

# On CR-Structure and F-Structure Satisfying Polynomial Equation

Manisha Kankarej

**Abstract**—The purpose of this paper is to show a relation between CR structure and F-structure satisfying polynomial equation. In this paper, we have checked the significance of CR structure and F-structure on Integrability conditions and Nijenhuis tensor. It was proved that all the properties of Integrability conditions and Nijenhuis tensor are satisfied by CR structures and F-structure satisfying polynomial equation.

**Keywords**—CR-submanifolds, CR-structure, Integrability condition & Nijenhuis tensor.

**2000 AMS Mathematics Subject Classification**—53C40, 53D10.

## I. INTRODUCTION

THE study of F structure and CR structure is done by many mathematicians. In this paper the study of these structures are considered with polynomial equations, the study of Integrability and Nijenhuis Tensor is also extended to polynomial equation. Yano [1] initiated the study of F structure. Nikie [8] and Das [9] further studied the properties of F structure.

Let  $F$  be a non zero tensor field of type (1,1) and of class  $C^\infty$  dimensional manifold  $M$  such that

$$a_n F^n + a_{n-1} F^{n-1} \dots a_2 F^2 + a_1 F^1 = 0 \quad (1)$$

where  $n$  is a fixed positive integer greater than or equal to 1. Such a structure on  $M$  is called an F-structure. If the rank of  $F$  is constant and  $r=F(r)$ , then  $M$  is called an  $F$  structure manifold of degree  $n$ .

Let us define the operator on  $M$  as:

$$l = - \left( \frac{a_n F^{n-1} + a_{n-1} F^{n-2} + \dots a_3 F^2 + a_2 F^1}{a_1} \right) \quad (2)$$

$$m = I + \left( \frac{a_n F^{n-1} + a_{n-1} F^{n-2} + \dots a_3 F^2 + a_2 F^1}{a_1} \right) \quad (3)$$

where  $I$  denotes the identify operator on  $M$ .

**Theorem 1.** Let  $M$  be an  $F(a_n, a_{n-1}, \dots a_1)$  structure manifold satisfying (1) then

- $l + m = I$
- $l^2 = l$
- $m^2 = m$
- $l.m = 0$

Manisha M. Kankarej is with the Department of Mathematics and Statistics, University College, Dubai, UAE (e-mail: Manisha.Kankarej@zu.ac.ae).

**Proof.**

$$\begin{aligned} \text{a) } l + m &= I \\ l + m &= - \left( \frac{a_n F^{n-1} + a_{n-1} F^{n-2} + \dots a_3 F^2 + a_2 F^1}{a_1} \right) + I \\ &+ \left( \frac{a_n F^{n-1} + a_{n-1} F^{n-2} + \dots a_3 F^2 + a_2 F^1}{a_1} \right) = I \\ &\Rightarrow l + m = I \end{aligned} \quad (4)$$

$$\begin{aligned} \text{b) } l^2 &= l \\ l^2 &= - \left( \frac{a_n F^{n-1} + a_{n-1} F^{n-2} + \dots a_3 F^2 + a_2 F^1}{a_1} \right) \\ &* - \left( \frac{a_n F^{n-1} + a_{n-1} F^{n-2} + \dots a_3 F^2 + a_2 F^1}{a_1} \right) = \\ &- \left( \frac{-a_1}{a_1} \right) * - \left( \frac{a_n F^{n-1} + a_{n-1} F^{n-2} + \dots a_3 F^2 + a_2 F^1}{a_1} \right) = \\ &- \left( \frac{a_n F^{n-1} + a_{n-1} F^{n-2} + \dots a_3 F^2 + a_2 F^1}{a_1} \right) = l \\ &\Rightarrow l^2 = l \end{aligned} \quad (5)$$

