On Problem of Parameters Identification of Dynamic Object

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Abstract—In this paper, some problem formulations of dynamic object parameters recovery described by non-autonomous system of ordinary differential equations with multipoint unshared edge conditions are investigated. Depending on the number of additional conditions the problem is reduced to an algebraic equations system or to a problem of quadratic programming. With this purpose the paper offers a new scheme of the edge conditions transfer method called by conditions shift. The method permits to get rid from differential links and multipoint unshared initially-edge conditions. The advantage of the proposed approach is concluded by capabilities of reduction of a parametric identification problem to essential simple problems of the solution of an algebraic system or quadratic programming.

Keywords—dynamic objects, ordinary differential equations, multipoint unshared edge conditions, quadratic programming, conditions shift

I. PROBLEM FORMULATION

LET's consider a problem of parameters identification of a linear non-autonomous dynamic system:

$$\dot{x}(t) = A(t)x(t) + B(t)p + C(t), \quad t \in [t_0, t_k], \tag{1}$$

where $x(t) \in E^n$ - phase state of system; $p \in E^l$ - required parameters; $A(t), B(t), C(t)$ - given matrixes with dimensions $(n \times n), (n \times l), (n \times 1)$ respectively, moreover $A(t) \not\equiv const$.

There are m initially-edge conditions of a system that can also depend on unknown parameters:

$$\sum_{\nu=0}^{k} \hat{\alpha}^{\nu} x(\hat{t}_\nu) + \hat{\xi} p = \hat{\beta}, \tag{2}$$

where $\hat{t}_\nu, \nu = 0,1,...,k$, -times moments from $[t_0, t_k]$, $\hat{t}_0 = t_0, \hat{t}_k = t_k$, the matrixes $\hat{\alpha}^\nu, \hat{\xi}, \hat{\beta}$ with respective dimensions $(m \times n), (m \times l), (m \times 1)$ are given.

Let's mark a general problem of linear systems of differential equations with the multipoint unshared edge conditions. The problem (1), (2), generally speaking, concerns to this class of problems at fixed values of parameter p. The problem is connected to the complexity of obtaining of constructive necessary and sufficient conditions of the solution existence of a boundary value problem such as (1), (2), that is studied by many scientists, starting from activities of Tamarkin, Valle-Poussin and other scientists ([1], [3]).

Let

$$rang\ A(t) = n, \quad rang\ B(t) = max(n,l), \quad t \in [t_0, t_k],$$

$$rang[\hat{\alpha}^0,...,\hat{\alpha}^k, \xi] = rang[\hat{\alpha}^0,...,\hat{\alpha}^k, \xi, \hat{\beta}] = m.$$

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Depending on a ratio between values of matrixes ranks, participating in (1), (2), the following cases, corresponding to the different problem formulations, are possible.

Case A: $m = n + l$. Then there is a single vector of parameters p and corresponding solution of a boundary value problem (1), (2) (problem A).

Case B: $m < n + l$. Then the system vector of parameters, satisfying (1), (2), is not unique and there are additional conditions on system parameters and status in the form of equality with the number no more than $n + l - m$ and inequality

$$\sum_{j=0}^{k_1} e^j x(\breve{t}_j) + fp \genfrac{}{}{0pt}{}{\leq}{=} g, \tag{3}$$

the total number which one let will be equal m_1. In that case the choice of optimal parameter values can be performed according to any criterion. For example, as criterion of parametrs optimization can be used the minimized functional:

$$J(p) = \sigma_1 \|p\|_{E^l} + \sum_{j=0}^{k_2} \sigma_{2j} \|x(\bar{t}_j) - X^j\|_{E^n}. \tag{4}$$

Here $\sigma_1, \sigma_{2j}, \quad j = 0,1,...,k_2$ - positive weight coefficients; matrixes $e^j, j = 0,...,k_1, f, g$ with dimention $(m_1 \times n), (m_1 \times l), (m_1 \times 1)$ respectively; the time moments $\breve{t}_i \in [t_0, t_k], \quad \bar{t}_j \in [t_0, t_k]$ and desired system status X^j on moment $\bar{t}_j$, $i = 0,1,...,k_1, \quad j = 0,1,...,k_2$ are given (problem B).

