International Journal of Mechanical, Industrial and Aerospace Sciences
ISSN: 2517-9950
Vol:5, No:11, 2011

Numerical Simulation of Deoilin Hydrocyclones

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Abstract**—**In this research the separation efficiency of deoiling hydrocyclone is evaluated using three-dimensional simulation of multiphase flow based on Eulerian-Eulerian finite volume method. The mixture approach of Reynolds Stress Model is also employed to capture the features of turbulent multiphase swirling flow. The obtained separation efficiency of Colman's design is compared with available experimental data and showed that the separation curve of deoiling hydrocyclones can be predicted using numerical simulation.

Keywords**—**Deoiling hydrocyclone, Eulerian-Eulerian Model, Numerical simulation, Separation efficiency, Reynolds Stress Model

I. INTRODUCTION

HAVING an efficient and reliable system for oil-water separation is of crucial importance especially for offshore oil and gas industry. Due to the platform movement, space, weight and operating limitations in offshore, the usage of common methods (gravity based vessels) for oil-water separation are ineffective. On the other hand producing oil is often accompanied by large amount of water that is discharged into the sea at offshore platforms. The amount of oil in water is confined from environmental standards.

Therefore the need to have a high efficiency compact separator during variable operating conditions attracts the interests of researchers to hydrocyclones. Special trait of hydrocyclones such as simple design, easy to install and operate, no moving parts, and low manufacturing and maintenance costs make hydrocyclones as an economical and effective system for produced water treatment [1], [2].

The first idea of using common hydrocyclones for oil-water separation was suggested by Simkin and Olney [3] and Sheng, Welker and Sliepcevich [4] but fundamental studies on deoiling hydrocyclones started from 1980 by Colman and Thew. Several experimental researches on deoiling hydrocyclones were conducted by Colman [5], Colman, Thew and Corney [6] and Colman and Thew [7]-[9]. Their results showed that the separation efficiency of hydrocyclones is independent of flow split between 0.5 to 10 percent. So the overflow diameter should be designed based on working conditions. Moreover for constant droplet size distribution in inlet, the size distribution in outlet is independent of flow split. The migration probability curves are also independent of flow split. The problems of using hydrocyclones for water treatment were investigated by Thew [10] and new design was proposed. The new design had specific advantages such as high swirl flow, small size to prevent large pressure drop and minimum instability and turbulence near the axis.

The operation curves, principle of operations and the first field study of hydrocyclones were presented by Meldrum [11]. In addition to work of Meldrum, several field tests of hydrocyclones also were conducted by various researchers [1]-[2], [12]-[16].

The first attempt on optimizing hydrocyclone was conducted by Young, Wakley, Taggart, Andrews, and Worrell [17]. They measured the flow behavior in a 35-mm hydrocyclone, designed by Colman and Thew (1980), and compared the results with a new modified design developed by them. They studied the effects of operational variables and geometrical parameters, such as inlet size, cylindrical diameter, cone angle, straight section length, flow rate, and droplet diameter on the separation efficiency. Based on their experimental results, a new geometry was proposed for hydrocyclones.

Recent investigations on hydrocyclones focus on operational parameters (Belaidi and Thew [18], Husveg, Rambeau, Drengstig and Bilstad [19]), velocity field (Bai, Wang and Tu [20]) and distribution of oil droplets (Zhou, Gao, An and Yang [21]) in deoiling hydrocyclones.

Although there are lots of experimental studies on deoiling hydrocyclones, due to the difficulty of numerical simulations of liquid-liquid hydrocyclones, only few studies have been conducted on numerical simulations of deoiling hydrocyclones.

