# On Maneuvering Target Tracking with Online Observed Colored Glint Noise Parameter Estimation

M. A. Masnadi-Shirazi, and S. A. Banani

**Abstract**— In this paper a comprehensive algorithm is presented to alleviate the undesired simultaneous effects of target maneuvering, observed glint noise distribution, and colored noise spectrum using online colored glint noise parameter estimation. The simulation results illustrate a significant reduction in the root mean square error (RMSE) produced by the proposed algorithm compared to the algorithms that do not compensate all the above effects simultaneously.

**Keywords**—Glint noise, IMM, Kalman Filter, Kinematics, Target Tracking.

### I. INTRODUCTION

 $\mathbf{I}^{ ext{N}}$  1995, Daeipour and Bar-Shalom utilized the IMM algorithm to implement the glint noise model in nonmaneuvering target tracking [1]. They applied two extended Kalman filters, one matched to the dynamic system with Gaussian measurement noise, and the other matched to the same dynamic system but with a high variance Laplacian noise. Later in 1998, E. Daeipour, and et al. in [2], and K. Heydari and et al. in [3]-[4], almost concurrently, extended the algorithm used in [1] to maneuvering targets. They applied a layered IMM (LIMM) algorithm to implement the target maneuvering model as well as the glint noise model. Although they pursued the same goal, they were different in methodology. The algorithm in [2] was based upon the extension of [1] with the dynamic system state equations in Cartesian and observation equations in spherical coordinates. But, the one in [3]-[4] was developed with the dynamic system state equations and observation equations both in spherical coordinates. In both [2] and [3]-[4] the filters were split into two parts, one matched to the Gaussian component and the other matched to the Laplacian component of the glint noise. However, in the former the extended Kalman filters (EKF) were exploited to deal with the Gaussian and Laplacian noise components, but in the latter, linear Kalman filter was

used for the Gaussian noise and Masrelize filter with efficient approximate score function ([5] and [6]) was used for the Laplacian noise to filter the components of the glint noise.

In all the aforementioned methods, the observation noise spectrum was assumed to be white. However, in high frequency measurement radar systems the successive samples of the measurement noise are not uncorrelated, and consequently the observed noise spectrum is not white. In 1996 Wu and Chang presented the subject of maneuvering target tracking with observed colored noise and unknown parameters [7]. The drawback of this approach was the assumption of Gaussian noise distribution rather than glint distribution. Earlier, W. R Wu in [8] had reported a maximum likelihood approach to identify the glint noise parameters from recorded data. However, he assumed the spectrum of the glint noise to be white rather than colored, and also ignored maneuvering effects of the target in noise parameter estimation.

In this paper a comprehensive algorithm is developed to recursively provide an online estimate of the colored glint noise parameters and cope with simultaneous effects of the following main four factors that may degrade the optimality of the Kalman filter in target tracking. The four degrading factors are: maneuvering of the target, glint and colored characteristics of the observation noise and lack of knowledge about the noise parameters.

# II. PROBLEM FORMULATION

### A. Target Model and System Equations

In radar target tracking the dynamic state equations are expressed in rectangular coordinate, while the observation equations are measured in spherical coordinate. Converting either of them to the other one will result in nonlinear equations.

One approach is to convert the dynamic equations from rectangular to spherical coordinate, and use approximate linearized spherical model which encounters simultaneous solution of three complicated nonlinear differential equations [6, 8]. For instance the range channel is used to be represented here. Then the corresponding approximate spherical representation for the second-order dynamic model in nonmaneuvering mode is:

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$$\begin{bmatrix} r \\ \dot{r} \end{bmatrix}_{k+1} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}_k \begin{bmatrix} r \\ \dot{r} \end{bmatrix}_k + \begin{bmatrix} T^2 / 2 \\ T \end{bmatrix} w_k^r \tag{1}$$

where  $w_k^r$  is the zero mean white Gaussian noise. We get the third-order model as

$$\begin{bmatrix} r \\ \dot{r} \\ \ddot{r} \end{bmatrix}_{k+1} = \begin{bmatrix} 1 & T & \frac{1}{2}T^2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r \\ \dot{r} \\ \ddot{r} \end{bmatrix}_k + \begin{bmatrix} \frac{1}{6}T^3 \\ \frac{1}{2}T^2 \\ T \end{bmatrix} w_k^M \tag{2}$$

where  $w_k^M$  stands for zero mean white Guassian noise in maneuvering case.

