Numerical Study for Structural Design of Composite Rotor with Crack Initiation

A. Chellil, A. Nour, S. Lecheb, H. Mechakra, A. Bouderba, H. Kebir

Abstract—In this paper, a coupled damage effect in the instability of a composite rotor is presented, under dynamic loading response in the harmonic analysis condition. The analysis of the stress which operates the rotor is done. Calculations of different energies and the virtual work of the aerodynamic loads from the rotor blade are developed. The use of the composite material for the rotor offers a good stability.

Numerical calculations on the model developed prove that the damage effect has a negative effect on the stability of the rotor.

The study of the composite rotor in transient system allowed determining the vibratory responses due to various excitations.

Keywords—Rotor, composite, damage, finite element, numerical.

I. INTRODUCTION

THE study of vibration problems associated with rotating systems is currently a major issue in the industrial field. To optimize the dynamic behavior of rotors and dimension at best such systems, it is necessary to understand precisely their vibrational behavior.

The essential studies of rotor dynamics concern the Campbell diagram which represents the evolution of natural frequencies depending on the speed of rotation, and the calculation of unbalance responses, mainly during the passage of critical speeds. To carry out such studies, we now have many modeling tools such as transfer matrix method; finite element generalized polynomial expansion or transfer functions distributed. There may be mentioned for example the existence gyroscopic moments responsible for the variation of the natural frequency as a function of the speed of rotation or the forces that make the circulatory movement unstable from a certain speed, even in purely linear.

It is known that the eigenmodes of a conservative system rotational mode are complex conjugate two to two purely imaginary eigenfrequencies. This is due to gyroscopic effects, induced by the rotation, that couple the horizontal and vertical displacements [1], [2]. They may be combined to obtain qualified real modes either direct, or retrograde, according to the precessional motion associated with them is in a direction identical or opposite to the rotational movement.

The essential point is the dependence of their eigenfrequency as a function of the rotation speed [3]. It can

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be shown that the part of the kinetic energy on the gyroscopic terms is positive (negative) modes for direct (retrograde), so that the gyroscopic effects tend to stiffen (flexible) structure [4]. Thus, the curves increasing (decreasing) the Campbell diagrams are relative to the direct mode (backward).

The study of the vibrational behavior of a cracked shaft covers several branches of mechanics and mathematics. This is a complex non-linear system requiring, for a relevant description, a fine and accurate modeling of the shaft as well as cracks and for identifying the calculating parameters characterizing their presence.

In this context, Sundermeyer and Weaver [5] have studied the response of a cracked beam simply supported, modeled by two elements of Euler-Bernoulli beam connected by a nonlinear spring representing the cracked section.

Hamidi [6] presented two analytical models, one based on the approach of Rayleigh-Ritz and the other on a finite element formulation [7] to study the influence of cracks rotating shafts on the modal parameters of the structure. This work confirmed the low sensitivity of the resonance frequencies in the presence of a crack.

Wauer [8] formulated the equations of motion of a shaft having a cracked section. The crack often modeled by geometric discontinuities was replaced with a discontinuity at the loading section cracked. The effects of respiration mechanism of crack were also considered.

II. MATHEMATICAL MODEL OF THE PLATE

Consider a composite rotor consisting of N layers of orthotropic material. If the stacking sequence is symmetric the rotor has a typical behavior of a beam and it can be modeled by the classical theory associated with the damping parameters homogenized.

If the stack is non-symmetric coupling effects such as mechanical flexion-extension, torsion and extension-shear-extension appears. In this study, we neglect the coupling effects.

The fields of displacement and strain for a beam element representative of the rotor are given by equations according to beam theory, for each p fold of the section.