Hargreaves and Silvester [22] simulated the oil-water flow in deoiling hydrocyclones using Eulerian-Lagrangian approach. They used Algebraic Stress Model and simulated the flow in 2D cylindrical coordinate system. In dispersed phase, they ignored effect of particle-particle interaction, slip and droplet coalescence. The obtained results showed acceptable agreements with experimental ones. The flow field, velocity distribution and separation efficiency of 10 mm deoiling hydrocyclone was obtained by Grady, Wesson, Abdullah and Kalu [23] using Algebraic Slip Mixture (ASM) multiphase model. In order to simulate the highly swirling flow (swirl number 8.4) the Reynolds Stress Model (RSM) was used. Simulation of miniature hydrocyclones for downhole separation was conducted by Petty and Park [24]. Direct numerical simulation showed that the 3g centrifugal acceleration is created in 5mm miniature hydrocyclones. The pressure drop and flow rates were estimated as 1lit/sec and 1 kPa respectively. Huang [25] simulated the three dimensional turbulent flow in deoiling hydrocyclones using Euler-Euler approach and Reynolds Stress Model. Obtained results

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showed the accumulation of oil near the axis. The separation efficiency was also determined based on phase concentration. Obtained separation curve for Colman type hydrocyclones was in a good agreement with measured one. It must be mentioned that no velocity distribution was reported by author. Noroozi and Hashemabadi [26]-[27] investigated the effect of various inlet types and inlet chamber body profile on the separation efficiency of deoiling hydrocyclones. The separation efficiency was improved 10% and 8% using helical inlet and exponential body profile respectively.

The literature reviews showed that nearly all conducted numerical researches on deoiling hydrocyclones mainly focused on separation efficiency of deoiling hydrocyclones. But in order to predict the separation efficiency, the velocity field for both phases should be obtained accurately. In other words, the more precise determination of velocity field, the more accurate the separation efficiency is. So the aim of this research is to introduce an appropriate multiphase model, demonstrate the capability of computational fluid dynamics in predicting separation efficiency, oil droplet distribution inside deoiling hydrocyclones using general Eulerian-Eulerian multiphase model. It should be mentioned that previous investigations [23], [26]-[27] considered the simple Algebraic Slip Mixture (ASM) model which could not correctly shows the distribution of oil droplets in hydrocyclones.

II. Mathematical Model

Multiphase flow generally contains dispersed particles and droplets. The modeling of instantaneous governing equations for continuous phase and each particle is a very difficult task except for the ideal flow at low Reynolds number. Therefore the local instantaneous equations should be averaged using appropriate operators [28]-[29]. The following governing equations derived based on Ensemble-Averaged of Navier-Stokes for each phase using the Favre-weighting method. Details of averaging procedure and assumptions could be found in [29]-[30].

A. Governing equations

Eulerian-Eulerian or Two-Fluid Model transport equations (two continuity and six momentum equations) are as follows [30]:

$$\frac{\partial\left(\alpha^{(k)}\rho^{(k)}\right)}{\partial t}+\nabla\cdot\left(\alpha^{(k)}\rho^{(k)}U^{(k)}\right)=0 \tag{1}$$

$$\frac{\partial\left(\alpha^{(k)}\rho^{(k)}U^{(k)}\right)}{\partial t}+\nabla\cdot\left(\alpha^{(k)}\rho^{(k)}U^{(k)}U^{(k)}\right)= \nabla\cdot\overline{\overline{\tau}}^{(k)}+\alpha^{(k)}\left(-\nabla p+B^{(k)}\right)+I_M^{(k)}+F^{(k)} \tag{2}$$

In the above equations, $\alpha^{(k)}$ stands for volume fraction of phase (k), $U^{(k)}$ denotes the averaged velocity of *k*th phase, $\overline{\overline{\tau}}^{(k)}$, p and $B^{(k)}$ are stress (tensor and pressure which is shared by all phases and body forces per unit volume of phase (k) respectively. The term $F^{(k)}$ includes all other forces such as lift and virtual mass. $I_M^{(k)}$ is the momentum transfer to the phase (k) due to the phase interaction and could be written as:

$$I_M^{(k)}=\sum_{m\neq k} g^{(km)}\left[\left(U^{(m)}-U^{(k)}\right)-\left(\frac{\overline{\alpha^{(m)}u'^{(m)}}}{\overline{\alpha^{(m)}}}-\frac{\overline{\alpha^{(k)}u'^{(k)}}}{\overline{\alpha^{(k)}}}\right)\right] \tag{3}$$