In both cases, the received observation data in the range channel is

$$z_k^r = r_k + v_k \tag{3}$$

where  $v_k$  the observation Glint noise.

# B. IMM Algorithm and the System Model

In the proposed algorithm, two filter banks are used: one is matched to the Gaussian noise and the other matched to the Laplacian noise components of the observed data. The Kalman filter is used to deal with the Gaussian component, and the Masrelize filter with approximate score function is used to face the Laplacian component of the glint noise. With each of the above filter banks there are two Kalman filters with different dynamic systems (see Fig. 1). The Kalman filter matched to the constant velocity mode deals with the target system in nonmaneuvering conditions and the one which is matched to the constant acceleration mode deals with the target system in maneuvering case. The same is performed for the Masrelize filter. The IMM algorithm will combine the outputs of the four filters, and the resultant output is a weighted sum of all the subfilter outputs.

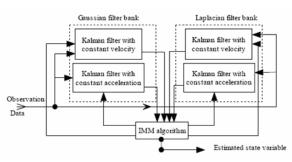


Fig. 1 Using Kalman filter and IMM algorithm to model the maneuver and Glint noise

For the Laplacian mode, the filtering and updating steps are performed by the approximate score function which are summarized in [[10], (39)-(45)]. These equations are only applicable to the glint white observation noise. Further

modifications has been performed in this paper to the case of glint colored noise including decorrelating the data before applying to the IMM algorithm.

# C. Colored Noise and Decorrelating Process

Since the observation noise parameters are unknown, one may not be able to use the conventional method of whitening process by incorporating the modal matrix corresponding to the noise. The other problem with using the conventional whitening process even with the case of known noise parameters is the ill-conditioned noise covariance matrix that one may encounter. Guu and Wei in [11] modeled the colored observation noise as an AR model with unknown parameters. Eventually Wu in [7] modeled the colored noise with Gaussian distribution as the following AR process to prevent all the above problems and get the benefit of less intensive computational effort:

$$v_k = \alpha v_{k-1} + \eta_k \tag{4}$$

 $v_k$  represents the colored noise output and  $\eta_k$  the white Gaussian noise input of the above AR process. The white noise  $\eta_k$  has zero mean and variance  $\sigma^2$ . The AR parameter  $\alpha$  is the unknown correlation parameter of the colored noise. To decorrelate the colored noise, we generate the following artificial measurement  $\bar{z}_k$  as suggested in [7]:

$$\bar{z}_k = z_k - \alpha z_{k-1} = \overline{H} r_k + \overline{\eta}_k \tag{5}$$

where

$$\overline{H} = H(I - \alpha \Phi^{-1}) \tag{6}$$

$$\overline{\eta}_k = \alpha \Phi^{-1} w_{k-1} + \eta_k \tag{7}$$

and I stands for the identity matrix. Since in practice  $\alpha\Phi^{-1}w_{k-1}$  in (7) is very small compared to  $\eta_k$ , the artificial data  $\overline{z}_k$  can be treated as a measurement data for the Kalman filter with white noise  $\overline{\eta}_k \approx \eta_k$  [7].

