilde{d}_k - \tilde{d}_i)] p_i + \sum_{i=k+1}^n [(p-c)\tilde{d}_k - s(\tilde{d}_i - \tilde{d}_k)] p_i \quad (2)$$

Here the α – level set of the fuzzy number \tilde{E}_p can be constructed as

$$E_p(\alpha) = E[P(\alpha)] = [E(P_L(\alpha)), E(P_R(\alpha))]; 0 \leq \alpha \leq 1$$

which provides different α – cut intervals for the fuzzy number \tilde{E}_p for different α between 0 and 1. Now using the method of graded mean representation of a fuzzy number, one can find a defuzzified representative of the fuzzy expected value \tilde{E}_p by the formula:

$$G(\tilde{E}_p) = \int_0^w \alpha \left(\frac{L^{-1}(\alpha) + R^{-1}(\alpha)}{2} \right) d\alpha \Bigg/ \int_0^w \alpha d\alpha \quad (3)$$

where $w \in (0, 1]$ is the grade of \tilde{E}_p and $L^{-1}(\alpha)$ and $R^{-1}(\alpha)$ are the representative of $E(P_L(\alpha))$ and $E(P_R(\alpha))$, respectively.

As the data are imprecise with triangular fuzzy number $(\underline{d}_i, d_i, \bar{d}_i)$, without loss of generality, we consider \tilde{D} and \tilde{E}_p have the same shape of the membership function with grade $w = 1$. So the fuzzy expected profit function $\tilde{E}_p = (\underline{E}_p, E_p, \bar{E}_p)$ can be rewritten as

$$\tilde{E}_p = \sum_{i=1}^k [(p+h)\underline{d}_i - (c+h)\bar{d}_k] p_i + \sum_{i=k+1}^n [(p-c+s)\bar{d}_k - s\underline{d}_i] p_i$$

where

$$\begin{aligned} E_p &= E(P(\alpha = 1)) \\ &= \sum_{i=1}^k [(p+h)\underline{d}_i - (c+h)\bar{d}_k] p_i + \sum_{i=k+1}^n [(p-c+s)\bar{d}_k - s\underline{d}_i] p_i \end{aligned}$$

$$\underline{E}_p = E[P_L(\alpha = 0)]$$

$$= \sum_{i=1}^k [(p+h)\underline{d}_i - (c+h)\bar{d}_k] p_i + \sum_{i=k+1}^n [(p-c+s)\underline{d}_k - s\underline{d}_i] p_i$$

and

$$\bar{E}_p = E[P_R(\alpha = 0)]$$

$$= \sum_{i=1}^k [(p+h)\bar{d}_i - (c+h)\underline{d}_k] p_i + \sum_{i=k+1}^n [(p-c+s)\bar{d}_k - s\underline{d}_i] p_i$$

Hence, using (3) we get

$$G(\tilde{E}_p) = \frac{E_p + 4E_p + \bar{E}_p}{6} \quad (4)$$

This is the expected profit in the fuzzy-stochastic sense based on graded mean representation method. For maximum value of $G(\tilde{E}_p)$ at \tilde{d}_k , the following condition must be satisfied.

$$G(\tilde{E}_p(\tilde{d}_{k-1})) < G(\tilde{E}_p(\tilde{d}_k)) > G(\tilde{E}_p(\tilde{d}_{k+1})) \quad (5)$$

Now, taking (4) and the first condition of (5), we obtain,

$$(p-c+s) \sum_{i=1}^n p_i - (p+h+s) \sum_{i=1}^{k-1} p_i > 0$$

Similarly, the second condition of (5) procures

$$(p+h+s) \sum_{i=1}^k p_i - (p-c+s) \sum_{i=1}^n p_i > 0$$

Since $\sum_{i=1}^n p_i = 1$ consequently the optimal value of can be

obtained by the relationship

$$\sum_{i=1}^{k-1} p_i < \frac{p-c+s}{p+h+s} < \sum_{i=1}^k p_i \quad (6)$$

Thus using the above restrictions we can find the optimal fuzzy quantity \tilde{d}_k^* . That is, \tilde{d}_k^* is the most preferable information among all the experts opinion. In the next subsection, we will determine the optimal order quantity Q^* from \tilde{d}_k^* as a crisp decision.

