

# Numerical Modeling of Temperature Fields in Aviation Gas Turbine Elements

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**Abstract**—A mathematical model and a numerical method for computing the temperature field of the profile part of convectionally cooled blades are developed. The theoretical substantiation of the method is proved by corresponding theorems. To this end, convergent quadrature processes were developed and error estimates were obtained in terms of the Zygmund continuity moduli. The boundary conditions for heat exchange are determined from the solution of the corresponding integral equations and empirical relations. The reliability of the developed methods is confirmed by calculation and experimental studies of the thermohydraulic characteristics of the nozzle apparatus of the first stage of the gas turbine.

**Keywords**—Aviation gas turbine, temperature field, cooled blades, numerical modeling.

## I. INTRODUCTION

PERSPECTIVE of a significant increase of the efficiency coefficient of aircraft power units (PU) and the reduction of fuel consumption are directly related to the increase in parameters of working process of gas turbine engines (GTE) and primarily the temperature. Therefore, in the process of improving specific parameters of GTE, one should strive to master limiting stoichiometric combustion temperatures of the fuel-air mixture, because the specific thrust of GTE when other conditions being equal, increases almost in proportion to the growth of the gas temperature ahead of the turbine  $T_r^*$  [1], [3], [6], [13].

The release of high  $T_r^*$  in gas turbines (GT) of modern GTE goes in several directions [1], [6]-[10], [14], [15], [17], [18]: the first is a creation of new high-temperature alloys with improved properties; the second is a development of metal-ceramic, ceramic and sintered materials; the third is a creation of effective cooling systems (CO) for GT elements.

The priority direction of research on thermal protection of GTE turbine components up to date is the development of efficient cooling systems [3], [10], [13], which ensuring the required heat dissipation should maintain the metal temperature within acceptable limits and equal distribution to eliminate residual stresses [3], [5], [10], [13], [14], [15].

Due to the comparative simplicity of design and reliability of operation, the only practical application in GTE is an open air CO, with the use of which 3 directions of the organization for thermal protection of GT elements were identified: convective, convective-film (obstruction) and penetrating (porous) cooling [3], [5], [14], [15]. In this order, maximum

levels of cooling depths, achieved in the designed constructions of cooled blades, also increase.

It should be noted that obtaining the required cooling depths in modern GTEs within the range of 500-550 deg. is achieved by a rather high price - 3.5-4% of air flow rate within working blades and 7-8% in nozzle blades in relation to the gas flow through the turbine. In general, up to 16-18% of cyclic air is used for cooling the turbine of modern GTE, and in some cases even more. Along with this, the use of blades CO is associated with the emergence of additional losses which reduce efficiency coefficient of cooled turbine compared to uncooled. As a result, the gain in fuel efficiency from the increase in values  $T_r^*$  and  $\pi_\kappa^*$  in the engine are somewhat depreciated by losses caused by the use of CO.

Peculiarities of heat transfer conditions in GTE elements, (complex geometry of heat-stressed parts, large temperature differences, high speeds of working fluid motion, nonstationary heat exchange processes) do not allow to solve the problem of developing a rational CO in a strict formulation [10], [15]. In complex shape bodies with different configurations, number and arrangement of cooling channels, even a separate solution of problems of hydrodynamics and heat transfer is far from being an easy task. And this despite the fact that with increasing  $T_r^*$  requirements for the accuracy of final results increase.

Widely distributed became Methods of Finite Differences (MFD), Finite Element Method (FEM), Boundary Integral Equations Method (BIEM) (or its discrete analog — Boundary Element Method (BEM)). In addition to these, other methods are also used [20].

The most effective is BIEN (or the method of potential theory - MPT), which has proved itself well when considering multiply connected regions of a complex configuration due to a number of advantages, such as boundary (variable) conditions, expansion of class of functions describing the shape of the blade and cooling channels.

## II. THEOREM

In the classical formulation, the differential heat transfer equation, describing the non-stationary process of heat propagation in a multidimensional region with internal sources (sinks) of heat  $q_v$ , under complex boundary conditions, is described by the Fourier-Kirchhoff equation [8], [14], [15]:

$$\frac{\partial(\rho C_v T)}{\partial t} = \text{div}(\lambda \text{grad } T) + q_v, \quad (1)$$

where  $\rho, C_v$  и  $\lambda$  - respectively are the density, heat capacity and thermal conductivity of the material,  $q_v$  - internal source or heat sink, and  $T$  - desired temperature. According to the results of the investigations, it is established [4], [14], [15] that the temperature state of the profile part of the blade with radial cooling channels can be determined with a sufficient degree of accuracy as two-dimensional. In the absence of internal sources (sinks) of heat, the temperature field under steady-state conditions will depend only on the shape of the body and on the temperature distribution on the outline (boundaries) of the body [4], [14], [15]. In this case (1) will look like:

$$\Delta T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \quad (2)$$

To determine the specific temperature fields in the elements of gas turbines, boundary conditions of the third kind are set, which characterize the heat exchange between the body and the environment on the basis of Newton-Riemann hypothesis:

$$\alpha_0(T_0 - T_{\gamma_0}) = \lambda \frac{\partial T_{\gamma_0}}{\partial n}, \quad (3)$$

which characterizes the amount of heat transferred by convection from gas to the unit of surface of the blade and taken away by thermal conductivity into the body of the blade. Ratio:

$$-\lambda \frac{\partial T_{\gamma_i}}{\partial n} = \alpha_i(T_{\gamma_i} - T_i) \quad (4)$$

characterizes the amount of heat released by the convection of the cooler, which is transferred by the blade material to the surface of the cooling channels. Here:  $T_0$  - ambient temperature  $T_i$  on  $i=0$ , that is the temperature of the gas surrounding the blade ( $i = \overline{0, M}$  - number of outlines);  $T_i$  - ambient temperature on  $i = \overline{1, M}$ , that is cooler temperature ( $M$ -number of outlines);  $T_{\gamma_0}$  - outline temperature  $T_{\gamma_i}$  on  $i = 0$  (outer outline of the blade);  $T_{\gamma_i}$  - outline temperature on  $i = \overline{1, M}$  (cooling channel outline);  $\alpha_0$  - coefficient of heat transfer from gas to the blade surface (on  $i=0$ );  $\alpha_i$  - coefficient of heat transfer from blade to cooling air (on  $i = \overline{1, M}$ );  $\lambda$  - coefficient of thermal conductivity of the blade material;  $n$  - outer normal on the outline of the region under investigation.

Let us consider the use of BIEM to solve the problem of determining the temperature field of convectionally cooled blades of gas turbines of aircraft engines.

The function  $T=T(x,y)$ , continuous with the derivatives up to the second order, satisfying the Laplace equation in considered area, including its outline  $\Gamma = \bigcup_{i=0}^M \Gamma_i$ , is harmonic.

Consequence of the Grin integral formula for researched harmonic function  $T=T(x,y)$  is the ratio:

$$T(x, y) = \frac{1}{2\pi} \int_{\Gamma} [T_{\Gamma} \frac{\partial(\ell n R)}{\partial n} - \ell n R \frac{\partial T_{\Gamma}}{\partial n}] ds, \quad (5)$$

where  $R$  - variable at an integration of the distance between point  $K(x,y)$  and "running" on the outline  $k$ ,  $T_{\Gamma}$  - temperature on the outline  $\Gamma$ . The temperature value in some point  $k$  lying on the boundary is determined as a limiting at approach of point  $k(x,y)$  to the boundary

$$T_k = \frac{1}{2\pi} \left[ \int_{\Gamma} T_{\Gamma} \frac{\partial(\ell n R_k)}{\partial n} ds - \int_{\Gamma} \frac{\partial T_{\Gamma}}{\partial n} \ell n R_k ds \right] \quad (6)$$

With allowance of set boundary conditions (3)-(4), after collecting of terms and input of new factors, the ratio (6) can be presented as a linear algebraic equation, computed for the point  $k$ :

$$\varphi_{k1} T_{\gamma_{01}} + \varphi_{k2} T_{\gamma_{02}} + \dots + \varphi_{kn} T_{\gamma_{0n}} - \varphi_{k\gamma_0} T_0 - \varphi_{k\gamma_i} T_i - 2\pi T_k = 0 \quad (7)$$

where  $n$  - is the quantity of sites of a partition of outside outline of the blade  $\ell_{\gamma_0}$  ( $\ell_{\gamma_i}$  on  $i=0$ ) on small sections  $\Delta S_0$  ( $\Delta S_i$  on  $i=0$ ),  $m$  - is the quantity of sites of a partition of outside outlines of all cooling channels  $\ell_{\gamma_i}$  ( $i=1, M$ ) on small sections  $\Delta S_i$ .

Let us note, that unknowns in (7) except the unknown of true value  $T_k$  in the  $k$  point are also mean on sections of outlines partition  $\Delta S_0$  and  $\Delta S_i$  temperatures  $T_{\gamma_{01}}, T_{\gamma_{02}}, \dots, T_{\gamma_{0n}}, \dots, T_{\gamma_{im}}$  (total number  $n+m$ ).