# D. Noise Parameter Estimation

As the radar tracking process advances, the parameters  $\alpha$ ,  $\varepsilon$ ,  $\sigma$ , and  $\mu$  are estimated recursively by the online observed data received in the radar detection mode. The estimation process is based on the method discussed in [8], where the parameters  $\varepsilon$ ,  $\sigma$  and  $\mu$  are estimated under the assumption of white glint observation noise. Thus, it requires using (4) to decorrelate the colored data; however, parameter  $\alpha$  in (4) must be first estimated. By receiving a new observed glint noise data at kth iteration, we generate the signal  $\overline{u}_k = z_k - 2z_{k-1} + z_{k-2}$  and pass it through the filter

$$F(z) = \frac{1}{(1 - \rho z^{-1})^2}$$
 to obtain the new signal  $u_k$  [7]. Under

nonmaneuvering mode if we select  $\rho = 1$ ,  $u_k$  will satisfy the following AR process:

$$u_k = \alpha u_{k-1} + \eta_k \tag{8}$$

This process is similar to (4) and the parameter  $\alpha$  can easily be estimated from

$$\hat{\alpha} = \frac{\hat{r}(1)}{\hat{r}(0)} \tag{9}$$

where  $\hat{r}(.)$  is the estimated autocorrelation function of  $u_k$  and can be evaluated from the following fading memory approach [8]:

$$\hat{r}_{k}(0) = \beta \hat{r}_{k-1}(0) + (1-\beta)u_{k}^{2}$$

$$\hat{r}_{k}(1) = \beta \hat{r}_{k-1}(1) + (1-\beta)u_{k}u_{k-1}$$
(10)

where  $0 < \beta < 1$  is the forgetting factor.

Choosing the best value of  $\rho$  has been discussed and simulated in [7]. Considering the simulation results in nonmaneuvering cases, the best estimation is with  $\rho$ =1. But in maneuvering condition  $\rho$ =1 will breakdown the algorithm. The corresponding simulation results show that as the value of  $\rho$  decreases the estimation of the glint noise parameters would be better, and the convergence would also be faster. Based on the discussion in [7] a good choice would be  $\rho$ =0.9, where the estimates are almost not affected by maneuvering. Therefore,  $\rho$ =0.9 could be an almost optimum value for the whole tracking period including maneuvering and nonmaneuvering cases.

However, with  $\rho \neq 1$ ,  $u_k$  is no longer similar to  $v_k$ , and (8) fails. Consequently (9) will be biased. After removing the bias,  $\alpha$  will be estimated as [7]:

$$\hat{\alpha} = \frac{-(\xi_3 + \xi_2) - \sqrt{(\xi_3 + \xi_2)^2 - 4\xi_3(\xi_3 + \xi_2 + \xi_1)}}{2\xi_3}$$
(11)

where,

$$\xi_{3} = (\rho + 1)^{3}$$

$$\xi_{2} = (-2\rho^{2} + 10\rho - 4) + (-6\rho - 2)\alpha^{b}$$

$$\xi_{1} = (-\rho^{3} - 3\rho^{2} + 5\rho + 7) + (8\rho + 8)\alpha^{b}$$

$$\xi_{0} = (2\rho^{2} + 2\rho - 4) + (-2\rho - 6)\alpha^{b}$$

$$\alpha^{b} = \frac{r(1)}{r(0)}$$

Having k samples of the glint white noise at iteration k, the iterative algorithm in [8] will yield the estimation of the other parameters, such as the glint noise parameters,  $\theta = (\varepsilon, \sigma, \mu)$ . However, in the system under study the real noise samples are glint but not white. We know that the noise in artificial measured data is white with glint distribution and it is the same as  $\eta_k$  in the AR process (8). Thus, the samples of  $\eta_k$  can be used to estimate the parameters  $\varepsilon$ ,  $\sigma$  and  $\mu$  of the glint noise instead of the samples of the colored glint noise

 $v_k$ . To perform the online process of the above parameter estimation, we generate k samples of the glint white noise  $\eta_k$  from (8) as follows:

$$\eta_k = u_k - \alpha u_{k-1} \tag{12}$$

In the first iterations of tracking where few observations are made, the onlline estimtion process of the glint noise parameters is performed with inadequate input information which results in large parameter estimation error. Consequently the corresponding tracking perforance will be poorer than the case with known noise parametr values. This has been shown in the simulation results. As the process proceeds, the number of observations used in the parameter estimators increases and the estimates get better. Consequently the tracking performance converges to its normal error performance in known noise parameters case.