The stress-strain relationship is written:

$$\{\sigma\}^p = \begin{cases} \sigma_{yy}^p = E_y^p \varepsilon_{yy} + \tilde{E}_y^p \dot{\varepsilon}_{yy} \\ \tau_{yz}^p = G_{yz}^p \gamma_{yz} + \tilde{G}_{yz}^p \dot{\gamma}_{yz} \\ \tau_{yx}^p = G_{yx}^p \gamma_{yx} + \tilde{G}_{yx}^p \dot{\gamma}_{yx} \end{cases} \qquad (1)$$

E_y^p, G_{yz}^p and G_{yx}^p are respectively the Young's modulus and transverse shear moduli, and $\tilde{E}_y^p$, $\tilde{G}_{yz}^p$ and $\tilde{G}_{yx}^p$ modules are

related to depreciation, for a layer p, along the axis of the rotor y. σ_{yy}^p and $\tilde{G}_{yy}^p$ represent the constraints in the normal section, $\tau_{yz}^p, \tilde{\tau}_{yz}^p$ et $\tau_{yx}^p, \tilde{\tau}_{yx}^p$ are the stresses due to transverse shear. Recall that the coupling effects induced by a sequence of non-symmetric stacking have been neglected.

$$U = \frac{1}{2}\int_0^L \int_S (\sigma_{yy}\varepsilon_{yy} + \tau_{yz}\gamma_{yz} + \tau_{yx}\gamma_{yx})dSdy \qquad (2)$$

$$\delta W = \int_0^L \int_S (\tilde{\sigma}_{yy}\delta\varepsilon_{yy} + \tilde{\tau}_{yz}\delta\gamma_{yz} + \tilde{\tau}_{yx}\delta\gamma_{yx})dSdy \qquad (3)$$

The terms of the potential energy and virtual work in (1) and (2) can be written in terms of shear forces and bending moments [5], [6] after integrating over the section as follows:

$$U = \frac{1}{2}\int_0^L \left[\frac{M_x^2}{\langle EI_x\rangle} + \frac{M_z^2}{\langle EI_z\rangle} + K_{C_{xx}}\frac{T_x^2}{\langle GS\rangle} + K_{C_{zz}}\frac{T_z^2}{\langle GS\rangle}\right] dy \qquad (4)$$

$$\delta W = \frac{1}{2}\int_0^L \left[\frac{\tilde{M}_x\delta\tilde{M}_x}{\langle \widetilde{EI}_x\rangle} + \frac{\tilde{M}_z\delta\tilde{M}_z}{\langle \widetilde{EI}_z\rangle} + K_{C_{xx}}\frac{\tilde{T}_x\delta\tilde{T}_x}{\langle \widetilde{GS}\rangle} + K_{C_{zz}}\frac{\tilde{T}_z\delta\tilde{T}_z}{\langle \widetilde{GS}\rangle}\right] dy \qquad (5)$$

The homogenized mechanical characteristics that appear in (4) and (5) are given by:

$$\langle EI_x\rangle = \int_S E_y^p z^2 dS = \sum_{p=1}^N E_y^p I_x^p \quad \text{with } I_x^p = \frac{R_p^4 - R_{p-1}^4}{4} \qquad (6)$$

$$\langle EI_z\rangle = \int_S E_y^p x^2 dS = \sum_{p=1}^N E_y^p I_z^p \quad \text{with } I_z^p = \frac{R_p^4 - R_{p-1}^4}{4}$$

$$\langle GS\rangle = G_{yz}\int_S dS = G_{yx}\int_S dS = \sum_{P=1}^N G_{12}^p S^p$$

The mechanical homogenized amortized are written about them:

$$\langle \widetilde{EI}_x\rangle = \int_S \tilde{E}_y^p z^2 dS = \sum_{p=1}^N \tilde{E}_y^p I_x^p \quad \text{with } I_x^p = \frac{R_p^4 - R_{p-1}^4}{4} \qquad (7)$$

$$\langle \widetilde{EI}_z\rangle = \int_S \tilde{E}_y^p x^2 dS = \sum_{p=1}^N \tilde{E}_y^p I_z^p \quad \text{with } I_z^p = \frac{R_p^4 - R_{p-1}^4}{4}$$

$$\langle \widetilde{GS}\rangle = \tilde{G}_{yz}\int_S dS = \tilde{G}_{yx}\int_S dS = \sum_{p-1}^N \tilde{G}_{12}^p S^p$$

where I^p is the contribution of p fold to the inertia of the section and R_p, R_{p-1} is the radius of the outer and inner envelope.