From the ratio (7) we shall receive the required temperature for any point, using (5):

$$T(x, y) = \frac{1}{2\pi} [\varphi_{k1} T_{\gamma_{01}} + \varphi_{k2} T_{\gamma_{02}} + \dots + \varphi_{kn} T_{\gamma_{0n}} + \dots + \varphi_{km} T_{\gamma_{m1}} - \varphi_{k\gamma_0} T_0 - \varphi_{k\gamma_i} T_{\gamma_i}], \quad (8)$$

where

$$\begin{aligned} \varphi_{k1} &= \int_{\Delta S_{01}} \frac{\partial(\ell n R_k)}{\partial n} ds - \frac{\alpha_{01}}{\lambda_1} \int_{\Delta S_{01}} \ell n R_k ds \\ &\quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ \varphi_{kn} &= \int_{\Delta S_{0n}} \frac{\partial(\ell n R_k)}{\partial n} ds - \frac{\alpha_{0n}}{\lambda_n} \int_{\Delta S_{0n}} \ell n R_k ds \\ \varphi_{k\gamma_0} &= \frac{\alpha_{01}}{\lambda_1} \int_{\Delta S_{01}} \ell n R_k ds + \dots + \frac{\alpha_{0n}}{\lambda_n} \int_{\Delta S_{0n}} \ell n R_k ds \\ \varphi_{k\gamma_{ii}} &= \frac{\alpha_{01}}{\lambda_1} \int_{\Delta S_{i1}} \ell n R_k ds + \dots + \frac{\alpha_{im}}{\lambda_m} \int_{\Delta S_{im}} \ell n R_k ds \end{aligned}$$

In contradistinction from [4], [5], the discretization of the outline  $\gamma_i$  ( $i = \overline{1, M}$ ) was carried out by a many discrete points and integrals that are included in the equations as logarithmic potentials, was calculated approximately with the following ratios:

$$\int_{\Delta S_{\gamma_i}} \frac{\partial(\ell n R_k)}{\partial n} ds \approx \frac{\partial(\ell n R_k)}{\partial n} \Delta S_{\gamma_i} \quad (9)$$

$$\int_{\Delta S_{\gamma_i}} \ell n R_k ds \approx \ell n R_k \Delta S_{\gamma_i}, \quad (10)$$

(here  $\Delta S_i \in L = \bigcup_{i=0}^M \gamma_i$ ;  $l_i = \int ds$ ) In contrast to [4] we offer to decide the given boundary value problem (2)-(4) as follows. We suppose that the temperature distribution  $T(x,y)$  should be retrieved as:

$$T(x, y) = \int_{\Gamma} \rho \ell n R^{-1} ds, \quad (11)$$

where  $\Gamma = \bigcup_{i=0}^M \gamma_i$  - smooth closed Jordan curves;  $M$  - quantity of cooled channels;  $\rho = \bigcup_{i=0}^M \rho_i$  - density of a logarithmic potential uniformly distributed on  $\gamma_i$ ;  $s = \bigcup_{i=0}^M s_i$  - arc coordinate from the point  $(x_i, y_i)$ . Thus curves  $\Gamma = \bigcup_{i=0}^M \gamma_i$  are positively oriented and are given in a parametric kind:  $x(s), y=y(s), s \in [0, L]$ ;  $L = \int_{\Gamma} ds$ .

Using Boundary Integral Equations Method (Boundary Element Method) and expression (11) we shall put problem (2)-(4) to the following system of boundary integral equations:

$$\rho(s) - \frac{1}{2\pi} \int_{\Gamma} (\rho(s) - \rho(\xi)) \frac{\partial}{\partial n} \ell n R(s, \xi) d\xi = \frac{\alpha_i}{2\pi\lambda} (T - \int_{\Gamma} \rho(s) \ell n R^{-1} ds), \quad (12)$$

where  $R(s, \xi) = ((x(s) - x(\xi))^2 + (y(s) - y(\xi))^2)^{1/2}$

For the singular integral operators evaluation, which are included in (12), the discrete operators of the logarithmic potential with simple and double layer are investigated, their connection and evaluations in modules term of the continuity (evaluation such as assessments by A. Zigmound are obtained) are shown.