### III. SUMMARY OF THE PROPOSED ONLINE ALGORITHM

- i) At each iteration, say the kth iteration, generate the signal  $u_k$  when the kth observed data,  $z_k$  is received as discussed in section 2.4. [7] and [11].
- ii) Estimate the parameter  $\alpha$  using (11).
- iii) Generate the kth sample of the glint white noise,  $\eta_k$  from (12) using estimated  $\alpha$  from step (ii).
- iv) Apply the preprocessor introduced in [8] to the samples of  $\eta_k$  to estimate the appropriate initial values of  $\theta = (\varepsilon, \sigma, \mu)$ .
- v) Estimate the parameters  $\theta = (\mathcal{E}, \sigma, \mu)$  using the algorithm discussed in [8]
- vi) Generate the artificial data  $\overline{z}_k$  from (5).
- vii) Apply the IMM algorithm in [3]-[4] to the artificial data  $\overline{z}_k$  for handling the problem of two independent discrete uncertainties (modes) of the tracking system: the target dynamic system, and the glint noise. The resultant output yields the estimated state variables.

**Remark:** In the standard IMM algorithm, updating the filter weights is set for the Gaussian noise. In the proposed algorithm some modifications must be imposed to set it for the Laplacian noise. To update the Laplacian filter weights, we use the approach presented in [10] with a change in the input data by using the artificial data  $\bar{z}_k$  instead of the glint colored observed data. Then, we use the general Bayes' rule with the Laplacian observation weights as

$$\hat{p}_{k}^{i} = f(m^{i} \mid \overline{Z}^{k}) = \frac{f(m^{i} \mid \overline{Z}^{k-1}) f(\overline{z}_{k} \mid m^{i}, \overline{Z}^{k-1})}{f(\overline{z}_{k} \mid \overline{Z}^{k-1})}$$
(13)

where  $\overline{Z}^k$  represents the artificial measured data  $\{\overline{z}_1,\overline{z}_2,...,\overline{z}_k\}$ , and  $m^i$  specifies the ith mode. In the proposed algorithm there are four modes where each one corresponds to a Kalman filter in the filter bank. In (13)  $f(\overline{z}_k \mid m^i,\overline{Z}^{k-1})$  can be obtained from convolution of  $f(\overline{H}(m^i)r_k \mid m^i,\overline{Z}^{k-1})$  and the white glint noise density function  $f(\eta_k)$ . Computing the direct convolution is difficult, so we use the approximate method introduced in [10] which yields

$$\hat{p}_k^i \approx c \frac{\overline{p}_k^i \phi(0)}{\alpha_k \sigma_k} \tag{14}$$

where  $\overline{p}_k^i$  represents  $f(m^i \mid \overline{Z}^{k-1})$  and c stands for the inverse of  $f(\overline{z}_k \mid \overline{Z}^{k-1})$ , and  $\alpha_k = e^{-K(T_k) + T_k \overline{z}_k}$ ,  $K(T) = \ln[M(T)]$  and  $M(T) = \int_{-\infty}^{\infty} e^{Tx} f(x) dx$  which is the moment generating function (MGF) of the probability density function f(.). The variable  $T_k$  is the value of T evaluated at the kth iteration, which is determined from the following equation [10]

$$K'(T_k) - \bar{z}_k = 0 \tag{15}$$

where  $\overline{Z}_k$  is the known artificial measured data in the kth iteration. Eventually, the probability density function  $f(\overline{z}_k \mid m^i, \overline{Z}^{k-1})$  in (13) at kth iteration of the algorithm will be evaluated as:

$$f(\overline{z}_k \mid m^i, \overline{Z}^{k-1}) \approx \frac{\phi(0)}{\alpha_k \sigma_k}$$
 (16)

For the normal expansion we obtain  $\phi(0) = 1/\sqrt{2\pi}$  and  $\sigma_k = \sqrt{K^{(2)}(T_k)}$  [9] and [10].

