Numerical Implementation of an Interfacial Edge Dislocation Solution in a Multi-Layered Medium

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Abstract—A novel method is presented for obtaining the stress field induced by an edge dislocation in a multilayered composite. To demonstrate the applications of the obtained solution, we consider the problem of an interfacial crack in a periodically layered bimaterial medium. The crack is modelled as a continuous distribution of edge dislocations and the Distributed Dislocation Technique (DDT) is utilized to obtain numerical results for the energy release rate (ERR). The numerical implementation of the dislocation solution in MATLAB is also provided.

Keywords—Distributed dislocation technique, Edge dislocation, Elastic field, Interfacial crack, Multi-layered composite.

I. INTRODUCTION

MULTI-LAYERED structures and components are widely utilized in engineering applications [1]-[5]. These structures are also prevalent in nature, ranging from the nano-scale building blocks of bio-materials such as nacre, tooth enamel and bone [6], [7] to the stratified rock formations in the earth's crust [8]-[12].

Accurate analysis of fracture problems in multi-layered laminates is of great practical interest, for e.g. in the study of delamination damage [13], [14], in the design of crack arresting interfaces [15]-[18], when describing the toughness behavior of natural composites [19] and in the modelling of hydraulic fracture propagation in oil/gas reservoirs [20]. The distributed dislocation technique, which is based on the pioneering work of Eshelby [21], can provide an efficient procedure for analyzing a variety of crack problems in such multi-layered structures. In this technique, the mixed boundary-value crack problem is reduced to a system of coupled singular integral equations of the Cauchy type with kernels formulated in terms of the unknown displacement discontinuities [22]-[25]. The literature is replete with solutions to crack problems in multi-layered composites obtained by the distributed dislocation approach [26]-[30]. However, in every case we find that there is some restriction on the geometry tackled; either because the crack is chosen to be normal or parallel to the interfaces.

In this paper, we consider a general multi-layered composite, composed of perfectly bonded isotropic elastic layers, and present the solution for the elastic field induced by an interfacial edge dislocation. There are three distinct

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advantages of the present approach, (1) the Fredholm kernels of the governing singular integral equations can be readily formulated in terms of the edge dislocation solution, which already satisfies the boundary conditions on the surfaces or interfaces, (2) the crack's orientation does not need to be restricted, and (3) both interfacial and embedded crack problems, as well as problems involving multiple interacting cracks can be considered within single framework.

Additionally, the present solution is more general than other available solutions for embedded or non-interfacial edge dislocations in: a film-substrate medium [31], a layer joining two substrates [32] and a general multilayered composite [33]. The solution is developed using the complex potential method of Muskhelishvili and the Fourier transform method for strip problems, based on the approach outlined by [32] for an embedded dislocation in an elastic layer.

II. PROBLEM FORMULATION

Consider the plane elasticity problem of a multi-layered medium, composed of $m + n$ layers, as shown in Fig. 1. The elastic properties of the layers are defined by the shear modulus, μ and Poisson's ratio, ν. The top and bottom layers are of infinite extent and the intermediate layers are of arbitrary thickness.
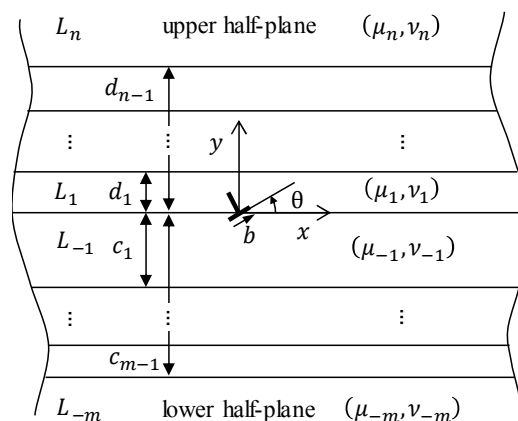

Fig. 1 An interfacial edge dislocation in a multi-layered medium

Finite composite structures can be modelled by setting the shear modulus of elasticity of the top and bottom layers to zero, while the embedded dislocation problem corresponds to the case when $\mu_{-1} = \mu_1$ and $\nu_{-1} = \nu_1$. The layers are perfectly bonded, except for an edge dislocation along the interface between layers L_1 and L_{-1}. The origin of the coordinate system lies at the location of the edge dislocation and the x-axis is

aligned with the interfaces.