Let us formulate the theorem. Suppose that the following condition holds:

where

$$(L_{\tau, \varepsilon} f)(z) = \sum_{z_{m, e} \in \tau(z)} \left( \frac{f(z_{k, e+1}) + f(z_{k, e})}{2} - f(z) \right) \cdot \frac{(y_{k, e+1} - y_{k, e})(x_{k, e} - x) - (x_{k, e+1} - x_{k, e})(y_{k, e} - y)}{|z - z_{k, e}|^2} + \pi f(z)$$

$(L_{\tau, \varepsilon} f)(z)$  - two-parameter quadrature formula (depending on  $\tau$  and  $\varepsilon$  parameters) for logarithmic double layer potential;  $\tilde{f}(z)$  - double layer logarithmic potential operator;  $C(\Gamma)$  - constant, dependent only from  $\Gamma$ ;  $\omega_f(x)$  - module of the

$$\int_0^{\infty} \frac{\omega_{\xi}(x)}{x} < +\infty$$

and let (12) have the solution  $f^* \in C_{\Gamma}$  (the set of continuous functions on  $\Gamma$ ). Then  $\exists N_0 \in \mathbb{N} = \{1, 2, \dots\}$  is such that the discrete system  $\forall N > N_0$  obtained from (12) by using the discrete double layer potential operator (its properties has been studied), has unique solution

$$f_{jk}^{(N)}, k = \overline{1, m}; j = \overline{1, n};$$

$$|f_{jk}^* - \hat{f}_{jk}^{(N)}| \leq C(\Gamma) \left( \int_0^{\varepsilon_N} \frac{\omega_{\xi}(x) \omega_{f^*}(x)}{x} dx + \varepsilon \int_{\varepsilon_N}^{L/2} \frac{\omega_{\xi}(x) \omega_{f^*}(x)}{x} dx + \omega_{f^*}(\|\tau_N\|) \int_0^{L/2} \frac{\omega_{f^*}(x)}{x} dx + \|\tau_N\| \int_{\varepsilon_N}^{L/2} \frac{\omega_{f^*}(x)}{x} dx \right),$$

where  $C(\Gamma)$  - is a constant, depending only on  $\|\tau_N\|_{N=1}^{\infty}$  - sequence of partitions of  $\Gamma$ ;  $\{\varepsilon_N\}_{N=1}^{\infty}$  - sequence of positive numbers such that the pair  $(\|\tau_N\|, \varepsilon_N)$  satisfies the condition  $2 < \varepsilon \|\tau\|^{-1} < p$ .

Let  $\varepsilon \in (0, d/2)$  where  $d$  - is diameter  $\Gamma$  and the splitting  $\tau$  is such, that satisfies the condition

$$p' \geq \frac{\delta}{\|\tau\|} \geq 2$$

Then for all  $\psi \in C_{\Gamma}$  ( $C_{\Gamma}$  - space for all functions continuous on  $\Gamma$ ) and  $z \in \Gamma$ , ( $z = x + iy$ )

$$|(L_{\tau, \varepsilon} f)(z) - \tilde{f}(z)| \leq C(\Gamma) \left( \|f\|_C \varepsilon \ln \frac{2d}{\varepsilon} + \omega_f(\|\tau\|) + \|\tau\| \ln \frac{2d}{\varepsilon} + \|f\|_C \omega_z(\|\tau\|) \right);$$

$$|(L_{\tau, \varepsilon} f)(z) - \tilde{f}(z)| \leq \left( C(\Gamma) \int_0^{\varepsilon} \frac{\omega_f(x) \omega_f(x)}{x^2} dx + \omega_f(\|\tau\|) \int_{\varepsilon}^d \frac{\omega_f(x)}{x} dx + \|\tau\| \int_{\varepsilon}^d \frac{\omega_f(x)}{x^2} dx \right)$$

continuity of function  $f$ ;

$$(I_{\tau, \varepsilon} f)(z) = \sum_{z_{m, e} \in \tau(z)} \frac{f(z_{k, j+1}) + f(z_{k, j})}{2} \cdot \ln \frac{1}{|z_{k, j+1} - z_{k, j}|}$$

$(I_{\tau,\varepsilon} f)(z)$  - two-parameter quadrature formula (depending on  $\tau$  and  $\varepsilon$  parameters) for logarithmic potential simple layer;  $f(z)$  simple layer logarithmic potential operator;

$$\begin{aligned} z_{k,e} \in \tau, z_{k,e} = x_{k,e} + iy_{k,e}, \\ \tau(z) = \left\{ z_{k,e} \mid |z_{k,e} - z| > \varepsilon \right\}, \\ \tau_k = \{z_{k,1}, \dots, z_{k,m_k}\}, z_{k,1} \leq z_{k,2} \leq \dots \leq z_{k,m_k}, \\ \|\tau\| = \max_{j \in (1, m_k)} |z_{k,j+1} - z_{k,j}| \end{aligned}$$

and are developed an effective from the point of view of realization on computers, numerical methods, based on constructed two-parametric quadrature processes for the discrete operators logarithmic potential of double and simple layer (their systematic errors are estimated, methods of quadratures are mathematically proved for the approximate solution of Fredholm I and II boundary integral equations using Tikhonov regularization and an appropriate theorems are proved) [13], [18], [19].