B. Determining Q^*

Since a fuzzy number is a collection of points with different degrees of belongingness, therefore, acquiring a final \tilde{d}_k^* , as desirable to obtaining a specific value, the DM can also compute the final order quantity Q^* . In order to determine the optimal Q^* , we now plan to formulate a profit function and then optimize it using graded mean integration representation method.

Suppose d be the actual realization of fuzzy demand \tilde{d}_k^* , then a related profit function $\Pi(Q)$ associated with each $d \in [\underline{d}_k, \bar{d}_k]$ is defined as $\Pi(Q) = p \min\{d, Q\} - cQ$. In describing the profit function, two situations may arise, viz.; over-stock ($d \leq Q$) or under-stock ($d > Q$). Since the information \tilde{d}_k is characterized imprecisely, for any feasible order quantity Q , it procures either a fuzzy over-stock profit or a fuzzy under-stock profit and hence the resultant profit function is given by

$$\tilde{\Pi}(Q) = (p\tilde{d}_{k,OS} - cQ) \cup (p-c)Q \quad (7)$$

Since $\tilde{\Pi}(Q)$ is a fuzzy quantity, using its several α – level sets, we first find out the expected resultant profit and then the optimal order quantity Q^* is determined by maximizing the expected value of this fuzzy profit function (7).

Let $\Pi(Q, \alpha) = [\Pi_L(Q, \alpha), \Pi_R(Q, \alpha)]$ be the α -level set of $\tilde{\Pi}(Q)$, where $\Pi_L(Q, \alpha)$ and $\Pi_R(Q, \alpha)$ are the left and right endpoints of $\Pi(Q, \alpha)$, respectively.

Lemma1. If $\Pi_L(Q, \alpha)$ and $\Pi_R(Q, \alpha)$ are continuous on the closed interval $[0, 1]$, they are also integrable in $[0, 1]$.

Now, the α -level graded mean integration of $\tilde{\Pi}(Q)$ is defined as

$$G(\tilde{\Pi}(Q)) = \int_0^1 \alpha(\Pi_L(Q, \alpha) + \Pi_R(Q, \alpha)) d\alpha \quad (8)$$

Since each \tilde{d}_k is assumed as a triangular fuzzy number, we now analyze the following two cases according to the position of Q in $[\underline{d}_k, \bar{d}_k]$ as follows:

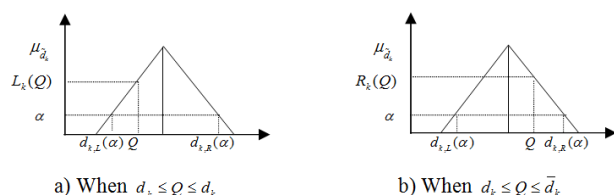


Fig. 1. Alpha cut of \tilde{d}_k

Case1. When Q lies in the range of \underline{d}_k and \bar{d}_k , from Fig. 1(a), we obtain the respective α -level sets of \tilde{d}_k in over stocking situation and under-stocking situation as

$$d_{k,OS}(\alpha) = \begin{cases} [d_{k,L}(\alpha), Q] & \alpha \leq L_k(Q) \\ \phi & \alpha > L_k(Q) \end{cases}$$

and

$$d_{k,US}(\alpha) = \begin{cases} [Q, d_{k,R}(\alpha)] & \alpha \leq L_k(Q) \\ [d_{k,L}(\alpha), d_{k,R}(\alpha)] & \alpha > L_k(Q) \end{cases}$$

Consequently, the left and right end points of $\Pi(Q, \alpha)$ are obtained as

$$\Pi_L(Q, \alpha) = \begin{cases} pd_L(\alpha) - cQ, & 0 \leq \alpha \leq L_k(Q) \\ (p-c)Q, & L_k(Q) \leq \alpha \leq 1 \end{cases}$$

and

$$\Pi_R(Q, \alpha) = (p-c)Q, \quad 0 \leq \alpha \leq 1$$

Therefore, using (8) the expected profit with respect to the opinion \tilde{d}_k is derived as

$$\begin{aligned} G(\tilde{\Pi}(Q)) &= \int_0^{L_k(Q)} \alpha((pd_{k,L}(\alpha) - cQ) + (p-c)Q) d\alpha \\ &+ \int_{L_k(Q)}^1 \alpha((p-c)Q + (p-c)Q) d\alpha \end{aligned}$$

$$= p \int_0^{L_k(Q)} \alpha d_{k,L}(\alpha) d\alpha + pQ(1 - 0.5L_k(Q)) - cQ \quad (9)$$