The last two terms in the above equation are outcome of averaging process and modeled using the eddy-diffusivity concept. The momentum exchange coefficient is defined as follows:

$$g^{(km)}=\frac{3}{4}\left(\frac{\rho^{(k)}\alpha^{(k)}V_r C_D}{d}\right)f\left(\alpha^{(k)}\right) \tag{4}$$

Where V_r is the slip velocity, d is the droplet diameter, C_D and function $f(\alpha)$ are obtained from works of Saboni and Alexandrova [31] and Zuber [32] respectively. The Stress tensor in equation (2) could be decomposed as:

$$\overline{\overline{\tau}}^{(k)}=\overline{\overline{\tau}}^{(k)}_{\text{Molecular}}+\overline{\overline{\tau}}^{(k)}_{\text{Turbulent}} \tag{5}$$

Where

$$\overline{\overline{\tau}}^{(k)}_{\text{Molecular}}=\alpha^{(k)}\mu^{(k)}\nabla U^{(k)} \tag{6}$$

$$\overline{\overline{\tau}}^{(k)}_{\text{Turbulent}}=\alpha^{(k)}\tilde{\tau} \tag{7}$$

The term ($\tilde{\tau}$) which is called mixture Reynolds stress tensor is modeled using appropriate turbulence model.

B. Turbulence Model

In order to simulate the complex swirling flow, employing an appropriate turbulence model with the high resolution scheme for discretizing governing equations is the key to have acceptable and accurate results. Previous numerical simulations of swirling flow showed the applicability of Reynolds Stress Model (RSM) for simulation of swirling flow inside hydrocyclones. So the mixture approach of RSM-LRR [33] model is adopted in this work. Further details of turbulence model can be found in [34].

III. Geometry of the Problem and Mesh generation

The considered geometry of deoiling hydrocyclone is shown in Fig. 1. The geometrical parameters are also shown in table I. Three non-uniform structured computational grids are used to

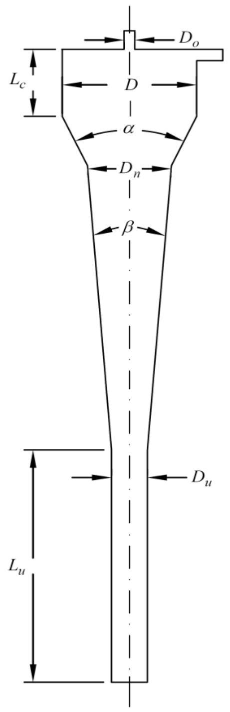


Fig. 1 Geometry of Colman's hydrocyclone

show the grid independency of results. The nodal distributions of geometries for coarse, medium and fine grid are shown in table II. The maximum differences in the tangential velocity for the radial line located in the middle of the domain between the coarse and medium grids and between the medium and fine grids are 5% and 1%, respectively. Therefore, the medium grid is selected for numerical simulation in order to reduce the computational costs. The generated mesh for simulating the flow inside the deoiling hydrocyclone is shown in Fig. 2. It can be seen that the mesh becomes finer in the region of high gradient especially near the wall and core of hydrocyclone.

TABLE II
NODAL DISTRIBUTION OF CONSIDERED GRID

	(r, θ, z)
Coarse	(20×48×260)
Medium	(25×60×300)
Fine	(30×64×400)

IV. BOUNDARY CONDITIONS

There are three types of boundaries (inlet, outlet and wall) which are considered as follows:

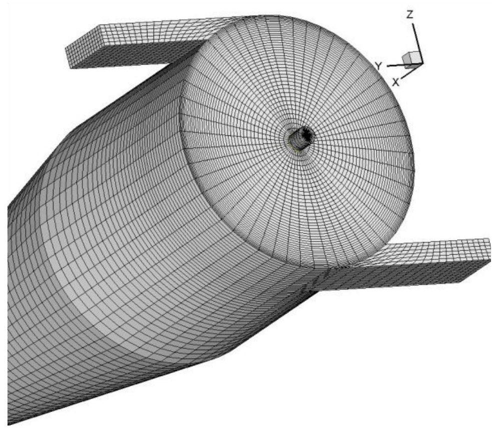


Fig. 2 Generated mesh for hydrocyclone

A. Inlet

All variables are known on the inlet region. Uniform velocity and volume fraction with the turbulence intensity of 5% are employed.