$$\begin{aligned} \text{m}^2 &= m \\ m^2 &= \left[ I + \left( \frac{a_n F^{n-1} + a_{n-1} F^{n-2} + \dots a_3 F^2 + a_2 F^1}{a_1} \right) \right] * \left[ I + \right. \\ &\left. \left( \frac{a_n F^{n-1} + a_{n-1} F^{n-2} + \dots a_3 F^2 + a_2 F^1}{a_1} \right) \right] = \\ I^2 + 2I \left( \frac{a_n F^{n-1} + a_{n-1} F^{n-2} + \dots a_3 F^2 + a_2 F^1}{a_1} \right) + \\ &\left( \frac{a_n F^{n-1} + a_{n-1} F^{n-2} + \dots a_3 F^2 + a_2 F^1}{a_1} \right)^2 = \\ I^2 + 2I \left( \frac{a_n F^{n-1} + a_{n-1} F^{n-2} + \dots a_3 F^2 + a_2 F^1}{a_1} \right) + l^2 = I^2 + \\ &2I \left( \frac{a_n F^{n-1} + a_{n-1} F^{n-2} + \dots a_3 F^2 + a_2 F^1}{a_1} \right) + l = \\ I^2 + 2I \left( \frac{a_n F^{n-1} + a_{n-1} F^{n-2} + \dots a_3 F^2 + a_2 F^1}{a_1} \right) + \\ &- \left( \frac{a_n F^{n-1} + a_{n-1} F^{n-2} + \dots a_3 F^2 + a_2 F^1}{a_1} \right) = \\ I + \left( \frac{a_n F^{n-1} + a_{n-1} F^{n-2} + \dots a_3 F^2 + a_2 F^1}{a_1} \right) = m \\ \text{So } m^2 &= m \end{aligned} \quad (6)$$

$$\begin{aligned} \text{d) } l.m &= l.(I-l) \\ &= l - l^2 = l - l \Rightarrow l.m = 0 \end{aligned} \quad (7)$$

For  $F \neq 0$  satisfying (1) there exist complimentary distributions  $D_l$  &  $D_m$  corresponding to the projection operator  $l$  &  $m$  respectively. If Rank  $F =$  constant and  $r = r(F)$ . Then,  $\dim D_l = r$  and  $\dim D_m = n - r$ .

**Theorem 2.** We have-

- (I)  $lF = Fl = F$ ,  
(II)  $mF = Fm = 0$
- (I)  $\left( \frac{a_n F^n + a_{n-1} F^{n-1} + \dots a_3 F^3 + a_2 F^2}{a_1} \right) * m = 0$

$$(II) \left( \frac{a_n F^n + a_{n-1} F^{n-1} + \dots + a_3 F^3 + a_2 F^2}{a_1} \right) * l = -l$$

**Proof.**

a) (I)  $lF = Fl = F$

$$lF = \left( \frac{a_n F^{n-1} + a_{n-1} F^{n-2} + \dots + a_3 F^2 + a_2 F^1}{a_1} \right) * F =$$

$$- \left( \frac{a_n F^n + a_{n-1} F^{n-1} + \dots + a_3 F^3 + a_2 F^2}{a_1} \right) = F$$

So  $lF = Fl = F$  (8)

(II)  $mF = Fm = 0$

$$mF = \left[ I + \left( \frac{a_n F^{n-1} + a_{n-1} F^{n-2} + \dots + a_3 F^2 + a_2 F^1}{a_1} \right) \right] * F = \left[ F + \left( \frac{a_n F^n + a_{n-1} F^{n-1} + \dots + a_3 F^3 + a_2 F^2}{a_1} \right) \right] = F + (-F) = 0$$

So  $mF = Fm = 0$  (9)

b) (I)  $\left( \frac{a_n F^n + a_{n-1} F^{n-1} + \dots + a_3 F^3 + a_2 F^2}{a_1} \right) * m = 0$

$$= -F * m = 0$$
 (10)

(II)  $\left( \frac{a_n F^n + a_{n-1} F^{n-1} + \dots + a_3 F^3 + a_2 F^2}{a_1} \right) * l$

$$= -F * l = -F$$
 (11)

Thus,  $F$  acts on  $D_l$  as an almost complex structure and on  $D_m$  as a null operator.