In case $m > n + l$, i.e. the number of linearly - independent initially-edge conditions exceeds the number of conditionally free parameters of a dynamic system, generally, as is known, boundary value problem (1), (2) will not have the solution at any value of vector of parameters p.

II. THE SOLUTION OF PROBLEM A

For a numerical solution of a problem A the following scheme of the transfer (shift) of multipoint unshared edge conditions (2), offered in [1,3], will be used.

Let's consider an expression at interval $[t_0, \hat{t}_1]$:

$$\alpha^0(t)x(t) + \sum_{\nu=1}^{k} \alpha^\nu(t)x(\hat{t}_\nu) + \xi(t)p = \beta(t), \tag{5}$$

where $x(t)$ is required solution of the boundary problem, the matrix functions $\alpha^\nu(t), \xi(t), \beta(t)$ with dimentions $(m \times n), (m \times l), (m \times 1)$ are still arbitrary satisfying only conditions:

$$\alpha^\nu(t_0) = \hat{\alpha}^\nu, \quad \nu = 0,...,k, \quad \xi(t_0) = \hat{\xi}, \quad \beta(\alpha_0) = \hat{\beta}. \tag{6}$$

The expression (5) at $t = t_0$ coincides with conditions (2). The matrix functions $\alpha^\nu(t)$, $\nu = 0,...,k$, $\xi(t)$, $\beta(t)$, satisfying a ratio (5) at $t \in [t_0, \hat{t}_1]$, are not unique. Let's speak, that they do shift of conditions (2) to the right, since the ratio (5) at $t = \hat{t}_1$ becomes:

$$\left[\alpha^0(\hat{t}_1) + \alpha^1(\hat{t}_1)\right]x(\hat{t}_1) + \sum_{\nu=2}^{k} \alpha^\nu(\hat{t}_1)x(\hat{t}_\nu) + \xi(\hat{t}_1)p = \beta(\hat{t}_1) ,$$

and after redenotation

$$\hat{\alpha}^1 = \alpha^0(\hat{t}_1) + \alpha^1(\hat{t}_1), \quad \hat{\alpha}^\nu = \alpha^\nu(\hat{t}_1),$$
$$\nu = 2,...,k, \quad \hat{\xi} = \xi(\hat{t}_1), \quad \hat{\beta} = \beta(\hat{t}_1)$$

we will get condition:

$$\sum_{\nu=1}^{k} \hat{\alpha}^\nu x(\hat{t}_\nu) + \hat{\xi}p = \hat{\beta} . \tag{7}$$

The condition (7) is equivalent (2), but differs from (2) by that in (7) values of a required trajectory $x(t)$ in the most left point $t = t_0 = \hat{t}_0$ will not be used.

Having repeated a similar procedure with a condition (7) on the following interval $[\hat{t}_1, \hat{t}_2]$ with the help of some matrix functions $\alpha^\nu(t)$, $\nu = 1,...,k$, $\xi(t)$, $\beta(t)$ it is possible to receive conditions, equivalent to (7), but not keeping values required trajectory $x(t)$ at $t = \hat{t}_1$. Step-by-step continuing shift to the right of edge conditions sequentially on subinterval $[t_{i-1}, t_i]$, $i = 2,...,k$ at the end we shall receive m ratio:

$$\hat{\alpha}^k x(t_k) + \hat{\xi}p = \hat{\beta} , \tag{8}$$

where the dimension of vector of unknowns $(x(t_k), p)$ is $n + l$. Considering, that for a problem A $m = n + l$, it is possible, having solved a system (8) to define $x(t_k)$ and the parameters vector p, then having solved a Cauchy problem (1), (2) from t_k up to t_0 to determine $x(t)$. By doing this the solution of a problem A is completed.