The given calculating technique of the blade temperature field can be applied also to blades with the plug-in deflector. On consideration blades with deflectors in addition to boundary condition of the III kind adjoin also interfaces conditions between segments of the outline partition as equalities of temperatures and heat flow:

$$\begin{aligned} T_v(x, y) &= T_{v+1}(x, y) \\ \frac{\partial T_v(x, y)}{\partial n} &= \frac{\partial T_{v+1}(x, y)}{\partial n} \end{aligned}$$

where  $v$ - number of segments of the outline partition of the blade cross-section;  $x, y$  - coordinates of segments.

At finding  $T$  optimal values, is necessary to solve the inverse problem of heat conduction. It is necessary at first to find a solution of the heat conduction direct problem with boundary condition of III kind from a gas part and boundary conditions of I kind from a cooling air part:

$$T_v(x, y)|_{\gamma_0} = T_{i_0},$$

where  $T_{i_0}$  - known optimum temperature of the blade wall on the part of cooling air.

The multiples computing experiments with the use of BIEM for calculation the temperature fields of gas turbine blades have showed that for practical calculations in this approach, the discretization of the integrations areas can be conducted with smaller quantity of discrete points. Thus the reactivity of developed algorithms rises.

The accuracy of temperature fields of cooling details is determined by the reliability of the boundary conditions for heat exchange that are laid down in the calculation.

To calculate the velocity of the gas flow along the contour of the blade profile, the methods of direct hydrodynamic problems of cascades are used, based on numerical realization of integral equations with some singularity. The problem is reduced to the solution of boundary integral equations for the components of complex flow potential - velocity potential  $\varphi$  and current function  $\psi$ , which differ from the existing [2] by efficiency in numerical realization.

The velocity field in the flow region of the airfoil cascades can be calculated by differentiating the values of the velocity potential along the contour, found from the solution of the integral equation:

$$\varphi(x_k, y_k) = V_\infty (x_k \cos \alpha_\infty + y_k \sin \alpha_\infty) \pm \frac{1}{2\pi} \Gamma \theta_B \mp \frac{1}{2\pi} \oint_{S^+} \varphi(S) d\theta$$

where  $\varphi(x_k, y_k)$  - velocity potential value;  $V_\infty$  - average vectoral velocity of the incident flow;  $\alpha_\infty$  - angle between vector  $\bar{V}_\infty$  and the axis of airfoil cascade;  $\Gamma$  - speed circulation;  $\theta_B$  - corresponds to an output profile edge.

The distribution of the velocity potential along the outline is obtained from the solution of the following system of linear algebraic equations:

$$\varphi_j \pm \sum_{i=1}^n \varphi_i (\theta_{j,i+1} - \theta_{j,i-1}) = V_\infty (x_{kj} \cos \alpha_\infty + y_{kj} \sin \alpha_\infty) \pm \frac{1}{2\pi} \Gamma \theta_{j,B}$$

where  $i = 2n - 1$ ,  $j = 2n$ ,  $n$  - number of sections.

The values of gas flow velocity are determined by differentiating the velocity potential along the outline  $S$ , that is  $V(s) = d\varphi/ds$  using the following formulas of numerical differentiation [21]:

$$V_i = \frac{1}{12\Delta s} (3\varphi_{i+1} + 10\varphi_i - 18\varphi_{i-1} + 6\varphi_{i-2} - \varphi_{i-3})$$

- for output and input edges;

$$V_i = \frac{1}{12\Delta s} ((\varphi_{i-2} - \varphi_{i+2}) - 8(\varphi_{i-1} - \varphi_{i+1}))$$

- for back and pressure side.

The velocity distribution along the profile contour, in contrast to [1], can be determined by solving also the integral equation obtained for the current function  $\psi$ :

$$\psi = V_\infty (y \cos \alpha_\infty - x \sin \alpha_\infty) \mp \frac{1}{2\pi} \oint_{S^+} V \ln \sqrt{sh^2 \frac{\pi}{t} (x - x_k) + \sin^2 \frac{\pi}{t} (y - y_k)} ds$$

leading it to the following algebraic form:

$$\psi = \psi_\infty \mp \frac{1}{2\pi} \sum_{i=1}^n V_i \ln \left\{ \sqrt{sh^2 \left[ \frac{2\pi}{t} (x - x_k) \right] - \sin^2 \left[ \frac{2\pi}{t} (y - y_k) \right]} \right\} \Delta s_i,$$

where  $\psi_\infty = V_\infty (y \cos \alpha_\infty - x \sin \alpha_\infty)$ . Calculated velocity distribution data along the contour are the initial ones for determining the external conditions of heat exchange.