### II. NIJENHUIS TENSOR

The Nijenhuis tensor  $N(X,Y)$  of  $F$  satisfying (1) in  $M$  is expressed as follows for every vector field  $X, Y$  on  $M$ .

$$N(X,Y) = [FX, FY] - F[FX, Y] - F[X, FY] + F^2[X, Y] \quad (12)$$

We state the following theorem without proof

**Theorem 3.** A necessary & sufficient condition for the  $f$ -structure to be integrable is that  $N(X,Y)=0$  for any vector field  $X$  &  $Y$  on  $M$ .

### III. LIE BRACKET

If  $X$  &  $Y$  are two vector fields in  $M$  then their lie bracket  $[X,Y]$  is defined by

$$[X, Y] = XY - YX \quad (13)$$

### IV. CR-STRUCTURE

A study of differential geometry of a CR submanifold has been initiated in [4]-[7]. Results on general theory of Cauchy Riemann manifolds have been obtained by [2].

Let  $M$  be a differentiable manifold and  $T_c(M)$  be its complex field on tangent bundle  $M$ . A CR-Structure on  $M$  is a complex sub bundle  $H$  of  $T_c(M)$  such that  $H_p \cap H_{\bar{p}} = 0$  &  $H$  is involutive i.e. for complex vector field  $X$  &  $Y$  in  $H$ ,  $[X, Y]$  is in  $H$ . In this case we say  $M$  is a CR-manifold.

Let  $F(a_n, a_{n-1}, \dots, a_1)$  be an integrable structure satisfying (1) of rank  $r = 2m$  on  $M$ . We define complex sub bundle  $H$  of  $T_c(M)$  by

$$H_p = \{X - \sqrt{-1}FX, X \in \chi(D_l)\} \quad (14)$$

where  $\chi(D_l)$  is the  $F(D_m)$  module for all differentiable sections of  $D_l$ . The  $Re(H) = D_l$  &  $H_p \cap H_{\bar{p}} = 0$ , where  $H_{\bar{p}}$  denotes the complex conjugate. Integrability conditions on such submanifolds have been investigated by [4].

**Theorem 4.** If  $P$  &  $Q$  are two elements of  $H$  then the following relation holds

$$[P, Q] = [X, Y] - [FX, FY] - \sqrt{-1}[X, FY] - \sqrt{-1}[FX, Y]$$

**Proof.** Let us define

$$P = X - \sqrt{-1}FX$$

$$Q = Y - \sqrt{-1}FY$$

then by direct calculation & on simplifying, we obtain-

$$[P, Q] = [X - \sqrt{-1}FX, Y - \sqrt{-1}FY]$$

$$= [X, Y] - \sqrt{-1}[X, FY] - \sqrt{-1}[FX, Y] - [FX, FY]$$

$$= [X, Y] - [FX, FY] - \sqrt{-1}[X, FY] - \sqrt{-1}[FX, Y] \quad (15)$$

**Theorem 5.** If  $F(a_n, a_{n-1}, \dots, a_2, a_1)$  structure satisfying (1) is integrable then we have

$$- \left( \frac{a_n F^{n-2} + a_{n-1} F^{n-3} + \dots + a_3 F^1 + a_2}{a_1} \right) \{ [FX * FY] + F^2[X * Y] \} = l$$

$$\{ [FX, Y] + [X, FY] \}$$

**Proof.** From (12) we have,

$$N(X, Y) = [FX, FY] + F^2[X, Y] - F[FX, Y] - F[X, FY]$$

Since  $N(X, Y) = 0$  we obtain

$$[FX, FY] + F^2[X, Y] = F[FX, Y] + F[X, FY]$$

Operating  $- \left( \frac{a_n F^{n-2} + a_{n-1} F^{n-3} + \dots + a_3 F^1 + a_2}{a_1} \right)$

$$= - \left( \frac{a_n F^{n-2} + a_{n-1} F^{n-3} + \dots + a_3 F^1 + a_2}{a_1} \right) \{ [FX, FY] + F^2[X, Y] \}$$