It is necessary to solve the problem of selection of matrix functions $\alpha^\nu(t)$, $\xi(t)$, $\beta(t)$ executing step-by-step shift of edge conditions. As it was already marked, they are not unique. In particular as such matrix functions it is possible to use that are adduced in the following theorem.

It is remain to solve the problem of selection of matrix functions $\alpha^\nu(t)$, $\xi(t)$, $\beta(t)$, executing step-by-step shift of edge conditions. As was noted above, they are not unique. In particular, as such matrix functions it is possible to use those given in the following theorem.

Theorem 1. Let functions $\alpha^\nu(t)$, $\nu = 0,...,k$, $\xi(t)$, $\beta(t)$ are determined by solution of following non-linear Cauchy problems:

$$\dot{\alpha}^0(t) = S^0(t)\alpha^0(t) - \alpha^0(t)A(t), \qquad \alpha^0(t_0) = \hat{\alpha}^0, \tag{9}$$

$$\dot{M}(t) = S^0(t)M(t), \qquad M(t_0) = I, \tag{10}$$

$$\dot{\xi}(t) = S^0(t)\xi(t) - \alpha^0(t)B(t), \qquad \xi(t_0) = \hat{\xi}, \tag{11}$$

$$\dot{\beta}(t) = S^0(t)\beta(t) + \alpha^0(t)C(t), \qquad \beta(t_0) = \hat{\beta}, \tag{12}$$

$$\alpha^\nu(t) = M(t)\hat{\alpha}^\nu, \qquad \nu = 1,...,k \quad , \tag{13}$$

where

$$S^0(t) = (\alpha_0 A \alpha_0^T + \alpha_0 B \xi^T - \alpha_0 C \beta^T)(\alpha_0 \alpha_0^T + \xi\xi^T + \beta\beta^T)^{-1},$$
$$rang\, \hat{\alpha}^0 = m,$$

T is the sign of matrix transposition, I is $m-$dimentional unit matrix. Then these matrix functions execute shift of conditions (2) to the right on an interval $[\hat{t}_0, \hat{t}_1]$, i.e. for them the (15), (7) is executed. Moreover, it takes place:

$$\left\|\alpha^0(t)\right\|^2 + \left\|\xi(t)\right\|^2 + \left\|\beta(t)\right\|^2 = const, \quad t \in [\hat{t}_0, \hat{t}_1], \tag{14}$$

whence follows the stability of a Cauchy problem (9) - (12) .

Proof. Let for $x(t)$, being solution of a problem (1), (2), the ratio takes place:

$$\alpha^0(t)x(t) + \sum_{\nu=1}^{k} \hat{\alpha}^\nu x(t_\nu) + \xi(t)p = \beta(t), \quad t \in \left[t_0, \hat{t}_1\right], \tag{15}$$

where $\alpha^0(t)$, $\xi(t)$, $\beta(t)$ while arbitrary differentiable functions. Let's differentiate (15) and take into account (1) (for short, argument t on functions is omitted):

$$\dot{\alpha}^0 x + \alpha^0 \dot{x} + \dot{\xi}p = \dot{\beta},$$
$$\dot{\alpha}^0 x + \alpha^0 A x + \alpha^0 B p + \alpha^0 C + \dot{\xi}p = \dot{\beta}.$$

Let's conduct a grouping:

$$(\dot{\alpha}^0 + \alpha^0 A)x + (\dot{\xi} + \alpha^0 B)p + (-\dot{\beta} + \alpha^0 C) = 0.$$

Using apparently checked by direct permutation in a problem conditions:

$$x(t) \not\equiv 0, \quad t \in [t_0, t_1], \quad p \neq 0$$

and arbitrary of functions $\alpha^0(t)$, $\xi(t)$, $\beta(t)$, we shall demand fulfilment of equalling to zero the expressions in brackets:

$$\dot{\alpha}^0 = -\alpha^0 A, \quad \dot{\xi} = -\alpha^0 B, \quad \dot{\beta} = \alpha^0 C, \tag{16}$$

provided that

$$\alpha^0(t_0) = \hat{\alpha}^0, \quad \xi(t_0) = \hat{\xi}, \quad \beta(t_0) = \hat{\beta}.$$