In the IMM algorithm the probability of transition between different modes are governed by the Markovian transition probability matrix. In the proposed algorithm, we will consider four different modes of operations as follows: constant velocity matched to Gaussian observation noise, constant acceleration matched to Gaussian observation noise, and constant acceleration matched to Laplacian observation noise.

Thus, the Markovian transition matrix ( IMM transition matrix )  $A \in \Re^{4\times 4}$  will be:

$$A = \begin{bmatrix} q(1-\varepsilon) & (1-q)(1-\varepsilon) & q\varepsilon & (1-q)\varepsilon \\ (1-q)(1-\varepsilon) & q(1-\varepsilon) & (1-q)\varepsilon & q\varepsilon \\ q(1-\varepsilon) & (1-q)(1-\varepsilon) & q\varepsilon & (1-q)\varepsilon \\ (1-q)(1-\varepsilon) & q(1-\varepsilon) & (1-q)\varepsilon & q\varepsilon \end{bmatrix}$$

$$(17)$$

q shows the probability of using the same dynamic system in two consecutive time samples, or the probability of no change in the dynamic of the system, and is very close to unity in practice.

### IV. SIMULATION RESULTS

In this section, we have presented some simulation results for the proposed algorithm. For simplicity as it was stated before we perform the results on one single channel or axis. Due to independency of the dimensions, the simulation process on the other channels will end up to similar results. The target is tracked for 1200 samples, and is assumed to move with constant velocity of 40 m/s between the first and the  $400^{th}$  samples. Then the velocity increases by a constant acceleration of 4  $m/s^2$  from the  $400^{th}$  sample up to the  $800^{th}$  sample. It continues moving with constant velocity between the  $800^{th}$  and  $1200^{th}$  samples. The acceleration used here is

$$\ddot{r_k} = 0$$
  $0 \le k \le 399$  and  $801 \le k \le 1200$   $\ddot{r_k} = 4$   $400 \le k \le 800$ 

The sampling time is considered to be T = 0.05 s. The variance of the process noise for constant velocity is assumed to be 0.1 and for the constant acceleration (maneuvering) would be 0.01. We assume that the dynamics of the target are known and the only unknowns are the parameters of the glint noise that would be estimated through the algorithm processes. Thus, choosing small process noise variance is consistent with having knowledge about the dynamics of the system. To analyze the performance of the proposed algorithm results an ideal system with following known parameters is taken into consideration: The correlation coefficient of the glint colored noise is assumed to be  $\alpha = 0.8$ , and the parameters of the white glint noise  $\eta$  are considered to be  $\varepsilon = 0.1$ ,  $\sigma = 50m$ , and  $\mu = 400m$ . The online estimation of the parameters is iteratively performed as the tracking process advances. The constants  $\rho = 0.9$ , and  $\beta = 0.99$  have been chosen in the estimation process of the parameter  $\alpha$ , and q is set to unity in the probability transition matrix.

100 Monte Carlo runs are carried out and the average is represented by the root mean square error (RMSE) criterion as a measure of the performance in this simulation:

$$RMSE(k) = \sqrt{\frac{1}{m} \sum_{i=1}^{m} (r_k - \hat{r}_k^i)^2} \qquad k = 1, 2, ..., 1200 \quad , \quad m = 100$$

where  $\hat{r}_k^i$  denotes the state estimate of the *i*th Monte Carlo run for the *k*th sample.

We first compare the proposed method error performance (RMSE) results with similar results obtained from the ideal system where all the parameters  $\alpha, \varepsilon, \sigma$ , and  $\mu$  are assumed to be known. The corresponding RMSE results for position, velocity and acceleration are shown in Figs. 2, 3, and 4. It is seen that the results of the proposed algorithm with unknown parameters are in very good agreement with the results obtained from the ideal system with known parameters.