$$= - \left( \frac{a_n F^{n-2} + a_{n-1} F^{n-3} + \dots + a_3 F^1 + a_2}{a_1} \right) \{ F[FX, Y] + F[X, FY] \}$$

$$= - \left( \frac{a_n F^{n-2} + a_{n-1} F^{n-3} + \dots + a_3 F^1 + a_2}{a_1} \right) F \{ [FX, Y] + [X, FY] \}$$

$$= - \left( \frac{a_n F^{n-1} + a_{n-1} F^{n-2} + \dots + a_3 F^2 + a_2 F}{a_1} \right) \{ [FX, Y] + [X, FY] \}$$

$$= - \left( \frac{a_n F^{n-1} + a_{n-1} F^{n-2} + \dots + a_3 F^2 + a_2 F}{a_1} \right) \{ [FX, Y] + [X, FY] \}$$

$$= l \{ [FX, Y] + [X, FY] \}$$

This proves the above theorem.

**Theorem 6.** The following identities hold

$$mN(X, Y) = m[FX, FY]$$

$$mN \left[ \left( \frac{a_n F^{n-2} + a_{n-1} F^{n-3} + \dots + a_3 F^1 + a_2}{a_1} \right) X, Y \right]$$

$$= m \left[ \left( \frac{a_n F^{n-1} + a_{n-1} F^{n-2} + \dots + a_3 F^2 + a_2 F}{a_1} \right), FY \right]$$

**Proof.**

a)  $mN(X, Y) = m\{ [FX, FY] + F^2[X, Y] - F[FX, Y] - F[X, FY] \}$

d)  $\Rightarrow b)$

$$mN(X, Y) = m\{ [F, F] + F^2[X, Y] - F[FX, Y] - F[X, FY] \}$$

$$= m[FX, FY] + m.F.F[X, Y] - mF[FX, Y] - mF[X, FY] =$$

$$m[FX, FY]$$

$$m\left[\left(\frac{a_n F^{n-2} + a_{n-1} F^{n-3} + \dots + a_3 F^1 + a_2}{a_1}\right) X, FY\right] = 0$$

$$\left(\frac{a_n F^{n-2} + a_{n-1} F^{n-3} + \dots + a_3 F^1 + a_2}{a_1}\right) = -F$$

$\Rightarrow mN(X, Y) = m[FX, FY]$  (16)

By (1)

$m[-FX, FY] = 0$  (20)

d)  $\Rightarrow b)$

b)  $mN\left[\left(\frac{a_n F^{n-2} + a_{n-1} F^{n-3} + \dots + a_3 F^1 + a_2}{a_1}\right) X, Y\right] = m$

$\left[\left(\frac{a_n F^{n-1} + a_{n-1} F^{n-2} + \dots + a_3 F^2 + a_2 F}{a_1}\right), FY\right]$

e)  $\Rightarrow b)$

$$mN\left[\left(\frac{a_n F^{n-2} + a_{n-1} F^{n-3} + \dots + a_3 F^1 + a_2}{a_1}\right) X, Y\right] =$$

$$mN\left[\left(\frac{a_n F^{n-2} + a_{n-1} F^{n-3} + \dots + a_3 F^1 + a_2}{a_1}\right) FX, FY\right] =$$

$$F^2\left[\left(\frac{a_n F^{n-2} + a_{n-1} F^{n-3} + \dots + a_3 F^1 + a_2}{a_1}\right) X, Y\right] -$$

$$F\left[F\left(\frac{a_n F^{n-2} + a_{n-1} F^{n-3} + \dots + a_3 F^1 + a_2}{a_1}\right) X, Y\right] -$$

$$F\left[\left(\frac{a_n F^{n-2} + a_{n-1} F^{n-3} + \dots + a_3 F^1 + a_2}{a_1}\right) X, FY\right]$$

$$m\left[\left(\frac{a_n F^n + a_{n-1} F^{n-1} + \dots + a_3 F^3 + a_2 F^2}{a_1}\right) LX, FY\right] = 0$$