Let's multiply (15) by an arbitrary matrix function $M(t)$ of dimension $(m \times m)$ such, that

$$M(t_0) = I, \tag{17}$$

$$M(t)\alpha^0(t)x(t) + \sum_{\nu=1}^{k} M(t)\hat{\alpha}^\nu x(\hat{t}_\nu) + \\ + M(t)\xi(t)p = M(t)\beta(t) . \tag{18}$$

Denoting

$$G_1(t) = M(t)\alpha^0(t), \quad G_2(t) = M(t)\xi(t),$$
$$G_3(t) = M(t)\beta(t), G_1^\nu(t) = M(t)\hat{\alpha}^\nu, \tag{19}$$
$$\nu = 1,...,k,$$

we obtain:

$$G_1(t)x(t) + \sum_{\nu=1}^{k} G_1^\nu(t)x(\hat{t}_\nu) + G_2(t)p = G_3(t),$$

$$G_1(t_0) = \hat{\alpha}^0, \quad G_2(t_0) = \hat{\xi}, \quad G_3(t_0) = \hat{\beta}, \tag{20}$$
$$G_1^\nu(t_0) = \hat{\alpha}^\nu, \quad \nu = 1,...,k$$

Let $M(t)$ satisfy the equality:

$$\|G_1(t)\|^2 + \|G_2(t)\|^2 + \|G_3(t)\|^2 = const .$$

Then

$$\dot{G}_1 G_1^T + G_1 \dot{G}_1^T + \dot{G}_2 G_2^T + G_2 \dot{G}_2^T + \dot{G}_3 G_3^T + G_3 \dot{G}_3^T = 0 . \quad (21)$$

Differentiating equation (19) and considering (16), we will receive:

$$\dot{G}_1 = \dot{M}\alpha^0 + M\dot{\alpha}^0 = \dot{M}M^{-1}G_1 - M\alpha^0 A = \dot{M}M^{-1}G_1 - G_1 A, \quad (22)$$

$$\dot{G}_2 = \dot{M}\xi + M\dot{\xi} = \dot{M}M^{-1}G_2 - M\alpha^0 B = \dot{M}M^{-1}G_2 - G_1 B, \quad (23)$$

$$\dot{G}_3 = \dot{M}\beta + M\dot{\beta} = \dot{M}M^{-1}G_3 + M\alpha^0 C = \dot{M}M^{-1}G_3 + G_1 C . \quad (24)$$

Transposing (22) - (24), we'll receive:

$$\dot{G}_1^T = G_1^T (M^{-1})^T \dot{M}^T - A^T G_1^T , \quad (25)$$

$$\dot{G}_2^T = G_2^T (M^{-1})^T \dot{M}^T - B^T G_1^T , \quad (26)$$

$$\dot{G}_3^T = G_3^T (M^{-1})^T \dot{M}^T + C^T G_1^T . \quad (27)$$

Allowing expressions (22) - (27) in (21), after simple transformations we'll receive:

$$\left[\dot{M}M^{-1}G_1 G_1^T - G_1 A G_1^T + G_1 G_1^T (M^{-1})^T \dot{M}^T - G_1 A^T G_1^T \right] +$$

$$\left[\dot{M}M^{-1}G_2 G_2^T - G_1 B G_2^T + G_2 G_2^T (M^{-1})^T \dot{M}^T - G_2 B^T G_1^T \right] +$$

$$\left[\dot{M}M^{-1}G_3 G_3^T + G_1 C G_3^T + G_3 G_3^T (M^{-1})^T \dot{M}^T + G_3 C^T G_1^T \right] = 0 ,$$