The solution to the plane elasticity problem shown in Fig. 1 is obtained using the approach outlined in [32]. First, we consider a bimaterial medium composed of two homogenous elastic half-planes, with the material interface along $y = 0$. The elastic properties of the material above the interface ($y > 0$) are denoted by μ_1, ν_1 and those of the material below the interface ($y < 0$) are denoted by μ_{-1}, ν_{-1}. The stress and displacement field due to an interfacial edge dislocation in this medium are well known.

The material in the region, $d_i < y < d_{i+1}$, is then allowed to transform to material μ_{i+1}, ν_{i+1} ($i = 1, \ldots, n-1$) and the material in the region, $-c_{j+1} < y < -c_j$, is allowed to transform to material $\mu_{-(j+1)}, \nu_{-(j+1)}$ ($j = 1, \ldots, m-1$). The transformation occurs in a manner which does not alter the stress state anywhere, but generates a displacement mismatch at the interfaces. The displacement jump at the interfaces is denoted by $\Delta u(x, d_i) + i\Delta v(x, d_i)$ for $y > 0$ and $\Delta u(x, -c_j) + i\Delta v(x, -c_j)$ for $y < 0$.

Finally, the problem of the multi-layered medium is considered, with the displacement jumps prescribed at the interfaces equal in magnitude but opposite in sign to those obtained previously. The corrective stress field required to generate these displacement jumps is then evaluated. The net stress field is obtained as a superposition of this corrective stress field and the known solution for stress field in a bimaterial due to an interfacial edge dislocation.

III. PROBLEM 1: EDGE DISLOCATION AT A BIMATERIAL INTERFACE

Consider a composite medium with a planar interface along $y = 0$, with an interfacial dislocation at the origin. The elastic properties of the material above the interface ($y > 0$) are denoted by μ_1, ν_1 and those of the material below the interface ($y < 0$) are denoted by μ_{-1}, ν_{-1}. The displacement field in the medium is continuous everywhere, except for the half-plane given by $x < 0$ and $y = 0$, along which it is discontinuous. The jump in displacement is given by

$$u(x, 0^+) + iv(x, 0^+) - u(x, 0^-) - iv(x, 0^-) = (b_x + ib_y)H(-x) \quad (1)$$

where b_x and b_y are the glide and climb components of the Burger's vector and $H(x)$ is the Heaviside step function.

A. Stress and Displacement Fields

The stress and displacement fields in each region can be conveniently expressed in terms of Muskhelishvili's complex potentials $\varphi(z)$ and $\psi(z)$ according to [34]

$$\sigma_{xx} + \sigma_{yy} = 2[\varphi'(z) + \overline{\varphi'(z)}] \quad (2)$$
$$\sigma_{yy} - \sigma_{xx} + 2i\sigma_{xy} = 2[\bar{z}\varphi''(z) + \psi'(z)] \quad (3)$$
$$2\mu(u + iv) = \kappa\varphi(z) - z\overline{\varphi'(z)} - \overline{\psi(z)} \quad (4)$$

where μ is the shear modulus, $\kappa = 3 - 4\nu$ is Kolosov's constant. For an interfacial edge dislocation, the complex potentials are given by [35], [36] as

$$\varphi_1(z) = \Gamma_1 \frac{b}{i\pi} \ln z, \quad \psi_1(z) = \Gamma_{-1} \frac{\bar{b}}{-i\pi} \ln z \quad (5)$$

in region 1 i.e. $y > 0$ and

$$\varphi_{-1}(z) = \Gamma_{-1} \frac{b}{i\pi} \ln z, \quad \psi_{-1}(z) = \Gamma_1 \frac{\bar{b}}{-i\pi} \ln z \quad (6)$$

in region 2 ($y < 0$), respectively. Here $z = x + iy$, $b = b_x + ib_y$ and the constants Γ_1 and Γ_{-1} are defined as

$$\Gamma_1 = \frac{\mu_1 \mu_{-1}}{\mu_1 + \mu_{-1}\kappa_1} = \frac{\mu_1}{\kappa_1 + 1} \frac{1 + \alpha}{1 - \beta}$$
$$\Gamma_{-1} = \frac{\mu_1 \mu_{-1}}{\mu_1 \kappa_{-1} + \mu_{-1}} = \frac{\mu_1}{\kappa_1 + 1} \frac{1 + \alpha}{1 + \beta} \quad (7)$$