$$m\left[\left(\frac{a_n F^{n+1} + a_{n-1} F^{n-1} + \dots + a_3 F^3 + a_2 F^2}{a_1}\right) X, FY\right] = 0$$

$$m\left[\left(\frac{a_n F^{n-1} F + a_{n-1} F^{n-2} F + \dots + a_3 F^2 F + a_2 F^1 F}{a_1}\right) X, FY\right] = 0$$

$$m\left[\left(\frac{a_n F^{n-1} F + a_{n-1} F^{n-2} F + \dots + a_3 F^2 F + a_2 F^1 F}{a_1}\right) X, FY\right] = 0$$

$$m\left[\left(\frac{a_n F^{n-1}(-F) + a_{n-1} F^{n-2}(-F) + \dots + a_3 F^2(-F) + a_2 F^1(-F)}{a_1}\right) X, FY\right] = 0$$

$$\left(\frac{a_n F^{n-1}(-F) + a_{n-1} F^{n-2}(-F) + \dots + a_3 F^2(-F) + a_2 F^1(-F)}{a_1}\right) = -F$$

By the equation  $mF = 0 = Fm$

$$mN\left[\left(\frac{a_n F^{n-2} + a_{n-1} F^{n-3} + \dots + a_3 F^1 + a_2}{a_1}\right) X, Y\right] = m$$

$$\left[\left(\frac{a_n F^{n-1} + a_{n-1} F^{n-2} + \dots + a_3 F^2 + a_2 F}{a_1}\right), FY\right]$$
 (17)

$\Rightarrow m[-FX, FY] = 0$  [By (1)]

$\Rightarrow m[FX, FY] = 0$  (21)

$\Rightarrow e) \Rightarrow b)$

**Theorem 7.** For any two vector field X & Y, the following condition are equivalent –

a)  $mN(X, Y) = 0$

b)  $m[FX, FY] = 0$

c)  $mN\left[\left(\frac{a_n F^{n-1} + a_{n-1} F^{n-2} + \dots + a_3 F^2 + a_2 F}{a_1}\right) X, Y\right] = 0$

d)  $m\left[\left(\frac{a_n F^{n-2} + a_{n-1} F^{n-3} + \dots + a_3 F^1 + a_2}{a_1}\right) X, FY\right] = 0$

e)  $m\left[\left(\frac{a_n F^n + a_{n-1} F^{n-1} + \dots + a_3 F^3 + a_2 F^2}{a_1}\right) LX, FY\right] = 0$

**Proof.** a)  $\Rightarrow$  b)

$mN(X, Y) = 0$

$\Rightarrow m\{ [FX, FY] = F^2[X, Y] - F[FX, Y] - F[X, FY] \} = 0$

$\Rightarrow m[FX, FY] = 0$  (18)

[since  $mF = Fm = 0$ ]

c)  $\Rightarrow$  a)

$$mN\left[\left(\frac{a_n F^{n-1} + a_{n-1} F^{n-2} + \dots + a_3 F^2 + a_2 F}{a_1}\right) X, Y\right] = 0$$

By (1)

$$\frac{a_n F^{n-1} + a_{n-1} F^{n-2} + \dots + a_3 F^2 + a_2 F}{a_1} = -I$$

$\Rightarrow mN[-X, Y] = 0$

$\Rightarrow mN[X, Y] = 0$

$\Rightarrow c) \Rightarrow a)$

(19)

**Theorem 8.** If  $F^n$  acts on  $D_1$  as an almost complex structure. Then

$$m\left[\left(\frac{a_n F^n + a_{n-1} F^{n-1} + \dots + a_3 F^3 + a_2 F^2}{a_1}\right) LX, FY\right] = m[-X, FY] = 0$$