Case2. When Q lies in the range of \underline{d}_k and \bar{d}_k , from Fig 1(b), we obtain the respective α -level sets of \tilde{d}_k in over-stocking situation and under-stocking situation as

$$d_{k,OS}(\alpha) = \begin{cases} [d_{k,L}(\alpha), Q] & \alpha \leq R_k(Q) \\ [d_{k,L}(\alpha), d_{k,R}(\alpha)] & \alpha > R_k(Q) \end{cases}$$

and

$$d_{k,US}(\alpha) = \begin{cases} [Q, d_{k,R}(\alpha)] & \alpha \leq R_k(Q) \\ \phi & \alpha > R_k(Q) \end{cases}$$

In this case, using (8) the expected profit with respect to the opinion \tilde{d}_k is derived as

$$\begin{aligned} G(\tilde{\Pi}(Q)) &= \int_0^{R_k(Q)} \alpha((pd_{k,L}(\alpha) - cQ) + (p-c)Q) d\alpha \\ &+ \int_{R_k(Q)}^1 \alpha((pd_{k,L}(\alpha) - cQ) + (pd_{k,R}(\alpha) - cQ)) d\alpha \\ &= p \int_0^{L_k(Q)} \alpha d_{k,L}(\alpha) d\alpha + p \int_{R_k(Q)}^1 \alpha d_{k,R}(\alpha) d\alpha \\ &+ 0.5pQR_k^2(Q) - cQ \end{aligned} \quad (10)$$

Having discussed these two situations, we may now introduce the following proposition.

Proposition2. There exists an optimal solution Q^* of problem (7) subject to the condition $2c - p \geq 0$ or < 0 according to the position of Q in $[\underline{d}_k, d_k]$ or $[d_k, \bar{d}_k]$, respectively.

Proof. To derive the optimality conditions for finding the crisp optimal order quantity, let us first consider that Q^* lies between \underline{d}_k and d_k .

In this case, the graded mean representation $G(\tilde{\Pi}(Q))$ is given in (9).

The optimal Q^* is determined by $\frac{dG(\tilde{\Pi}(Q))}{dQ} = 0$, which yields

$$L_k(Q^*) = \sqrt{\frac{2(p-c)}{p}}.$$

Since $\frac{d^2G(\tilde{\Pi}(Q))}{dQ^2} = -pL_k(Q)L'_k(Q) < 0$ and the left-shape function $L_k(\cdot)$ lies between 0 and 1, we have the necessary condition as $2c - p \geq 0$.

Similarly, if $2c - p < 0$, then the optimal Q^* lies between d_k and \bar{d}_k and is given by $R_k(Q^*) = \sqrt{\frac{2c}{p}}$ using (10).

From the above discussion, solution procedure to find the optimal value of \tilde{d}_k and Q can be summarized in following steps.

Step 1: Formulate the single-period profit function \tilde{P} with respect to the fuzzy random demand \tilde{D} . As \tilde{P} itself is a FRV, first find its fuzzy expectation and then use the graded mean representation method to determine a moderate representative $G(\tilde{E}_p)$.

Step 2: Following condition must be satisfied for maximization of $G(\tilde{E}_p)$ at \tilde{d}_k .

$$G(\tilde{E}_p(\tilde{d}_{k-1})) < G(\tilde{E}_p(\tilde{d}_k)) > G(\tilde{E}_p(\tilde{d}_{k+1}))$$

Identify the optimal \tilde{d}_k^* as the best expert's opinion out of all the fuzzy observations for the variable \tilde{D} .

Step 3: As \tilde{d}_k^* is available in linguistic phrase, DM's choice of the unambiguous optimal solution Q must lie between the support $[d_k, \bar{d}_k]$ of \tilde{d}_k . Formulate a dummy profit function $\tilde{\Pi}(Q)$.