$$\left[\dot{M}M^{-1}(G_1 G_1^T + G_2 G_2^T + G_3 G_3^T) + (-G_1 A G_1^T - \right.$$

$$\left. - G_1 B G_2^T + G_1 C G_3^T) \right]$$

$$\left[\dot{M}M^{-1}(G_1 G_1^T + G_2 G_2^T + G_3 G_3^T) + (-G_1 A G_1^T - G_1 B G_2^T + G_1 C G_3^T) \right]^T$$

$$= 0$$

From independence and arbitrary of matrixes G_1, G_2, G_3 for fulfilment of this equation it is enough to demand from $M(t)$, that the expressions in square brackets equals zero. Then

$$\dot{M}M^{-1} = (G_1 A G_1^T + G_1 B G_2^T - G_1 C G_3^T)(G_1 G_1^T +$$

$$+ G_2 G_2^T + G_3 G_3^T)^{-1} \equiv S^0 .$$

Here the notation of a right part of differential equation by S^0 is introduced. Then

$$\dot{M} = S^0 M . \quad (28)$$

Differentiating equation (19) and considering (28), (16), we'll receive:

$$\dot{G}_1 = \dot{M}\alpha^0 + M\dot{\alpha}^0 = S^0 M\alpha^0 - M\alpha^0 A = S^0 G_1 - G_1 A, \quad (29)$$

$$\dot{G}_2 = \dot{M}\xi + M\dot{\xi} = S^0 M\xi - M\alpha^0 B = S^0 G_2 - G_1 B, \quad (30)$$

$$\dot{G}_3 = \dot{M}\beta + M\dot{\beta} = S^0 M\beta + M\alpha^0 C = S^0 G_3 + G_1 C. \quad (31)$$

Adding the condition (20), we'll receive

$$G_1(t_0) = \hat{\alpha}_0 , \quad G_2(t_0) = \hat{\xi} , \quad G_3(t_0) = \hat{\beta} . \quad (32)$$

Again having renamed matrix functions $G_1(t)$ through $\alpha^0(t)$, $G_1^\nu(t)$ through $\alpha^\nu(t)$, $\nu = 1,...,k$, $G_2(t)$ through $\xi(t)$, and $G_3(t)$ through $\beta(t)$, we shall receive functions, executing shift of conditions (2) to the right, about which one there is a speech in the theorem.

Remark. It is important to note following. As it was indicated above, matrix functions executing shift of conditions (2), are not determined uniquely. For example, the functions defined by linear Cauchy problems (16), formally meet

definition (15) of shifting to the right of initially-edge conditions (2), but as is known [1] one of linear problems

$$\dot{x} = Ax + f(t) , \quad x(t_0) = x_0 ,$$

$$\dot{\alpha}^0 = -A\alpha^0 , \quad \alpha^0(t_0) = \hat{\alpha}^0 ,$$

or both simultaneously depending on eigenvalues of a matrix A are unstable. The fulfillment of a condition (14) provides that the auxiliary Cauchy problems (9) - (12) will have stable solution and it is very important at realization of practical calculations.

The proof method of the theorem can easily be applied to particular cases of condition (14):

$$\|\alpha^0(t)\|^2 \equiv \|\hat{\alpha}^0\|^2 = const ,$$

$$\|\alpha^0(t)\|^2 + \|\beta(t)\|^2 \equiv \|\hat{\alpha}^0\|^2 + \|\hat{\beta}\|^2 = const ,$$

$$\|\alpha^0(t)\|^2 + \|\xi(t)\|^2 \equiv \|\hat{\alpha}^0\|^2 + \|\hat{\xi}\| = const ,$$

under which the condition of the theorem will differ only by kind of a function $S^0(t)$.