To calculate local values  $\alpha_\Gamma$ , was used Central Research and Design Boiler and Turbine Institute method developed by L.M. Zysina - Molojen, which uses the ratio of integral energy for thermal boundary layer, written in the variables of AA. Dorodnitsyn, which allows in a unified form to present solutions for laminar, transition and turbulent boundary layers [3], [5], [14], [15]. To make corrections to the basic value  $\alpha_\Gamma$ , recommendations of Central Research and Design Boiler and Turbine Institute and Kharkiv Polytechnic Institute, confirmed by calculation and experiment, were used [5], [14].

In determining the internal boundary conditions of heat exchange, the interrelation of internal geometric and hydrodynamic models with thermal ones, which characterize the temperature field of the blade body, is used. The complex of parameters, which combines thermal and hydraulic characteristics of the cooling system, has a form [5], [14]:

$$\alpha_B \cdot F_B = f(\alpha_\Gamma, Q_\Gamma, T_{\Gamma\Gamma}, T_{B\Gamma}, \lambda_B, \mu_B, \lambda_\Gamma).$$

At the same time, in fact, an optimization task is performed with a preliminary specification of permissible according to the strength temperature conditions of walls with gas  $T_{\Gamma\Gamma}$  and air  $T_{B\Gamma}$  sides, taking into account its extreme unevenness.

The problem of internal hydrodynamics of the cooling system is considered in the example of a blade with plug-in deflector.

For thin-walled deflector blades with a transverse flow of air, the search for an optimal design of the cooling system previously is carried out by detecting of superheated sections. To determine the local heat transfer coefficients of the coolant  $\alpha_B$  a preliminary distribution of the flow in the cooling channels should be provided. The value of the air flow  $G_B$  for cooling of individual sections and entire thin-walled shell of the deflector blade can be determined from the following dependence:

$$G_B = \frac{\mu_B F_B}{d_B} \left( \frac{d_B}{\lambda_B \cdot C} \right)^{\frac{1}{n}} \left[ \frac{\alpha_\Gamma (\psi_\Gamma - 1) \kappa_\phi}{1 - \frac{2Bi(\psi_\Gamma - 1) \kappa_\phi}{1 + \kappa_\phi} - \psi_B} \right]^{\frac{1}{n}},$$

where  $\psi_\Gamma$ ,  $\psi_B$  - temperature coefficients of gas and air;  $\kappa_\phi$  - coefficient of shape;  $d_B$  - characteristic size in the formula  $Re_B$ ;  $\mu_B$ ,  $\lambda_B$  - coefficients of dynamic viscosity and thermal conductivity of the cooler;  $Bi$  - Bio criterion for the blade wall;  $F_B$  - total area of air passage;  $C$ ,  $n$  - coefficient and exponent in the criterial ratio of heat transfer

$Nu_B = C Re_B^n$  for the cooling section under consideration.

To determine the flow distribution in the cooling system of the blade, an equivalent hydraulic outline (EHC) is carried out.

When compiling EHC, the entire flow path of the cooler is divided into a set of interconnected sections - typical elements (channels), for each of which there is the possibility uniquely to determine the values of the coefficients of hydraulic resistance. The places of connection of typical elements in EHC are replaced by nodal points in which the flow, fusion and separation of the coolant flow presumably occurs without pressure changing. Typical elements and nodal points are interconnected in the same sequence as the sections of CS.

The flow of the coolant in branched circuits is described by the 1st Kirchhoff law [5], [14], [15]:

$$f_1 = \sum_{j=1}^m G_{ij} = \sum_{j=1}^m \text{sign}(\Delta p_{ij}) k_{ij} \sqrt{\Delta p_{ij}}; \quad i = 1, 2, 3 \dots n, \quad (13)$$

where  $G_{ij}$  - cooler consumption on the branch,  $i - j$ ,  $m$  - number of branches attached to  $i$ -node,  $n$  - number of internal nodes of the hydraulic circuit,  $\Delta p_{ij}$  - differential pressure of the coolant on the branch  $i - j$ . In this formula, the coefficient of hydraulic conductivity of the branch ( $i - j$ ) is defined as follows [5], [9]:

$$k_{ij} = \sqrt{2 f_{ij}^2 \cdot p_{ij} / \xi_{ij}}, \quad (14)$$

where  $f_{ij}$ ,  $p_{ij}$ ,  $\xi_{ij}$  - respectively, average cross-sectional area of the channel ( $i - j$ ), coolant flow density in this section, and total coefficient of hydraulic resistance of the branch.