**Proof.**

$$m\left[\left(\frac{a_n F^n + a_{n-1} F^{n-1} + \dots + a_3 F^3 + a_2 F^2}{a_1}\right) LX, FY\right]$$

$$= m\left[\left(\frac{a_n F^{n-1} + a_{n-1} F^{n-2} + \dots + a_3 F^2 + a_2 F^1}{a_1}\right) FLX, FY\right]$$

$$= m[-FLX, FY] = [-X, FY] \text{ {By (8)}}$$

**Theorem 9.** For X, Y  $\in x(D_1)$  we have

$$l([X, FY] + [FX, FY]) = [X, FY] + [FX, Y]$$

**Proof.**

$$l([X, FY] + [FX, Y]) = l\{X.FY - FY.X + FX.Y - Y.FX\}$$

{By (5)}

$$= X.FY - FY.X + FX.Y - Y.FX$$

{By (13)}

$$= [X, FY] + [FX, Y]$$

**Theorem 10:** The integrable  $F(a_n, a_{n-1}, \dots, a_1)$  structure satisfying (1) on  $M$  defines a CR-structure  $H$  on it. Such that  $ReH = D_1$ .

**Proof.** From theorem 4 we have,

$$[P, Q] = [X, Y] - [FX, FY] - \sqrt{-1}[X, FY] - \sqrt{-1}[FX, Y]$$

$$l[P, Q] = l[X, Y] - l[FX, FY] - \sqrt{-1} ([X, FY] + [FX, Y])$$

{By theorem (9)}

$$= [X, Y] - [FX, FY] - \sqrt{-1} ([X, FY] + [FX, Y]) = [P, Q]$$

{By theorem (4)}

Since  $l[P, Q] = [P, Q] \Rightarrow [P, Q] \in \chi(Dl)$ . Then,  $F(a_n, a_{n-1}, \dots, a_1)$  structure satisfying (1) on  $M$  defines a CR-structure.

V. MORPHISM OF VECTOR BUNDLES

Let  $\bar{K}$  be the complementary distribution of  $Re(H)$  to  $TM$ . We define a morphism of vector bundles  $F: TM \rightarrow TM$  given by

$$F(X) = 0 \quad \forall X \in \chi(\bar{K}) \text{ such that}$$

We have

$$F(X) = \frac{1}{2} \sqrt{-1} (P - \bar{P})$$

where  $P = X + \sqrt{-1} Y \in \chi(HP)$  and  $\bar{P}$  is the complex of  $P$ .

**Corollary 1.** If  $P = X + iY$  and  $\bar{P} = X - iY$  belong to  $H_p$  and  $F(X) = \frac{1}{2} \sqrt{-1} (P - \bar{P}), F(Y) = \frac{1}{2} \sqrt{-1} (P + \bar{P})$  and  $F(-Y) = \frac{1}{2} \sqrt{-1} (P + \bar{P})$  then  $F(X) = \frac{1}{2} \sqrt{-1} (P - \bar{P}) = -Y, F^2(X) = -X$  and  $F(-Y) = -X$ .

**Proof.**  $P = X + \sqrt{-1} Y$  and  $\bar{P} = X - \sqrt{-1} Y \Rightarrow \frac{(P + \bar{P})}{2}, Y = \frac{(P - \bar{P})}{2\sqrt{-1}}$ . Since  $P + \bar{P} = 2X$  and  $P - \bar{P} = 2\sqrt{-1} Y, F(X) = F[\frac{P + \bar{P}}{2}] = \frac{1}{2} \sqrt{-1} (P - \bar{P}) = -Y$  from the definition of morphism

$$F(-Y) = F\left[-\frac{P - \bar{P}}{2\sqrt{-1}}\right] = -X$$

**Theorem 11.** If  $M$  has a CR-structure  $H$ , then we have  $a_n F^n + a_{n-1} F^{n-1} \dots a_2 F^2 + a_1 F^1 = 0$  and consequently  $F(a_n, a_{n-1}, \dots, a_2, a_1)$  structure satisfying (1) is defined on  $M$  such that the distribution  $D_1$  and  $D_m$  coincide with  $Re(H)$  and  $\bar{K}$  respectively.