At definite specificity of conditions (2), for example, when a part m_0 from conditions (2) $(m_0 < m)$ have local nature and are determined on the left-hand end of an interval at $t = t_0$:

$$\overline{\eta}x(t_0) + \overline{\xi}p = \overline{\gamma} , \quad (33)$$

where $\overline{\eta}, \overline{\xi}, \overline{\gamma}$ are given matrixes of dimension $(m_0 \times n), (m_0 \times l), (m_0 \times 1)$ respectively, more effective is the implementation of a left-shift of conditions (2), beginning from an interval $[\hat{t}_{k-1}, \hat{t}_k]$, the number which one is in this case equal $m = n - m_0 + l$. In this case instead of (5) we'll consider a ratio on a section $[\hat{t}_{k-1}, \hat{t}_k]$:

$$\alpha^k(t)x(t) + \sum_{\nu=0}^{k-1} \alpha^\nu(t)x(\hat{t}_\nu) + \xi(t)p = \beta(t), \quad (34)$$

where $x(t)$ is the solution of a boundary value problem; $\alpha^\nu(t), \xi(t), \beta(t)$ are arbitrary matrixes of dimension $(m \times n), (m \times l), (m \times 1), \quad \nu = 0...k$, obeying:

$$\alpha^\nu(\hat{t}_k) = \hat{\alpha}^\nu, \quad \xi(\hat{t}_k) = \hat{\xi}, \quad \beta(\hat{t}_k) = \hat{\beta}, \quad \nu = 0...k \quad (35)$$

Let's speak, that the matrix functions execute shift of conditions (2) to the left, since from (34) at $t = \hat{t}_{k-1}$ we'll receive:

$$\sum_{\nu=0}^{k-1} \hat{\alpha}^\nu x(\hat{t}_\nu) + \hat{\xi}p = \hat{\beta} , \quad (36)$$

here following redesignations are used:

$$\hat{\alpha}^{k-1} = \alpha^k(\hat{t}_{k-1}) + \alpha^{k-1}(\hat{t}_{k-1}),$$

$$\hat{\alpha}^\nu = \alpha^\nu(\hat{t}_{k-1}), \quad \nu = 0,...,k-2 ,$$

$$\hat{\xi} = \xi(\hat{t}_{k-1}) .$$

The conditions (36) and (2) are equivalent, but in (36) the value of a required function in the most right point $x(\hat{t}_k)$ does not participate:. Further, repeating a left-shift of conditions on series time frames $[\hat{t}_{s-1}, \hat{t}_s], \quad s = k-1, k-2,...,1$, in the end we shall receive locally given condition on the left-hand end:

$$\hat{\alpha}^0 x(t_0) + \hat{\xi} p = \hat{\beta} \qquad (37)$$

From $(n+l)$ conditions (33), (37) it is possible to define $(n+l)$ of unknowns $x_0 = x(t_0) \in E^l$ and $p \in E^n$, then to solve a Cauchy problem concerning (1) with the obtained initial conditions $x(t_0)$ and already known values of parameters vector p in a right part of (1).

The template functions $\alpha^\nu(t), \nu = 0,...,k, \xi(t), \beta(t)$ executing shift of conditions (2) to the left are not unique, the following theorem takes place.

Theorem 2. Functions $\alpha^\nu(t), \quad \nu = 0,...,k, \quad \xi(t), \quad \beta(t)$ defined by the solution of following non-linear Cauchy problems:

$$\dot{\alpha}^k(t) = S^k(t)\alpha^k(t) - \alpha^k(t)A(t), \qquad \alpha^k(t_0) = \hat{\alpha}^k,$$

$$\dot{M}(t) = S^k(t)M(t), \qquad M(\hat{t}_k) = I,$$

$$\dot{\xi}(t) = S^k(t)\xi(t) - \alpha^k(t)B(t),$$

$$\dot{\beta}(t) = S^k(t)\beta(t) + \alpha^k C(t),$$

$$\alpha^\nu(t) = M(t)\hat{\alpha}^\nu, \qquad \nu = 0,...,k-1 \qquad ,$$

$$S^k(t) = (\alpha^k A \alpha^{kT} + \alpha^k B \xi^T - \alpha^k C \beta^T)(\alpha^k \alpha^{kT} + \xi \xi^T + \beta \beta^T)^{-1},$$

execute shift of conditions (2) to the left on a section $[\hat{t}_{k-1}, \hat{t}_k]$, and takes place

$$\left\|\alpha^k(t)\right\|^2 + \left\|\xi(t)\right\|^2 + \left\|\beta(t)\right\|^2 = const, \quad t \in [\hat{t}_{k-1}, \hat{t}_k],$$