B. Wall

No slip condition ($U^{(k)} = 0$) are assumed at wall. Turbulence quantities near the wall are handled by the wall function.

C. Outlet

At the outlets (overflow and underflow), the gauge static pressure is assumed to be fixed at the minimum radius of the outlet. The overflow and underflow pressures determined due to the desired split ratio. Velocities also calculated from local continuity equation for the boundary cells [35].

The operational parameters for hydrocyclone are shown in table III.

TABLE III
OPERATIONAL PARAMETERS OF HYDROCYCLONE

Q_i (m^3/h)	R %	α_i	ρ_{oil}	μ_{oil}
4.84	0.5 ~ 10	0.05	850	0.00332

V. SOLUTION METHODOLOGY

For numerical investigation on flow field inside deoiling hydrocyclones a new finite volume code, EULER-CALC, has been developed based on CALC-BFC [36]. The solution algorithm is developed using Mass Conservation-Based Algorithms (MCBA) method that is proposed by Moukalled and Darwish [37]. For calculating velocities in faces of control volumes, the Rhie–Chow [38] interpolation method is used and the modification of SIMPLEC for multifluid systems [39] handles the linkage between velocities and pressure.

TABLE I
GEOMETRICAL PARAMETERS OF COLMAN'S DESIGN

	$D(m)$	$D_n(m)$	L_c/D	L_u/D	L_v/D	D_o/D	D_u/D	D_i/D	α	β
Colman's Design	0.070	0.035	1.0	13.0	-	0.042	0.363	0.226	20	1.5

The Partial Elimination Algorithm (PEA) [39] is used to reduce the linkage between phases and accelerate the convergence. The global continuity equation which is used for calculating shared pressure between phases, is normalized using ($\rho^{(k)}$) as a weighting factor to reduce the continuity error. The convergence rate is accelerated by solving two implicit volume fractions and then enforcing the geometric conservation constraints ($\sum \alpha^{(k)} = 1$) [39]. The Carver procedure [40] is also employed for bounding the volume fractions between 0 and 1.

To calculate the convection and diffusion terms the high resolution SMART within the context of NVSF methodology [41] and second order central difference scheme are used respectively. All simulations were performed in unsteady mode using implicit Three Time Level (TTL) method. The time steps were changed from 10^{-3} to 5×10^{-5} during the convergence sequence. The convergence is assessed by comparing the normalized sum of the absolute residual source over all control volumes with the some reference value. The residual in a convergence state is on the order of 10^{-4} for continuity and 10^{-5} for all other equations. The under relaxation factors are assumed as 0.2-0.4 for pressure equation, 0.3-0.5 for momentum equations, 0.2-0.5 for volume fraction equations and 0.1-0.5 for turbulent equations. The greater values of under relaxations are implemented when simulations starts.