Shear forces and bending moments are written depreciated:

$$\begin{cases} T_x = \frac{1}{K_{C_{xx}}}\langle GS\rangle \left(\frac{\partial u^*}{\partial y} + \theta_z\right) \\ T_z = \frac{1}{K_{C_{zz}}}\langle GS\rangle \left(\frac{\partial \omega^*}{\partial y} - \theta_x\right) \\ M_x = \langle EI_x\rangle \frac{\partial \theta_x}{\partial y} \\ M_z = \langle EI_z\rangle \frac{\partial \theta_z}{\partial y} \end{cases} \qquad (8)$$

With the mechanical homogenized damped in (8) becomes:

$$\begin{cases} \tilde{T}_x = \frac{1}{K_{C_{xx}}}\langle \widetilde{GS}\rangle \left(\frac{\partial u^*}{\partial y} + \theta_z\right) \\ \tilde{T}_z = \frac{1}{K_{C_{zz}}}\langle \widetilde{GS}\rangle \left(\frac{\partial \omega^*}{\partial y} - \theta_x\right) \\ \tilde{M}_x = \langle \widetilde{EI}_x\rangle \frac{\partial \theta_x}{\partial y} \\ \tilde{M}_z = \langle \widetilde{EI}_z\rangle \frac{\partial \theta_z}{\partial y} \end{cases} \qquad (9)$$

Therefore, the potential energy and virtual work can be expressed by:

$$U = \frac{1}{2}\int_0^L \left(EI_x\left(\frac{\partial \theta_x}{\partial y}\right)^2 + EI_z\left(\frac{\partial \theta_z}{\partial y}\right)^2\right)dy + \frac{1}{2}\int_0^L \left[\frac{GS}{K_{C_{xx}}}\left(-\theta_x + \frac{\partial \omega^*}{\partial y}\right)^2 + \frac{GS}{K_{C_{zz}}}\left(\theta_z + \frac{\partial u^*}{\partial y}\right)^2\right]dy \qquad (10)$$

$$\delta W = \int_0^L \left(\widetilde{EI}_x\frac{\partial \theta_x}{\partial y}\frac{\partial \delta\theta_x}{\partial y} + \widetilde{EI}_z\frac{\partial \theta_z}{\partial y}\frac{\partial \delta\theta_z}{\partial y}\right)dy + \int_0^L \left[K_x\widetilde{GS}\left(-\dot{\theta}_x + \frac{\partial \dot{w}^*}{\partial y}\right)\left(-\delta\theta_x + \frac{\partial \delta w^*}{\partial y}\right)\right]dy + \int_0^L \left[K_z\widetilde{GS}\left(\dot{\theta}_z + \frac{\partial \dot{u}^*}{\partial y}\right)\left(\delta\theta_z + \frac{\partial \delta u^*}{\partial y}\right)\right]dy \qquad (11)$$

$$K_x = \frac{1}{K_{C_{xx}}} \qquad (12)$$

$$K_z = \frac{1}{K_{C_{zz}}} \qquad (13)$$

where k is the shear correction factor, the section of the rotor is circular then homogenized flexural inertia is $EI = EI_x = EI_z$ and homogenized damped inertia $\widetilde{EI} = \widetilde{EI}_x = \widetilde{EI}_z$.

III. NUMERICAL SIMULATION OF THE COMPOSITE ROTOR

This part concerned a numerical simulation with abaqus software to obtain a different property of plate with crack growth.

A. Ply plan

The composite rotors are generally obtained by filament winding on a mandrel, Fig. 1. Each fiber layer can be regarded as a unidirectional ply tooth constitutive law is orthotropic.
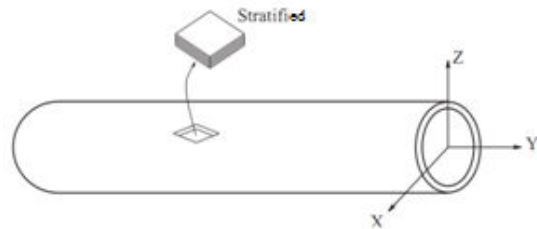