**Proof.** Suppose  $M$  has a CR-structure. Then in view of definition of CR manifold & corollary 1 we have-

$$F(X) = -Y;$$

operating above equation by  $\frac{a_n F^{n-1} + a_{n-1} F^{n-2} \dots a_2 F^1}{a_1}$  on both sides we get

$$\left(\frac{a_n F^{n-1} + a_{n-1} F^{n-2} \dots a_2 F^1}{a_1}\right)(F(X)) = \left(\frac{a_n F^{n-1} + a_{n-1} F^{n-2} \dots a_2 F^1}{a_1}\right)(-Y)$$

on making use of Corollary 1 the right hand side of the above equation becomes

$$\left(\frac{a_n F^{n-1} + a_{n-1} F^{n-2} \dots a_2 F^1}{a_1}\right)F(X) =$$

$$\left(\frac{a_n F^{n-2} + a_{n-1} F^{n-3} \dots a_2}{a_1}\right)F(-Y)$$

which can be written as -

$$\begin{aligned} \left(\frac{a_n F^{n-1} + a_{n-1} F^{n-2} \dots a_2 F^1}{a_1}\right)F(X) &= \frac{a_n F^{n-2} + a_{n-1} F^{n-3} \dots a_2}{a_1} (-X) \\ &= -\frac{a_n F^{n-2} + a_{n-1} F^{n-3} \dots a_2}{a_1} (X) = -\frac{a_n F^{n-3} + a_{n-1} F^{n-4} \dots a_2 F^{-1}}{a_1} \\ F(X) &= -\frac{a_n F^{n-3} + a_{n-1} F^{n-4} \dots a_2 F^{-1}}{a_1} (-Y) = \\ &= -\frac{a_n F^{n-4} + a_{n-1} F^{n-5} \dots a_2 F^{-2}}{a_1} F(-Y) = \\ &= -\frac{a_n F^{n-4} + a_{n-1} F^{n-5} \dots a_2 F^{-2}}{a_1} (-X) \\ &= (-)^2 \frac{a_n F^{n-4} + a_{n-1} F^{n-5} \dots a_2 F^{-2}}{a_1} (X) \end{aligned}$$

We continue simplifying in this manner n times. We get

$$\left(\frac{a_n F^{n-1} + a_{n-1} F^{n-2} \dots a_2 F^1}{a_1}\right)F(X) = -F(X)$$

On simplifying the above equation we get

$$a_n F^n + a_{n-1} F^{n-1} \dots a_2 F^2 + a_1 F^1 = 0$$

REFERENCES

- [1] K. Yano: On structure defined by a tensor field  $f$  of type  $(1,1)$   $f^3 + f = 0$ , Tensor, N.S. 14 (1963), 99-109.
- [2] K. Yano & S. Ishihara; On Integrability of a structure  $f$  satisfying  $f^3 + f = 0$  Quart J. Math Oxford 25 PP. 217-222 (1964).
- [3] S.I. Goldberg; On the existence of manifold with an  $f$  structure, Tensor N.S. 26 P.P. 323-329 (1972).
- [4] B.Y. Chen; Geometry of submanifold, Marcel Dekker, New York (1973).
- [5] A. Bejancu, CR submanifold of a Kaehler manifold, 1, Proc. Amer. Math. Soc. 69 (1978), 135-142.
- [6] D.E. Blair & B.Y. Chen; on CR-submanifold of Hermitian manifolds, Israel Journal of Mathematics vol. 34, No. 4, PP 353-363 (1979).
- [7] K. Yano & Masaturo Kon; Differential Geometry of CR-submanifolds, Geom. Dedicata Vol. 10, PP 369-391 (1981).
- [8] J. Nikie;  $F(2K+1,1)$  structure on the lagrangian space, filomat (Nis) 9:2 P.P. 161-167 (1995).
- [9] Love Joy S. Das; On CR-structure and  $F(2K+1,1)$  structure satisfying  $F^{2k+1} + F = 0$ , Journal of the Tensor society of India Vol. 22, (2004) p.p.1-7.
- [10] Lovejoy S. Das; On CR-structure & F-structure satisfying  $F^{k+1} + (-1)^{k+1} F = 0$ , Rocky Mountain, Journal of Mathematics, Vol. 36, Number 3, 2006.