VI. RESULTS AND DISCUSSION

A. Velocity Distribution

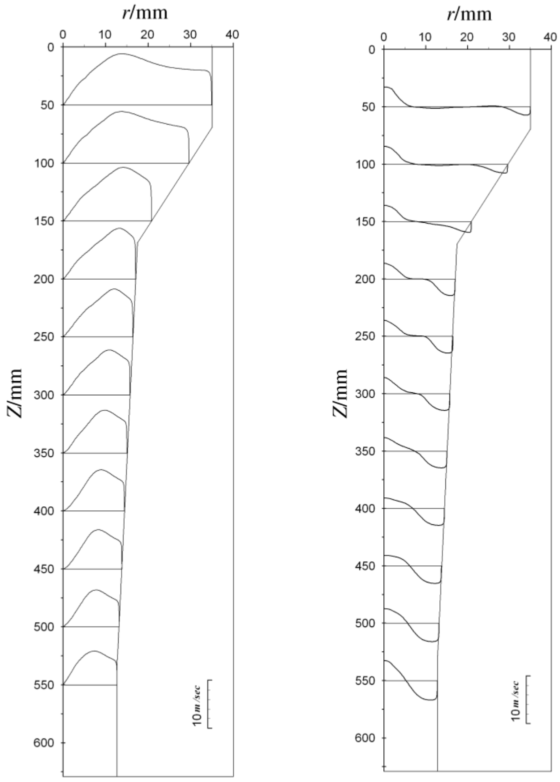
Fig. 3 (a) demonstrates the tangential velocity distribution for the Colman's design. Tangential velocity has a shape of Rankine vortex (forced vortex near the axes of rotation and free vortex in outer region) which is also reported by other researchers [42]. The widths of free and forced regions highly depend on swirl intensity distribution along the hydrocyclone axis.

Fig. 3 (b) shows the radial variation of axial velocity at different axial position for the split ratio of 5%. The outer vortex moves downward to the underflow of hydrocyclone and the inner vortex moves upward toward the over flow. The maximum axial distance that has negative upward velocity occurs at the location of 1000.

B. Separation Efficiency

Separation efficiency of standard Colman's design is compared with available experimental data [17] in Fig. 4. It can be seen that the numerical simulations could predict the separation efficiency curve satisfactorily. A slight over predicting the separation efficiency occurred because of two reasons:

1- In numerical simulation only the median diameter of droplets considered to calculate the interphase forces (equation (4)) and assumed that all droplets have the same



a) Tangential Velocity b) Axial Velocity

Fig. 3 Velocity Distribution inside deoiling hydrocyclone

diameter. But in experimental measurements droplet with different diameter (larger or smaller than median) could be found. Separation efficiency of small diameter oil droplets is less than large one. Therefore the efficiency of hydrocyclones over predicted using numerical simulations.

2- The wall shear stress and pressure drop in numerical simulations are smaller than real operating condition and swirling flow could keep its swirl motion in a longer axial distance. So the location of negative axial velocity occurred in a longer axial distance in numerical simulations compare with experimental data and more droplets captured by upward reverse flow.

C. Oil Distribution

Distribution of oil inside hydrocyclones for is shown in Fig. 5. The radial pressure gradient which is generated by swirling flow, causes to migrate the lighter phase toward the center. The migration velocity which also called slip velocity is function of density difference between dispersed and continuous phase, radial pressure gradient, relaxation time of droplets and the residence time of flow [43]. If the migration velocity is great enough that the oil droplets arrive at locus of zero axial velocity, before the bulk flow leaves the hydrocyclone (measured by flow residence time) oil droplets could be separated [43]. The accumulation of oil droplets near the axis could be seen in Fig. 5.

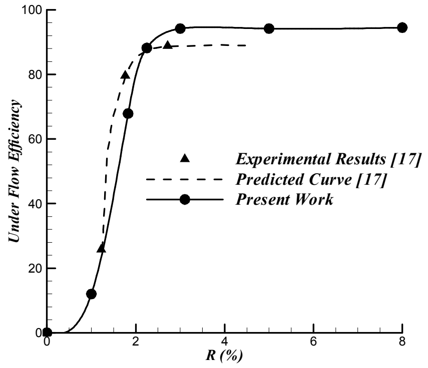


Fig. 4 Comparison between numerical efficiency curve and experimental data in [17]