Step 4: Use graded mean integration representation method to defuzzify the fuzzy profit function $\tilde{\Pi}(Q)$. Compute the expected profit $G(\tilde{\Pi}(Q))$ and then use Proposition 2 to compute the ultimate Q^* .

Step 5: Computed (\tilde{d}_k^*, Q^*) is the optimal solution of this model.

Thus, we can determine the required optimal order quantity for a single-period 'newsvendor type' inventory model under mixed uncertainty of customer demand. A detailed application of the proposed fuzzy-stochastic inventory model is illustrated below with a numerical example.

III. NUMERICAL EXAMPLE

To illustrate the above hybrid inventory system, let us consider the following data: $c = \$4$ per unit purchase cost, $p = \$6$ per unit selling price, $h = \$2$ per unit holding cost, $s = \$5$ per unit short and the customer demand is prescribed fuzzy randomly i.e.; the values of \tilde{D} are not numeric but linguistic and are characterized by the phrases "demand is about 30", "around 35", "nearly 50" etc.

For this purpose, the experts are asked about their opinion for the variable \tilde{D} . Assume that out of fifty experts 3 have been "around 20", 6 "about 25", 9 "about 30", 11 "around

35", 10 "around 40", 7 "more or less 45" and 4 "nearly 50" with support contained in $[15, 55]$. Table I depicts the feature of \tilde{D} .

TABLE I
INPUT DATA OF CUSTOMER DEMAND \tilde{D}

i	1	2	3	4	5	6	7
Demand	(15,20,25)	(20,25,30)	(25,30,35)	(30,35,40)	(35,40,45)	(40,45,50)	(45,50,55)
Prob.	0.06	0.12	0.18	0.22	0.20	0.14	0.08

The solution procedure proceeds as follows. Here the least information for demand \tilde{D} is $\tilde{d}_1 = (15, 20, 25)$ and the upper most information is $\tilde{d}_7 = (45, 50, 55)$. So the optimum value of \tilde{d}_k ; $k \in \{1, 2, \dots, 7\}$ must be obtained from (6).

Since $(p - c + s)/(p + h + s) = 0.5385$, substituting the values of p_i ($i=1$ to n) into (6) we obtain $\sum_{i=1}^4 p_i = 0.58$.

That is, the inequalities of (6) hold for $k = 4$. Hence the optimum $\tilde{d}_k^* = (30, 35, 40)$ is the best information among the entire expert-opinions.

We now obtain the required Q^* from this selected \tilde{d}_k^* using the results of (9) and (10) and Proposition 2. Since $2c - p = 2 > 0$, the optimum Q^* lies between 30 and 35 and is given by $L(Q^*) = 0.8165$. Consequently, the value of optimal quantity $Q^* = 34.08$ and the associated expected resultant profit $G(\tilde{\Pi}(Q^*)) = 65.44$. Thus we can conclude that the optimal solution $(\tilde{d}_k^*, Q^*) = ((30, 35, 40), 34.08)$ can be achieved when the customer demand is characterized fuzzy randomly.

IV. CONCLUSION

Incorporating fuzziness in single-period inventory modeling is of interest not only in theoretical contexts but also in practical contexts. There are numerous goods such as fashion items of seasonal changes, innovative technology goods (e.g. mobile phones, personal computers), new newspapers and magazines etc. which is of great economic importance and input parameters (demand and cost data) for modeling for these goods is available only in linguistic expressions. The paper develops a SPIM in presence of fuzzy random demand to determine an unambiguous optimal solution from a set of n fuzzy observations. In SPIM, the optimization totally depends on the attitude of the DM. That is, the manner of investigation to derive a significant Q^* may vary from DM to DM if there is any knowledge-based parameter what is put into it. In subsection 2, we considered a simple profit function in

determining Q^* , however, one may include the associated cost parameters into the profit function. Again, instead of fuzzy probability of a fuzzy event, we have considered crisp probability as the chance. Fortunately, these considerations do not affect the solution procedure of the model.

Finally, we conclude that in such fuzzy random phenomena, the DM can select one of the expert's opinions as the best and as it fluctuates in a range of possibilities, therefore, a compromise decision can also be suggested. This extension is essential not only in theoretical context, but also from a managerial point of view.

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