The constants α and β are Dundur's elastic mismatch parameters, which are defined as:

$$\alpha = \frac{\mu_1(\kappa_{-1} + 1) - \mu_{-1}(\kappa_1 + 1)}{\mu_1(\kappa_{-1} + 1) + \mu_{-1}(\kappa_1 + 1)}$$
$$\beta = \frac{\mu_1(\kappa_{-1} - 1) - \mu_{-1}(\kappa_1 - 1)}{\mu_1(\kappa_{-1} + 1) + \mu_{-1}(\kappa_1 + 1)} \quad (8)$$

By substituting (5) into (2), (3), the stress field in the upper half-space, $y > 0$, can be obtained as

$$\sigma_{xx}(x, y > 0) = \frac{b_x}{\pi}[-\frac{y}{r^4}\{(5\Gamma_1 + \Gamma_{-1})x^2 + (\Gamma_1 + \Gamma_{-1})y^2\}] + \frac{b_y}{\pi}[+\frac{x}{r^4}\{(3\Gamma_1 - \Gamma_{-1})x^2 - (\Gamma_1 + \Gamma_{-1})y^2\}] \quad (9)$$

$$\sigma_{yy}(x, y > 0) = \frac{b_x}{\pi}[+\frac{y}{r^4}\{(\Gamma_1 + \Gamma_{-1})x^2 - (3\Gamma_1 - \Gamma_{-1})y^2\}] + \frac{b_y}{\pi}[+\frac{x}{r^4}\{(\Gamma_1 + \Gamma_{-1})x^2 - (5\Gamma_1 + \Gamma_{-1})y^2\}] \quad (10)$$

$$\sigma_{xy}(x, y > 0) = \frac{b_x}{\pi}[+\frac{x}{r^4}\{(\Gamma_1 + \Gamma_{-1})x^2 - (3\Gamma_1 - \Gamma_{-1})y^2\}] + \frac{b_y}{\pi}[+\frac{y}{r^4}\{(3\Gamma_1 - \Gamma_{-1})x^2 - (\Gamma_1 + \Gamma_{-1})y^2\}] \quad (11)$$

Similarly, the displacement field in region 1 ($y > 0$) can be obtained by substituting (5) into (4), which yields

$$2\mu_1 u(x, y > 0) = \frac{1}{\pi}\left[\left\{(\kappa_1\Gamma_1 + \Gamma_{-1})\tan^{-1}\frac{y}{x} + \Gamma_1\frac{2xy}{x^2 + y^2}\right\}b_x + \left\{\left(\frac{\kappa_1\Gamma_1 - \Gamma_{-1}}{2}\right)\ln(x^2 + y^2) - \Gamma_1\frac{x^2 - y^2}{x^2 + y^2}\right\}b_y\right] \quad (12)$$

$$2\mu_1 v(x, y > 0) = \frac{1}{\pi}\left[-\left\{\left(\frac{\kappa_1\Gamma_1 - \Gamma_{-1}}{2}\right)\ln(x^2 + y^2) + \right.\right.$$
$$\left.\Gamma_1\frac{x^2 - y^2}{x^2 + y^2}\right\}b_x + \left\{(\kappa_1\Gamma_1 + \Gamma_{-1})\tan^{-1}\frac{y}{x} - \right. \tag{13}$$
$$\left.\left.\Gamma_1\frac{2xy}{x^2 + y^2}\right\}b_y\right].$$

In (12), (13), κ is Kolosov's constant, which is defined in terms of the Poisson's ratio v as

$$\kappa = \begin{cases} 3 - 4v & \text{in plane strain,} \\ \dfrac{3 - v}{1 + v} & \text{in plane stress.} \end{cases} \tag{14}$$

The stresses and displacements in the lower half-space, i.e. $y < 0$, can be obtained by interchanging the subscripts 1 and -1 in (9)-(13) or by replacing β with $-\beta$ if Dundur's parameters are used. In the case when $\mu_1 = \mu_{-1} = \mu_0$ and $\kappa_1 = \kappa_{-1} = \kappa_0$, then $\Gamma_1 = \Gamma_{-1} = \mu_0/(\kappa_0 + 1)$ and (9)-(13) yield the standard results for the elastic fiend due to an edge dislocation in an infinite homogenous medium.