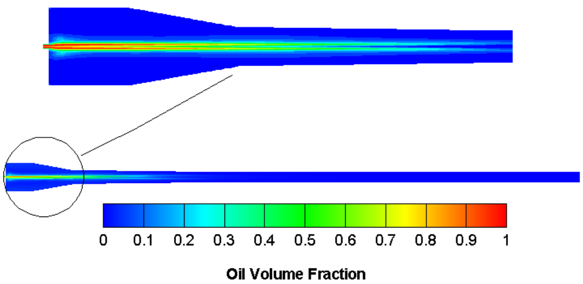


Fig. 5 Distribution of oil in deoiling hydrocyclone

The radial distribution of oil volume fraction in three axial distances from the top wall of hydrocyclones is shown in Fig. 6. The variation of axial velocity also affects the distribution of oil inside hydrocyclone. The great negative upward velocity in Colman's design creates a concentration valley near the axis.

It should be mentioned that previous researchers due to the weakness of employed multiphase models (ASM model), could achieve neither the accurate oil distribution inside deoiling hydrocyclones nor the separation efficiency.

VII. Conclusion

Velocity and oil distributions inside deoiling hydrocyclones are obtained using a general code based on Eulerian-Eulerian multiphase model. The turbulent stresses are approximated using mixture approach of Reynolds Stress Model. The results of separation efficiency is validated against experimental data and showed that the RSM model is appropriate choice for modeling multiphase flow inside deoiling hydrocyclone. A slight over predict is also seen in the results of separation efficiency due to the fact that in the drag coefficient only the median diameter of oil droplets is considered. Also the distribution of oil droplets in various axial distances from the top wall of hydrocyclone is also obtained.

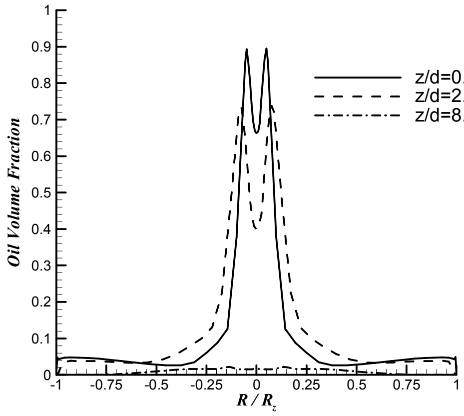


Fig. 6 Radial distribution of oil droplets in different axial distance from the top wall

Acknowledgement

I would like to thanks Marwan Darwish, Professor in American University of Beirut / Mechanical Engineering Department, for his guidance and help in developing the multiphase part of this research.

References

[1] W. M. G. T. Van den Broek, R. Plat, "Characteristics and Possibilities of Some Techniques for De-Oiling of Production Water", *in Proc. 1st Int. Conf. on Health, Safety and Environmental*, Netherlands, 1991, SPE 23315

[2] W. M. G. T. Van den Broek, R. Plat, M. J. Van der Zande, "Comparison of Plate Separator, Centrifuge and Hydrocyclone", *Proc. SPE Int. Conf. and Exhibition*, Beijing, China, SPE 48870, 1998

[3] D. J. Simkin, R. B. Olney, "Phase Separation and Mass Transfer in a Liquid-Liquid Cyclone", *AIChE*, Vol. 2, pp. 545-551, 1956

[4] Sheng H. P., Welker J. R., Sliepcevich C. M., "Liquid-Liquid Separation in a Conventional Hydrocyclone", *The Canadian J. of Chem. Eng.*, Vol. 52, pp.487-491, 1974

[5] D. A. Colman, "The Hydrocyclone for Separating Light Dispersions", Ph.D. Thesis, Southampton University, UK, 1981

[6] D. Colman, M. Thew, D. Corney, "hydrocyclones for Oil/Water Separation". i*n Int. Conf. on Hydrocyclones*, BHRA, Cambridge, United Kingdom, paper 11, pp. 143-165, 1980

[7] D. Colman, M. Thew, "Hydrocyclone to Give a Highly Concentrated Sample of a Lighter Dispersed Phase". *In Int. Conf. on Hydrocyclones*, BHRA, Cambridge, United Kingdom, paper 15, pp. 209-223, 1980

