

Numerical Study of Fluid Mixing in a Grooved Micro-Channel with Wavy Sidewalls

Yu-Sin Lin, Chih-Yang Wu, and Yung-Ching Chu

Abstract—In this work, we perform numerical simulation of fluid mixing in a floor-grooved micro-channel with wavy sidewalls which may impose perturbation on the helical flow induced by the slanted grooves on the channel floor. The perturbation is caused by separation vortices in the recesses of the wavy-walled channel as the Reynolds number is large enough. The results show that the effects of the wavy sidewalls of the present micromixer on the enhancement of fluid mixing increase with the increase of Reynolds number. The degree of mixing increases with the increase of the corrugation angle, until the angle is greater than 45 degrees. Besides, the pumping pressure of the micromixer increases with the increase of the corrugation angle monotonically. Therefore, we would suggest setting the corrugation angle of the wavy sidewalls to be 45 degrees.

Keywords—Fluid mixing, grooved channel, microfluidics, separation vortex.

I. INTRODUCTION

THE micromixer is an important component in the microfluidic systems for chemistry and biology [1], [2]. The mixing of liquids at a small Reynolds number is typical for the applications involving microfluidic systems, and so the flow of the microfluidic systems is always laminar. The Reynolds number is usually defined as $Re = \bar{u} d_h \nu^{-1}$, where \bar{u} , d_h and ν denote the characteristic value of flow velocity, the hydraulic diameter of the main channel and the kinematic viscosity of the fluid, respectively. To enhance mixing efficiency, two main types of designs, active and passive micromixers, have been developed. Time and length of mixing for active micromixers are generally less than those for passive mixers. However, active micromixers utilize external forces to enhance fluid mixing and require complicated device packaging and system integration. Therefore, passive micromixers are widely developed.

Among the passive micromixers proposed, the micromixers with grooved surfaces have attracted intensive interests and have been used in some applications.

The grooved micromixers use a series of oblique grooves on the channel surface to generate transverse component of flow which stretches and folds volumes of fluids to be mixed over the cross section of the channel and, as a result, diffusion mixing is

enhanced [3]. In addition to validation of the design, recent work has been focused on the optimization of slanted groove micromixers [4]-[6]. Some designs have evolved from the original design based on slanted grooves on the channel floor. Kim et al. [7] placed alternating barriers above the slanted grooves on the channel floor to enhance mixing, because the inserted barriers may impose alternating perturbation on the helical flow induced by the slanted grooves. Sato et al. [8] fabricated PDMS micro-channels with slanted grooves on the sidewalls as well as the channel floor to achieve increases in bulk helical flow. When a fluid is forced through a channel with a number of recesses at a Reynolds number great than a certain value, separation vortices occur [9]. Several studies on heat/mass transfer enhancement using channels with recesses constructed by wavy sidewalls have been reported [10], [11]. In this work, we perform numerical simulation of fluid mixing in a floor-grooved micro-channel with wavy sidewalls which may impose alternating perturbation on the helical flow induced by the slanted grooves on the channel floor. The perturbation is caused by separation vortices in the recesses of the wavy-walled channel. The effects of the corrugation angle and the Reynolds number on the degree of mixing are investigated.

II. PROBLEM STATEMENT AND SOLUTION METHOD

The proposed micromixer is shown schematically in Fig. 1. The design includes a floor-grooved channel with two symmetrical wavy sidewalls, two straight inlets, a straight channel from the inlets to the wavy-walled channel and an exit channel. The inlets have a rectangular cross section of $W_i = 110 \mu\text{m}$ by $H = 100 \mu\text{m}$ and a length L_i of $1040 \mu\text{m}$. The average width and the total length of the channel from the inlets to the exit are $W_m = 260 \mu\text{m}$ and $L = 10920 \mu\text{m}$, respectively. The lengths of the straight channel from the inlets to the front edge of the first groove is $L_e = 3 W_m$. The floor-grooved channel with two wavy sidewalls is the mixing section. The depth (h) and the width (a) of grooves are $30 \mu\text{m}$ and $65 \mu\text{m}$, respectively. The published studies [3]-[6] agree on the conclusion that non-axial flow is maximized over a slanted angle $\theta_i = 45^\circ$. The distance between two adjacent grooves is $65 \mu\text{m}$. There are eight groups of grooves and each of the groups has six grooves.