B. Displacement Discontinuities Induced at the Interfaces

The material in the region, $d_i < y < d_{i+1}$, is then allowed to transform to material μ_{i+1}, v_{i+1} $(i = 1, \dots, n - 1)$ and the material in the region, $-c_{j+1} < y < -c_j$, is allowed to transform to material $\mu_{-(j+1)}, v_{-(j+1)}$ $(j = 1, \dots, m - 1)$. However, the Muskhelishvili potentials given by (5), (6) are kept fixed. As a result, the stresses given by (9)-(11) remain unaltered but displacement jumps are registered at the interfaces when using (4) across a material interface. The resulting displacement jump is defined as

$$\Delta u(x, d_i) + i\Delta v(x, d_i) = u(x, d_i^+) + iv(x, d_i^+) - \\ u(x, d_i^-) - iv(x, d_i^-) \tag{15}$$

for an interface located above the origin, and

$$\Delta u(x, -c_j) + i\Delta v(x, -c_j) = u(x, -c_j^+) + iv(x, -c_j^+) \\ -u(x, -c_j^-) - iv(x, -c_j^-) \tag{16}$$

for an interface located below the origin, i.e. $y < 0$. The usual convention is adopted for evaluating the one-sided limits, i.e. $y \to d_i^+$ means that y approaches d_i from above and $y \to d_i^-$ means that y approaches d_i from below. The jump in displacement gradient can be evaluated substituting (12)-(13) into (15), (16). In boundary matching problem, such as the present one, the displacement mismatch is expressed more conveniently in differential from, thus avoiding the integration constants associated with rigid-body motions. The jumps in displacement gradient due to the dislocation at an interface above the origin are specified by

$$\Delta\frac{\partial u}{\partial x}(x, d_i) = -\frac{b_x}{\pi}\left[\left(\frac{\kappa_{i+1}\Gamma_1 + \Gamma_{-1}}{2\mu_{i+1}} - \frac{\kappa_i\Gamma_1 + \Gamma_{-1}}{2\mu_i}\right)\right.$$
$$\frac{d_i}{(x^2 + d_i^2)} + \left(\frac{\Gamma_1}{2\mu_{i+1}} - \frac{\Gamma_1}{2\mu_i}\right)\frac{2d_i(x^2 - d_i^2)}{(x^2 + d_i^2)^2}\right] +$$
$$\frac{b_y}{\pi}\left[\left(\frac{\kappa_{i+1}\Gamma_1 - \Gamma_{-1}}{2\mu_{i+1}} - \frac{\kappa_i\Gamma_1 - \Gamma_{-1}}{2\mu_i}\right)\frac{x}{(x^2 + d_i^2)} - \right.$$
$$\left.\left(\frac{\Gamma_1}{2\mu_{i+1}} - \frac{\Gamma_1}{2\mu_i}\right)\frac{4xd_i^2}{(x^2 + d_i^2)^2}\right] \tag{17}$$

$$\Delta\frac{\partial v}{\partial x}(x, d_i) = -\frac{b_x}{\pi}\left[\left(\frac{\kappa_{i+1}\Gamma_1 - \Gamma_{-1}}{2\mu_{i+1}} - \frac{\kappa_i\Gamma_1 - \Gamma_{-1}}{2\mu_i}\right)\right.$$
$$\frac{x}{(x^2 + d_i^2)} + \left(\frac{\Gamma_1}{2\mu_{i+1}} - \frac{\Gamma_1}{2\mu_i}\right)\frac{4xd_i^2}{(x^2 + d_i^2)^2}\right] -$$
$$\frac{b_y}{\pi}\left[\left(\frac{\kappa_{i+1}\Gamma_1 + \Gamma_{-1}}{2\mu_{i+1}} - \frac{\kappa_i\Gamma_1 + \Gamma_{-1}}{2\mu_i}\right)\frac{d_i}{(x^2 + d_i^2)} - \right.$$
$$\left.\left(\frac{\Gamma_1}{2\mu_{i+1}} - \frac{\Gamma_1}{2\mu_i}\right)\frac{2d_i(x^2 - d_i^2)}{(x^2 + d_i^2)^2}\right], \tag{18}$$

where $i = 1, \dots, n - 1$. The jump in displacement gradient at an interface located in the region $y < 0$, can be evaluated in a similar manner as