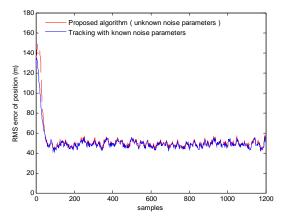


Fig. 2 Position estimation RMSE

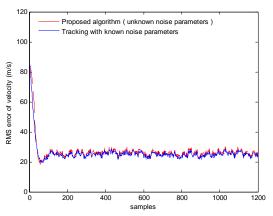


Fig. 3 Velocity estimation RMSE

The slight changes between the two results at the first 20 iterations, is the effect of the imperfect parameter estimations due to insufficient received data samples. As the tracking process advances, the estimation results get better and better and eventually reach their steady state values, where the differences between the two corresponding results, are very small.

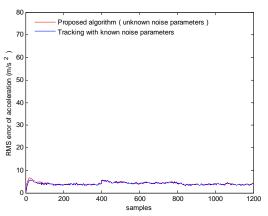


Fig. 4 Acceleration estimation RMSE

To study the effect of different types of observed data noises on performance of the proposed algorithm the following tests have been carried out:

- i) Disabling the decorrelating process.
- ii) Disabling the glint noise process.

In the first test we treat the observed data noise as a white noise process, and ignore its colored characteristic. Thus, there is no need to decorrelate the noise and consequently, we disable the decorrelating process of the algorithm and leave the signal undecorrelated. The corresponding results for the target position, velocity and acceleration are illustrated in Figs. 5, 6, and 7.

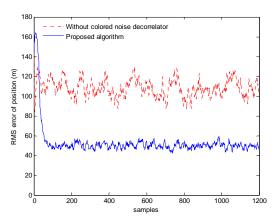


Fig. 5 Position estimation RMSE without Colored noise decorrelator

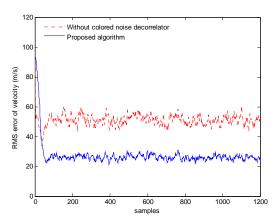


Fig. 6 Velocity estimation RMSE without Colored noise decorrelator

It is quite obvious that the error performance has increased compared to the proposed algorithm results. Therefore, this test represents the capability of the proposed algorithm in canceling the effect of the colored noise.

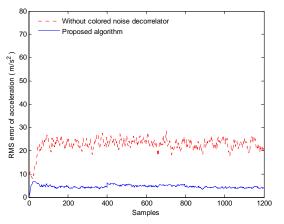


Fig. 7 Acceleration estimation without Colored noise decorrelator

In the second test we neglect the glint characteristic of the observed data noise. In other words we treat the noise as colored Gaussian noise with uknown parameters. In this case there is no need to use the Laplacian filters in place of the Gaussian filters to model the glint noise in the IMM algorithm. So, we run the algorithm all with Gaussian filters. The corresponding results for position, velocity, and acceleration are shown in Figs. 8, 9, and 10.

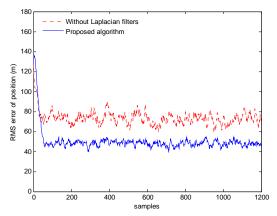


Fig. 8 Position estimation RMSE with no Laplacian filter

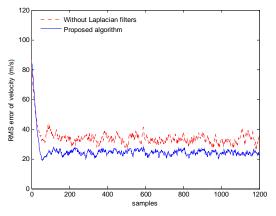


Fig. 9 Velocity estimation RMSE with no Laplacian filter

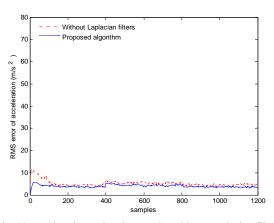


Fig. 10 Acceleration estimation RMSE with no Laplacian filter

Again, the error performance has increased compared to the proposed algorithm results.. Therefore, the second test represents the capability of the proposed algorithm in eliminating the effect of the glint noise data.

The overall results show that the proposed algorithm is able to simultaneously deal with the problem of glint and colored noise, and also estimate the unknown glint noise parameters in manuevering target tracking.

# V. CONCLUSION

The simulation results show the remarkable strength of the proposed algorithm in simultaneous removal of all the undesired degrading factors in optimal operation of the Kalman filter for target tracking. The tests carried out in this paper illustrate that disabling the effect of any of the degrading factors in the algorithm will result in a significant increase in the corresponding RMSE values.

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