Fig. 1 Composite rotor

B. Properties Material of the Model

After the creation the model we have to enter the material properties of the part which are:

TABLE I
PROPERTIES MATERIAL OF THE CASE OF COMPOSITE ROTOR

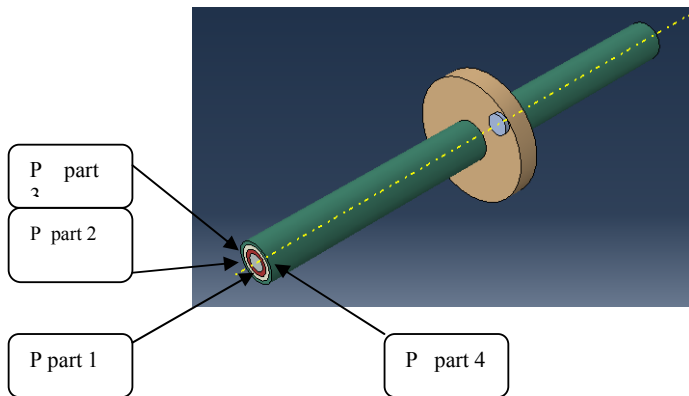|	Part 1 epoxy carbon	Part 2 epoxy glass	Part 3 Special alloy	Part 4 E glass
Young's modulus (Gpa)	210	172.7	247	72.7
Poison's ratio	0.36	0.3	0.26	0.32
Density (kg/m3)	1967	1446.2	1967	1446.2

Fig. 2 Properties material in the part

C. Frequency Study

In this section simulation can identify and indicate a set of modal analysis of the rotor to determine the eigen modes and eigen frequency.
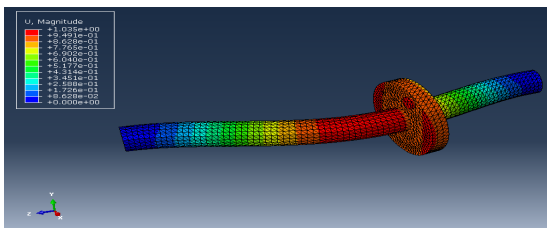
Fig. 3 2nd mode shapes Freq = 0.48805Hz → U$_{max}$=1.035mm
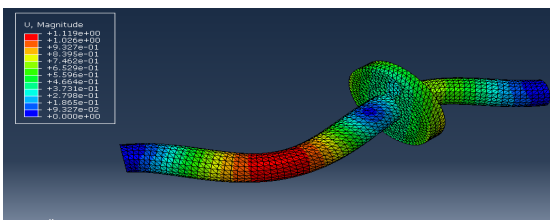
Fig. 4 5th mode shapes, Freq = 1.4024 Hz → U$_{max}$=1.119mm

The previous figures have shown the second and five mode shapes of the specimen. The visualization of this modes improve that the top of the specimen is the zone the more solicited by the force applied such bending and torsion.

D. The Von Mises Stress

The Von Mises stress allows us to observe critical areas to determine the points of maximum solicitation.

E. Tresca Stress

We deduce that the stresses of Teresa are higher than Von Mises stresses which equal to (4.405E4Pa).

F. The Strain

We observe from the figure that the connection area between the shaft and the disc is the most solicited in strain area, so the maximum value of the strain is equal to 0.00704.
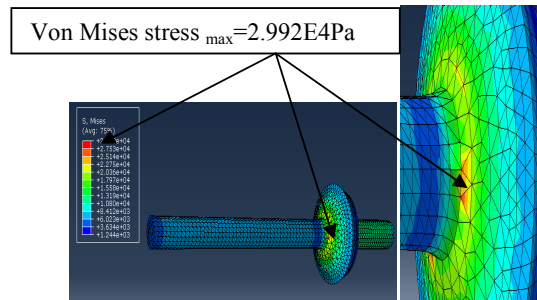
Von Mises stress $_{max}$=2.992E4Pa