III. THE SOLUTION OF PROBLEM B

For the solution of problem B it is basically used the procedure of conditions shift (2), described above, permitting to get rid from differential links (1) and to reduce a problem of parameters optimization to a problem of quadratic programming.

Let's introduce new variables:

$$z_\nu^1 = x(\hat{t}_\nu), \quad \nu = 0,...,k, \quad z_j^2 = x(\check{t}_j), \quad j = 0,...,k_1,$$

$$z_i^3 = x(\bar{t}_i), \quad i = 0,...,k_2,$$

$$Z = (z^1, z^2, z^3) \in E^N, \quad N = (k + k_1 + k_2 + 3)n. \qquad (38)$$

It is clear that executing series shift to the right of conditions (2) at intervals $[\hat{t}_{j-1}\hat{t}_j), j = 1,...,k$, i.e. solving Cauchy problems (9) - (12) and determining values of matrix functions in time moments $\check{t}_j, \bar{t}_i, j = 0,...,k_1, i = 0,..,k_2$ in addition to $(k + 1)m$ relations of a kind (7):

$$\sum_{\nu=j}^{k} \alpha^\nu(\hat{t}_\nu) z_\nu^1 + \xi(\hat{t}_j)p = \beta(\hat{t}_j), \quad j = 0,...,k, \qquad (39)$$

we receive the $(k_1 + 1)m$ and $(k_2 + 1)m$ relations:

$$\alpha^{\mu_j}(\check{t}_j)z_j^2 + \sum_{\nu=\mu_j+1}^{k} \alpha^\nu(\check{t}_j)z_\nu^1 + \xi(\check{t}_j)p = \beta(\check{t}_j), \quad j = 0,...,k_1, (40)$$

$$\alpha^{\chi_i}(\bar{t}_i)z_i^3 + \sum_{\nu=\chi_i+1}^{k} \alpha^\nu z_\nu^1 + \xi(\bar{t}_i)p = \beta(\bar{t}_i), \quad i = 0,...,k_2, \quad (41)$$

where μ_j, χ_i are the numbers of subintervals, keeping instants accordingly $\check{t}_j$ and $\bar{t}_i$, i.e.

$$\check{t}_j \in [\hat{t}_{\mu_j}, \hat{t}_{\mu_{j+1}}), \quad \bar{t}_i \in [\hat{t}_{\chi_i}, \hat{t}_{\chi_{i+1}}), \quad j = 0,...,k_1, \quad i = 0,...,k_2$$

It is clear, that the number of restrictions as equalities in (34) - (36) equals $M = (k + k_1 + k_2 + 3)m$.

Using notations (38), limitations (3) and target functionals (4) we'll write as:

$$\sum_{j=0}^{k_1} e^j z_j^2 + fp\left(\genfrac{}{}{0pt}{}{\leq}{=}\right)g, \qquad (42)$$

$$J(Z, p) = \sigma_1 \|p\|_{E^l}^2 + \sum_{j=0}^{k_2} \sigma_{2j} \left\|z_j^3 - X^j\right\|_{E^n}^2. \qquad (43)$$

The problem (39) - (43) is a problem of quadratic programming with optimized vector (Z, p) of dimension $N + l$ and the number of limitations as equalities (39) - (41) is M and mixed type (37) is m_1, that generally can be presented as:

$$RZ + Fp\left(\genfrac{}{}{0pt}{}{\leq}{=}\right)T,$$

$$J(Z, p) = \sigma \|p\|_{E^l}^2 + \sigma_2 \|Z - Q\|_{E^N}^2,$$

where the matrixes R, F and vectors T, Q are formed by limitations (39) - (42) with considering notation (38).