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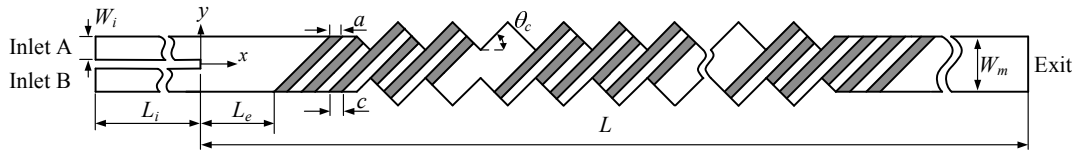


Fig. 1 Schematic diagram of the micromixer; the shadow strips represent grooves

The present structure can be fabricated with two steps of photolithography. The mixing behaviors of fluids in the micromixer are described by the continuity equation, the momentum equation and the species convection-diffusion equation for isothermal steady incompressible flow. These equations are expressed as

$$\nabla \cdot \vec{V} = 0, \quad (1)$$

$$\rho(\vec{V} \cdot \nabla) \vec{V} = -\nabla p + \mu \nabla^2 \vec{V}, \quad (2)$$

$$(\vec{V} \cdot \nabla) c = D \nabla^2 c, \quad (3)$$

where \vec{V} , ρ , p , μ , c and D denote the velocity vector, the density of the fluid, the pressure, the dynamic viscosity of the fluid, the species concentration and the diffusion coefficient, respectively. The no-slip condition is set on the solid walls and the pressure at the exit is set to be 1atm. Furthermore, the fluids entering the two inlets are assumed to possess properties of water and we consider equal flow rate at both inlets. The mole fractions of solutions are set to be 1 (fluorescent solution) at inlet A and 0 (DI water) at inlet B, respectively. The density, the viscosity and the diffusion coefficient of Rhodamine B (Fluka, Germany) in DI water are 997 kg m^{-3} , $0.00097 \text{ kg m}^{-1} \text{ s}^{-1}$ and $3.6 \times 10^{-10} \text{ m}^2 \text{ s}^{-1}$ [12], respectively. The present problem is solved by the computational fluid dynamics software CFD-ACE (CFD Research Corporation, CA, USA). Here, the SIMPLEC algorithm is used for pressure-velocity coupling and the second order upwind scheme with limiter is adopted for the velocity and concentration calculations. To minimize the effect of the numerical diffusion on simulation results, we use the cells with varying sizes. The stopping criterion of iterative computation is that the relative residual of each variable is less than 0.0001.

In this simulation, the velocity at inlets is set to be a uniform profile corresponding to the Re 's of 0.033, 0.1, 0.33, 1, 3, 9, 27, 46.8 and 81. Those values of Re are selected to cover the range of flows from the diffusion domination to the convection domination of the mixing flow. To compare the mixing efficiency of micromixers with various flow conditions and geometrical parameters, the degree of mixing (M) [13], is calculated by

$$M = 1 - \frac{\sigma}{\sigma_0} \quad (4)$$

where

$$\sigma^2 = \frac{1}{n} \sum_{i=1}^n (c_i - \bar{c})^2 \quad (5)$$

with n denoting the total number of sampling, c_i denoting the concentration at a position on the cross section considered and \bar{c} denoting the average value of c_i , and σ_0 is the σ at the beginning of the mixing process.

III. RESULTS AND DISCUSSION

The main flow direction in a wavy-walled channel with a flat bottom is approximately parallel, while the direction of local flow around the sidewall is always changing. The separation vortices in the recesses of the channel appear as the Re is large enough, as shown in Fig. 2 (a). The separation vortices do not enhance fluid mixing, because they do not stretch and/or fold the interface of the fluids to be mixed, as shown in Fig. 2 (b).

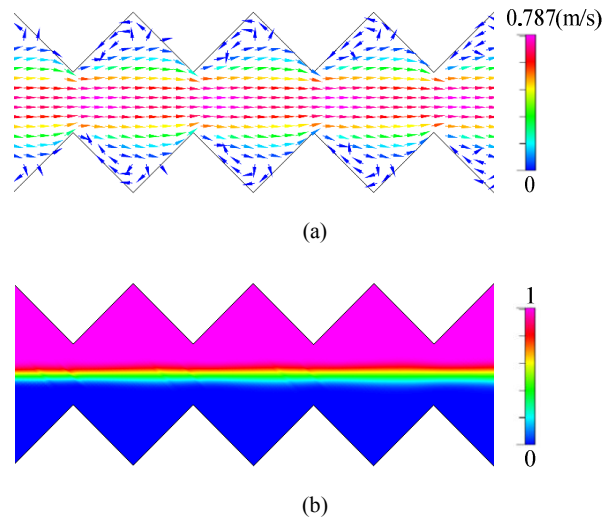
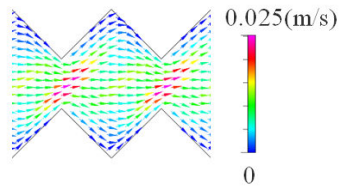