The system of nonlinear algebraic equations (13) is solved by the Zeidel method with acceleration according to the following formula [5], [9], [22]:

$$p_i^{k+1} = p_i^k - f_i^k / (\partial f / \partial p)^k,$$

where  $k$  - iteration number,  $p_i^k$  - coolant pressure in  $i$  - section of the hydraulic circuit. The coefficients of hydraulic resistance  $\xi_{ij}$ , within (14), can be determined from the empirical correlation found in special literature [5].

To calculate heat transfer in the channels of cooling systems of the blades, criterion correlation are mainly used. Value  $\alpha_B$  in the region of the leading edge of the blade with internal segment finings, air blown by one row of round jets through the holes in the nozzle of the deflector, is calculated by the dependence [5], [14]:

$$Nu = C Re^{0.98} Pr^{0.43} / (L / b_{equ}),$$

$b_{equ} = \pi d_0^2 / 2t_0$  - width of an equivalent in terms of area gap;

$d_0$ ,  $t_0$  - diameter and pitch of the holes in the deflector nozzle. Criterion  $Re$  in this formula is determined by the jet speed at the outlet through the deflector holes, as a characteristic size, the length of entering bypass of leading edge  $L$  is used.

In areas of jet blasting of surfaces, except leading edge zone, one can use an empirical dependence [5], [14]:

$$Nu = 0.018 \left( 0.36\bar{\delta}^2 - 0.34\bar{\delta} + 0.56 - 0.1\bar{h} \right) \bar{S}_x \cdot (G_c f_k / G_k f_c)^k \cdot Re^{0.8}, \quad (15)$$

where  $\bar{\delta} = \delta/d$  - deflector relative thickness  $\bar{h} = h/d$  - relative height of the channel between the deflector and the blade wall;  $\bar{S} = S/d$  - relative pitch of the jet system;  $d$  - perforation diameter;  $k = 0.25 + 0.5\bar{h}$ . Criterion  $Re$  in (15) is determined by hydraulic diameter of the transverse channel  $L = 0.75 - 0.45\bar{\delta}$  and coolant flow rate behind the deflector perforation zone.

In calculations in each iterative process, it is necessary to check the capacity of the cooling path to the total outlet pressure, calculated, respectively, through the loss of total pressure and the reduced air flow rate from the blade, taking into account its heating. Thus, the necessary heat transfer coefficient  $\alpha_B$  is carried out by varying the complex of geometrical parameters of the cooling circuit and regime parameters of the cooler [5], [14], [15].

### III. CONCLUSION

- 1) The developed methods for profiling, calculating the temperature fields and cooling parameters in cooling systems were realized during the computationally experimental studies of the thermal state of the nozzle apparatus of the 1st stage of the high-pressure turbine of combustion turbine unit GTN-6U OAO "UralTurbo" (ГТН-6У ОАО "УралТурбо") (Yekaterinburg, Russia). The following geometrical and regime parameters of the gas flow, obtained by calculation, were used: cascade step -  $t = 41.5 \text{ mm}$ , inlet gas speed to cascade -  $V_1 = 156 \text{ m/c}$ , outlet gas speed to cascade -  $V_2 = 512 \text{ m/c}$ , outlet superficial gas speed -  $\lambda_{1ao} = 0.891$ ; inlet gas speed vector angle -  $\alpha_1 = 0.7^\circ$ , gas flow temperature and pressure: on entrance to the stage -  $T_z^* = 1333 \text{ K}$ ,  $p_z^* = 1.2095 \cdot 10^6 \text{ Pa}$ , on exit from the stage -  $T_{z1} = 1005 \text{ K}$ ,  $p_{z1} = 0.751 \cdot 10^6 \text{ Pa}$ .
- 2) The geometric model of the blade is obtained (Fig. 1), as well as diagrams of speed distributions  $V$  and convective heat exchange local coefficient of gas  $\alpha_z$  along the profile contour (Fig. 2) are received.
- 3) The geometrical model (Fig. 3) and the cooling tract equivalent hydraulic scheme (Fig. 4) are developed. Cooler basic parameters in the cooking system and the temperature field of the blade cross section (Fig. 5) were determined.