[8] D. Colman, M. Thew, "Correlation of Separation Results From Light Dispersion Hydrocyclones", *Chem. Eng. Res. Des.*, Vol. 61, pp. 233-240, 1983

[9] D. Colman, M. Thew, "Cyclone Separator", U.S. Patent 4764287, 1988

[10] M. T. Thew, "Hydrocyclone Redesign for Liquid-Liquid Separation", *The Chemical Engineer*, pp. 17-23, 1986, July/August

[11] N. Meldrum, "Hyrocyclones: A Solution to Produced-Water Treatment", *SPE Production Engineering*, SPE 16642, 1988

[12] M. S. Choi, "Hydrocyclone Produced Water Treatment for Offshore Developments", *Proc. 65th Annual Technical Conf. and Exhibition of the Society of Petroleum Engineers*, New Orleans, Los Angeles, SPE 20662, 1990

[13] F. J. Noort, J. P. Etten, R. S. Donders, “Reduction of Residual Oil Content in Produced Water at Offshore Gas Production Platform P/6A”, *Proc. Europe’s 90 Conf.*, The Hague, Netherlands, SPE 20882, 1990
[14] D. A. Falnigan, J. E. Stolhand, E. Shimoda, F. Skllbeck, “Use of Low-Shear Pumps and Hydrocyclones for Improved Performance in the Cleanup of Low-Pressure Water”, *SPE Production Engineering*, SPE 19743, 1992
[15] P. S. Jones, “A Field Comparison of Static and Dynamic Hydrocyclones”, *SPE Production and Facilities*, SPE 20701, 1993
[16] J. C. Ditria, M. E. Hoyack, “The Separation of Solids and Liquids with Hydrocyclone Based Technology for Water and Crude Processing”, *Proc. of SPE Asia and Pacific Oil & Gas Conf.*, SPE 28815, 1994
[17] G. A. B. Young, W. D. Wakley, D. L. Taggart, S. L. Andrews, J. R. Worrell, “Oil-Water Separation Using Hydrocyclones: An Experimental Search for Optimum Dimensions”, *J. of Petroleum Sci. and Eng.*, Vol. 11, pp. 37-50, 1994
[18] A. Belaidi, M. T. Thew, “The Effect of Oil and Gas Content on the Controllability and Separation in a De-Oiling Hydrocyclone”, *Trans. IChemE*, Part A, Vol. 81, pp. 305-314, 2003
[19] T. Husveg, O. Rambeau, T. Drengstig, T. Bilstad, “Performance of a Deoiling Hydrocyclone during Variable Flow Rates”, *Minerals Eng.*, , Vol. 20, pp. 368-379, 2007
[20] Z. Bai, H. Wang, S. Tu, “Experimental study of flow patterns in deoiling hydrocyclone”, *Minerals Eng.*, Vol. 22, Issue 4, March, pp. 319-323, 2009
[21] N. Zhou, Y. Gao, W. An, M. Yang, “Investigation of velocity field and oil distribution in an oil–water hydrocyclone using a particle dynamics analyzer”, *Chem. Eng. J.*, Vol. 157, Issue 1, pp. 73-79, 2010
[22] H. J. Hargreaves, R. S. Silvester, “Computational fluid dynamics applied to the analysis of deoiling hydrocyclone performance”, *Chem. Eng. Res. and Des.*, Vol. 68, pp. 365-83. July 1990
[23] S. A. Grady, G. D. Wesson, M. Abdullah, E. E. Kalu, “Prediction of 10-mm Hydrocyclone Separation Efficiency Using Computational Fluid Dynamics”, *Filtration & Separation*, Vol. 40, Issue 9, November, pp. 41-46, 2003
[24] C. A. Petty, S. M. Parks, “Flow Structures within Miniature Hydrocyclones”, *Mineral Eng.*, Vol. 17, pp. 615-624, 2004