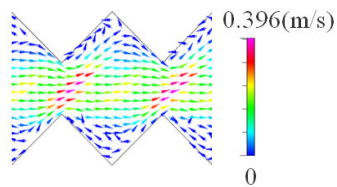
Fig. 2 (a) Flow pattern and (b) concentration distribution on the horizontal plane at $z = 35 \mu\text{m}$ in a micromixer with a flat bottom and $\theta_c = 45^\circ$ for the case with $Re = 46.8$

Comparison of Figs. 3 (a) and (b) shows that the wavy sidewalls of the micromixer considered in this work form a number of recesses which cause flow separation and generate vortices as the Re is large enough. Fig. 3 also shows the transverse component of flow driven by a steady longitudinal pressure gradient in the microchannel with slanted grooves placed on the floor at an oblique angle $\theta_i = 45^\circ$ with respect to

the long axis (x) of the channel. The transverse component of flow causes a helical flow which enhances fluid mixing at low Re [3]. Fig. 4 shows the strength of the vortices caused by the slanted grooves placed on the floor of the channel increases with the increase of Re . This sort of vortices is in direction different from that of the separation vortices. The multi-directional vortices may stretch and fold the interface between the fluids to be mixed, as shown in Figs. 5 and 6.

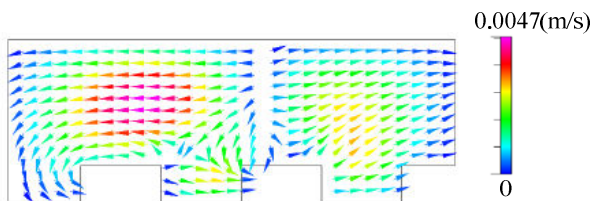


(a)

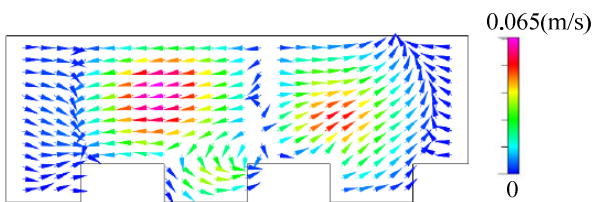


(b)

Fig. 3 Flow patterns on the horizontal plane at $z = 5 \mu\text{m}$ of the micromixer with grooved bottom and $\theta_c = 45^\circ$: (a) $Re = 3$, (b) $Re = 46.8$

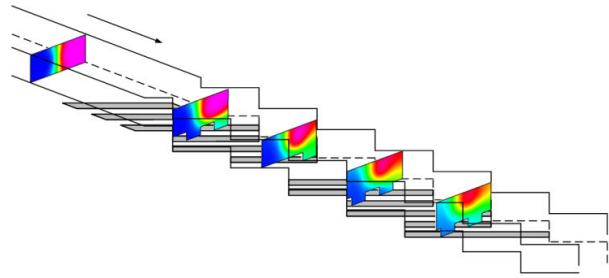


(a)

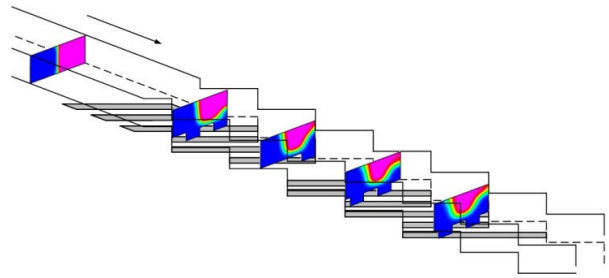


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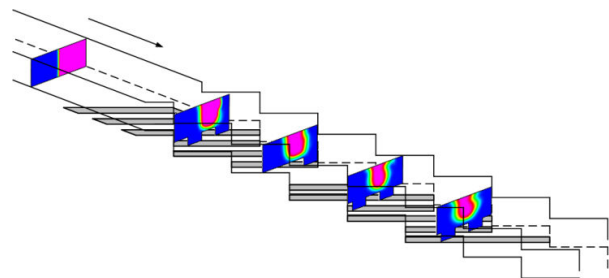
Fig. 4 Flow patterns on the vertical plane at $x = 2000 \mu\text{m}$ of the micromixer with grooved bottom and $\theta_c = 45^\circ$: (a) $Re = 3$, (b) $Re = 46.8$



(a)

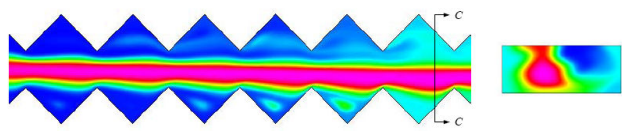


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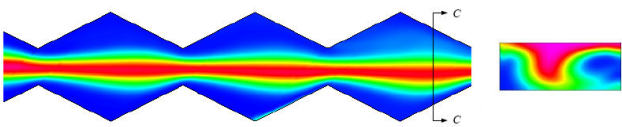


(c)