Thus, for a numerical solution of a problem B it is necessary to execute following. Using the numerical methods to solve Cauchy problems (9) - (12), to store values of matrix functions in all instants, participating in a formulation of problem B, namely:

$$\hat{t}_j, \check{t}_i, \bar{t}_\nu, \quad j = 0,...,k, \quad i = 0,...,k_1, \quad \nu = 0,...,k_2.$$

Then it's necessary to form matrixes R, F, vector T for the reference to the standard programs of the solution of a quadratic programming problem. Having received values of vector p and $x(t_0) = z^1(\hat{t}_0)$, it is possible numerically to decide a Cauchy problem concerning a system (1) and to receive values $x(t)$ on all interval $t \in [t_0, t_k]$ and to complete the solution of a problem B.

As example, we shall consider outcomes of parameters recovery in a following problem:

$$\dot{x}_1 = 3tx_2 + t^2 p_1 + 3tp_2 + 4t, \quad t \in [0;2],$$

$$\dot{x}_2 = tx_1 - 2t^2 x_2 - tp_1 - 2t^2 p_2 - 2t - 1,$$

with multipoint boundary conditions

$$2x_1(0) - x_2(0) + x_2(2) - 2p_1 = 6,$$

$$x_2(0) + x_1(2) - p_1 + p_2 = 12,$$

with minimized criteria of quality:

$$J_\sigma(p) = \sigma(p_1^2 + p_2^2) + (x_1(1) - 1)^2 + x_2^2(1).$$

In the table 1 the problem solution results at different values of parameter σ are adduced. Let's mark, that the precise solution of a problem at $\sigma = 0$, as is simply to verify, is reached at value of parameters $p^* = (-3;1)$, at which $x_1(t) = 2t^2 - 1, x_2(t) = t - 1$. The numerical solution of Cauchy

problems was conducted singleprecise by the fourth order Runge-Kutta method at number of splittings equal 100.

TABLE I
THE RESULTS OF THE PROBLEM SOLUTION AT DIFFERENT
VALUES OF PARAMETERS σ

NN	σ	p_1^*	p_2^*	$J_\sigma(p^*)$
1	1	-1.49770	0.50174	4.98484
2	0.1	-2.72551	0.89845	0.91156
3	0.01	-2.97014	0.98872	0.11097
4	0	-3.00240	1.00057	0.0001042

In a problem of quadratic programming obtained after series two shifts in edge conditions to the right, the vector $Z = (x(1), x(2), p) \in E^6$ is unknown and there are four limitations as equalling: two of them are obtained after the first shift:

$$\hat{\alpha}^1 x(1) + \hat{\alpha}^2 x(2) + \hat{\xi}p = \hat{\beta},$$

and two after the second shift:

$$\hat{\hat{\alpha}}^1 x(2) + \hat{\hat{\xi}}p = \hat{\hat{\beta}}.$$

Four problems of quadratic programming with the same limitations, but different target functions at the expense of values of σ were decided. As a whole on all variants of calculation on IBM the Pentium-I was required 7 seconds.

IV. CONCLUSION

With the application of the approach, offered in paper, the plenty of numerical experiments concerning solving the problem A, B is conducted.

With usage of a method of a linearization offered by Pshenichniy B.N. [4], the stated technique utilised also for a numerical solution of a non-linear problem of a dynamic system parameters recovery:

$$\dot{x}(t) = f(x(t), p), \quad t \in [t_0, t_k],$$

$$g_j(x(\hat{t}_0), x(\hat{t}_1),..., x(\hat{t}_k), p) = 0, \quad j = 1,...,n+l.$$

It is necessary to mark convenience of application of the offered approach. Its programmatic implementation does not produce problems, as basically the standard procedures, solving of Cauchy problem, system of algebraic equations and problems of quadratic programming, available, in particular, in software package Matlab, will be used.

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