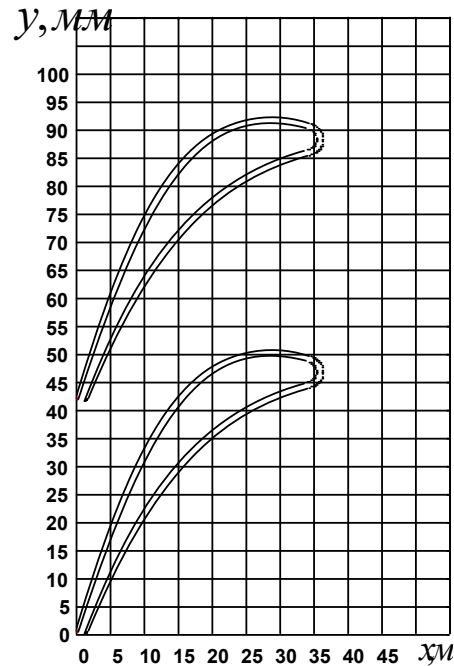


Fig. 1 Geometrical model of the blade

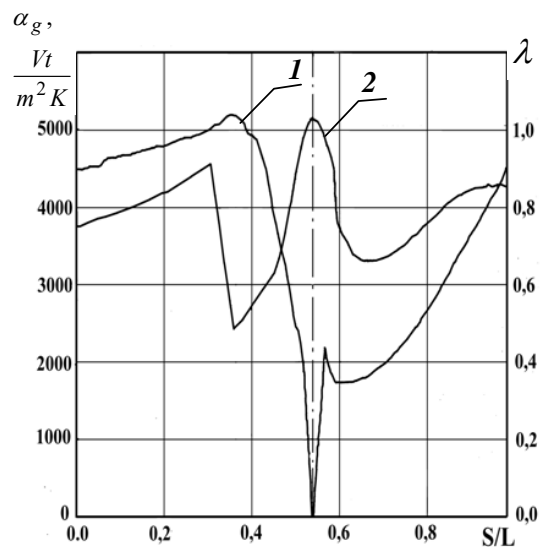


Fig. 2 Distribution graph

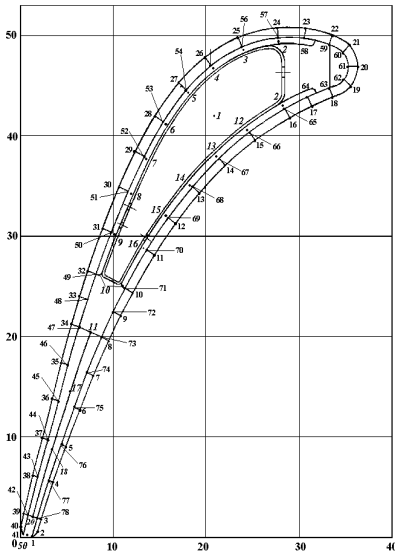


Fig. 3 Geometrical model

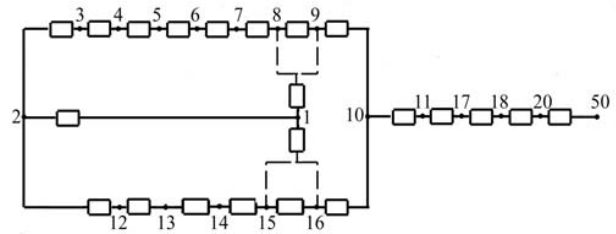


Fig. 4 Scheme of experimental nozzle blade cooling system

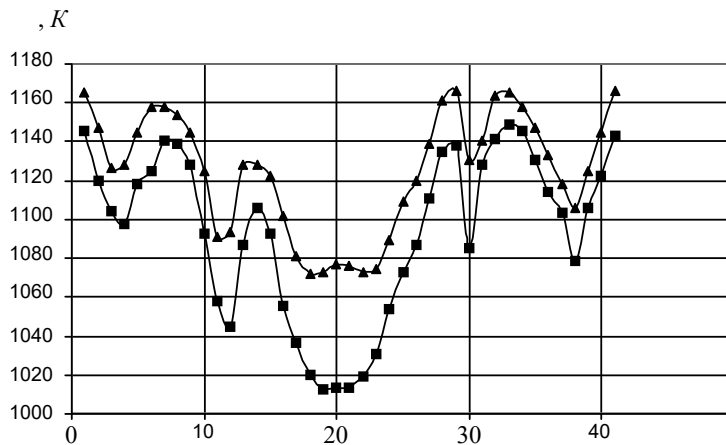


Fig. 5 Temperature field of blade cross section

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