Fig. 5 The stress of Von Mises

Propagation of crack in the shaft
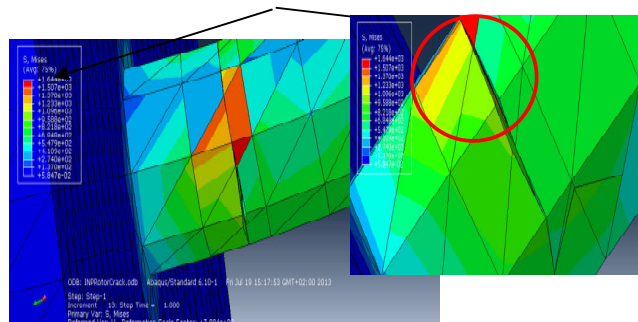
Fig. 6 Composite rotor with crack propagation in the shaft
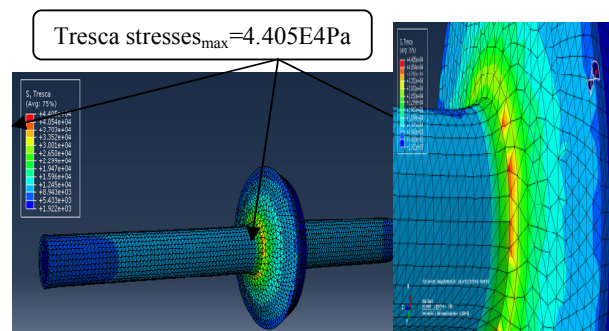
Tresca stresses$_{max}$=4.405E4Pa
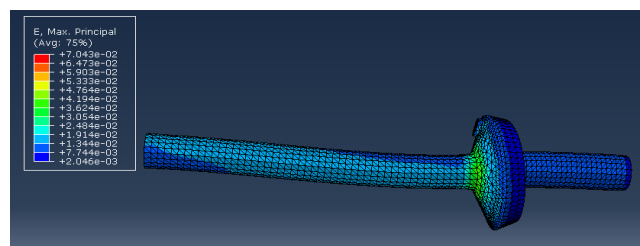

Fig. 7 Representation of Tresca stress

Fig. 8 Representation of the strain distribution

G. The Frequency and Displacement According the Mode Number

The graph below shows the frequencies and Von Mises stress according the mode number of the rotor.
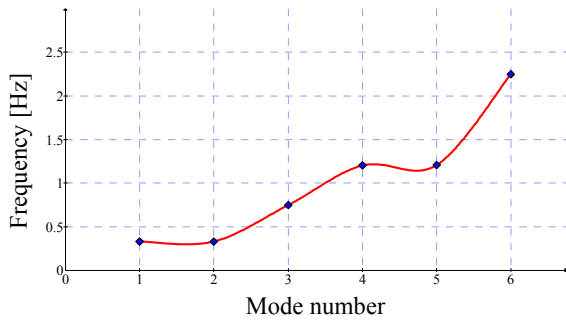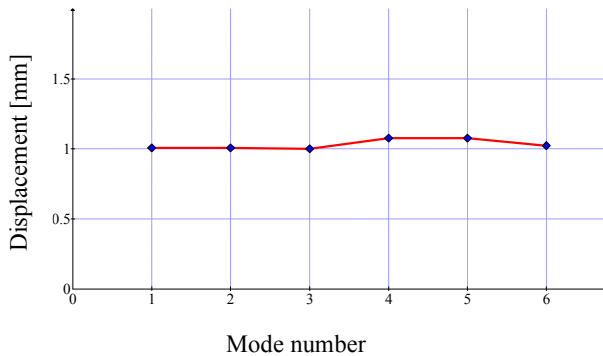
Fig. 9 Frequency function in mode number

Fig. 10 Displacement function in mode number

These results represent the frequencies and displacements for all modes respectively.

IV. CONCLUSION

The rotors are subject to various external forces besides the weight, we find the imbalance forces, which are specific to the rotating machines. For this the composite materials offer the best solutions for the design of the rotors.

A theory of homogenization simplified beam adapted to take account of the internal damping in each ply constituting the stack is presented.

We see the effect of the composite material in the rotor, and we simulate the rotor composite by ABAQUS software which can give the distribution of the stress, strain and field of the displacement and the mode shapes, from the analysis of results, the use of composite materialsoffers a good stability for the rotor.

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