Fig. 5 Concentration distributions at $x = 2W_m$, $4.5W_m$, $6W_m$, $7.5W_m$ and $9W_m$ of the micromixer with grooved bottom and $\theta_c = 45^\circ$: (a) $Re = 0.1$, (b) $Re = 3$, (c) $Re = 46.8$



(a)



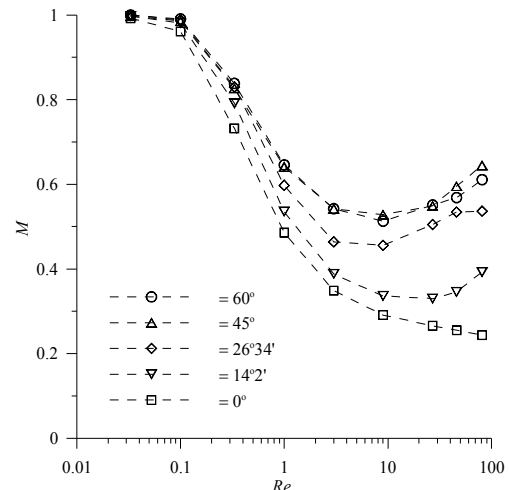
(b)

Fig. 6 Concentration distributions on the horizontal plane at $z = 35 \mu\text{m}$ of the flow in a micromixer with a grooved bottom, (a) $\theta_c = 45^\circ$ and (b) $\theta_c = 26^\circ 34'$ for the case with $Re = 46.8$

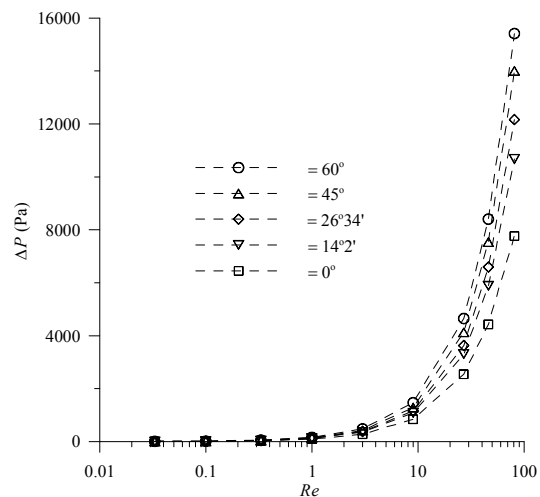
It can be seen from Fig. 6 that the separation vortices caused by flow forced through the wavy-walled channel are considered to be dominated by the corrugation angle, θ_c . Fig. 7 (a) shows that the degree of mixing increases with the increase of the corrugation angle, until the θ_c is greater than 45° . The increase of Re corresponds to the increase of contribution of lateral advection to the fluid mixing, as shown in Fig. 5. Besides, the wavy sidewalls may induce separation vortices, and so are important for the performance of the proposed micromixer in the relatively high Reynolds number region, as shown in Fig. 6. It is worth noting the present design with $\theta_c = 0^\circ$ is identical to the original slanted groove micromixer. Thus, Fig. 7 (a) also shows that the effects of the wavy sidewalls of the present micromixer on the enhancement of fluid mixing increase with the increase of Re . For cases with low Reynolds numbers, the mixing relies mainly on diffusion. The smaller value of Re allows more time for mixing by pure diffusion and results in a thicker interface between the fluids to be mixed, as shown in Fig. 5. Thus, the degree of mixing increases with the decrease of the Re , as shown in Fig. 7 (a). Besides, the pumping pressure of the micromixer increases with the increase of the corrugation angle monotonically, as shown in Fig. 7 (b). Therefore, we would suggest setting the corrugation angle of the wavy sidewalls to be 45° .

IV. CONCLUDING REMARKS

In this work, we investigate the influence of the θ_c and the Re on the mixing efficiency of a floor-grooved micro-channel with wavy sidewalls by numerical simulation. The following trends may be observed from the concentration distribution and flow pattern of the flow in the micromixers considered and degrees of mixing at the exit and the pressure drops of micromixers for various values of Re . (i) The combination of the helical flow induced by the slanted grooves placed on the floor of the channel and the separation vortices generated in the recesses of the wavy-walled channel enhances fluid mixing, especially for the case with a relatively high Re . (ii) The degree of mixing increases with the increase of the θ_c , until the θ_c is greater than 45° . Besides, the pumping pressure of the micromixer increases with the increase of the θ_c monotonically. Therefore, we would suggest setting the θ_c of the wavy sidewalls to be 45° . (iii) For the cases with low Reynolds numbers, the degree of mixing increases with the decrease of the Re .



(a)



(b)

Fig. 7 (a) Degrees of mixing at the exit and (b) the pressure drops of micromixers for the cases with $Re = 0.033, 0.1, 0.33, 1, 3, 9, 27, 46.8$ and 81

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