[25] S. Huang, “Numerical Simulation of Oil-water Numerical Simulation of Oil-water Hydrocyclone Using Reynolds-Stress Model for Eulerian Multiphase Flows”, *The Canadian J. of Chem. Eng.*, Vol. 83, Issue 5, pp. 829–834, 2005
[26] S. Noroozi, S. H. Hashemabadi, “CFD Simulation of Inlet Design Effect on Deoiling Hydrocyclone Separation Efficiency”, *Chem. Eng. Technol.*, 32, No. 12, 1885–1893, 2009
[27] S. Noroozi, S. H. Hashemabadi, “CFD analysis of inlet chamber body profile effects on de-oiling hydrocyclone efficiency, *Chem. Eng. Res. Des.*, doi:10.1016/j.cherd.2010.11.017, 2011
[28] M. Ishii, *Thermo-fluid Dynamic Theory of Two-Phase Flow*. Paris: Eyrolles, 1975
[29] D. A. Drew, Mathematical Modeling of Two-Phase Flow, *Ann. Rev. Fluid Mech.*, Vol. 15, pp. 261-91, 1983
[30] H. Enwald, E. Peirano, A. E. Almstedt, “Eulerian two-phase flow theory applied to fluidization”, *Int. J. of Multiphase Flow*, Vol. 22, pp. 21-66, 1996
[31] A. Saboni, S. Alexandrova, “Numerical study of the drag on a fluid sphere”, AIChE J., Vol. 48, No. 12, DOI: 10.1002/aic.690481225, 2002
[32] N. Zuber, “on the dispersed two phase flow in the laminar regime”, Chem. Eng. Sci., Vol. 19, pp. 897-917, 1964
[33] B. E. Launder, G. J. Reece, W. Rodi, “Progress in the development of a Reynolds-stress turbulence closure”, *J. of Fluid Mech.*, vol. 68, Apr. 15, pp. 537-566, 1975
[34] R. Maddahian, B. Farhanieh, M. Asadi, “Numerical Investigation of Velocity Field and Separation Efficiency of Deoiling Hydrocyclones”, unpublished.
[35] S. V. Patankar, *Numerical Heat Transfer and Fluid Flow*, 1st ed., Taylor and Francis, Washington DC, 113–135, 1980
[36] B. Farhanieh, L. Davidson, Manual of CALC-BFC, Chalmers University of Technology, Gothenburg, Sweden, 1991
[37] F. Moukalled, M. Darwish, “Pressure-Based Algorithms for Multifluid Flow at All Speeds – Part I: Mass Conservation Formulation”, *Numer. Heat Transfer*, Part B, Vol. 45, pp. 495–522, 2004
[38] C. M. Rhie, L. W. Chow, “Numerical study of the turbulent flow past an airfoil with trailing edge separation”, *AIAA*, Vol. 21, pp. 1527–1532, 1983
[39] M. Darwish, F. Moukalled, B. Sekar, “A Unified Formulation of The Segregated Class of Algorithms for Multifluid Flow at All Speeds”, *Numer. Heat Transfer*, Part B, Vol. 40, pp. 99-137, 2001
[40] M. B. Carver, “A Method of Limiting Intermediate Values of Volume Fraction in Iterative Two-Fluid Computations”, J. Mech. Eng. Sci., vol. 24, pp. 221-224, 1982
[41] M. S. Darwish and F. Moukalled, “Normalized Variable and Space Formulation Methodology for High-Resolution Schemes”, *Numer. Heat Transfer B*, vol. 26, pp. 79–96, 1994.
[42] R. Maddahian, B. Farhanieh, B. Firoozabadi, “Turbulent flow in converging nozzles, part one: boundary layer solution”, *Appl. Math. Mech. -Engl. Ed.*, vol. 32, No. 5, pp. 645–662, 2011
[43] D. Wolbert, B. F. Ma, Y. Aurelle, J. Seureau, “Efficiency Estimation of Liquid-Liquid Hydrocyclones Using Trajectory Analysis”, *AIChE* Journal, Vol. 41, No. 6, pp. 1395-1402, 1995.