$$\Delta\frac{\partial u}{\partial x}(x, -c_j) = \frac{b_x}{\pi}\left[\left(\frac{\kappa_{-j}\Gamma_{-1} + \Gamma_1}{2\mu_{-j}} - \frac{\kappa_{-(j+1)}\Gamma_{-1} + \Gamma_1}{2\mu_{-(j+1)}}\right)\right.$$
$$\frac{c_j}{(x^2 + c_j^2)} + \left(\frac{\Gamma_{-1}}{2\mu_{-j}} - \frac{\Gamma_{-1}}{2\mu_{-(j+1)}}\right)\frac{2c_j(x^2 - c_j^2)}{(x^2 + c_j^2)^2}\right] +$$
$$\frac{b_y}{\pi}\left[\left(\frac{\kappa_{-j}\Gamma_{-1} - \Gamma_1}{2\mu_{-j}} - \frac{\kappa_{-(j+1)}\Gamma_{-1} - \Gamma_1}{2\mu_{-(j+1)}}\right)\frac{x}{(x^2 + c_j^2)} - \right.$$
$$\left.\left(\frac{\Gamma_{-1}}{2\mu_{-j}} - \frac{\Gamma_{-1}}{2\mu_{-(j+1)}}\right)\frac{4xc_j^2}{(x^2 + c_j^2)^2}\right] \tag{19}$$

$$\Delta\frac{\partial v}{\partial x}(x, -c_j) = -\frac{b_x}{\pi}\left[\left(\frac{\kappa_{-j}\Gamma_{-1} - \Gamma_1}{2\mu_{-j}} - \right.\right.$$
$$\left.\frac{\kappa_{-(j+1)}\Gamma_{-1} - \Gamma_1}{2\mu_{-(j+1)}}\right)\frac{x}{(x^2 + c_j^2)} + \left(\frac{\Gamma_{-1}}{2\mu_{-j}} - \frac{\Gamma_{-1}}{2\mu_{-(j+1)}}\right)$$
$$\frac{4xc_j^2}{(x^2 + c_j^2)^2}\right] + \frac{b_y}{\pi}\left[\left(\frac{\kappa_{-j}\Gamma_{-1} + \Gamma_1}{2\mu_{-j}} - \frac{\kappa_{-(j+1)}\Gamma_{-1} + \Gamma_1}{2\mu_{-(j+1)}}\right)\right.$$
$$\frac{c_j}{(x^2 + c_j^2)} - \left(\frac{\Gamma_{-1}}{2\mu_{-j}} - \frac{\Gamma_{-1}}{2\mu_{-(j+1)}}\right)\frac{2c_j(x^2 - c_j^2)}{(x^2 + c_j^2)^2}\right], \tag{20}$$

where $j = 1, \dots, m - 1$.

IV. PROBLEM 2: DISLOCATION FREE STRIP PROBLEM

The displacement field should be continuous across a perfectly bonded interface. However, using the same Muskhelishvili complex potential for layers with different material properties introduces discontinuity or jump in the displacement field at the bimaterial interfaces. In this section, we consider the bonded system shown in Fig. 1, having stress-free infinite boundaries and prescribed displacement jumps at the interfaces, equal in magnitude, and of opposite sign to those obtained previously. The latter problem is treated with the Airy stress function formulation. The unknown Airy stress function, $\varphi(x, y)$ is determined by matching the free boundary conditions in the far field and by satisfying the traction equilibrium and displacement jump conditions at the interfaces. The corrective stresses and displacement field can be obtained in terms of the solution for $\varphi(x, y)$.

A. General Solution for Elastic Field in a Strip

In the absence of body forces, the Airy's stress function, $\varphi(x, y)$, satisfies the biharmonic equation

$$\nabla^4 \varphi = 0 \tag{21}$$

Fourier transform techniques can be used to obtain a solution to the biharmonic equation when the dependence on one or both of the coordinates is harmonic [37]. The resulting general solution for $\varphi(x, y)$ can be decomposed into its even and odd components according to [32]

$$\varphi(x, y) = \varphi_{b_x}(x, y) + \varphi_{b_y}(x, y), \tag{22}$$

where

$$\varphi_{b_x}(x, y) = b_x \int_0^\infty \left[\left(\frac{A_x}{\xi^2} + \frac{B_x}{\xi} y \right) e^{-\xi y} + \left(\frac{C_x}{\xi^2} + \frac{D_x}{\xi} y \right) e^{\xi y} \right] \cos(\xi x)\, d\xi, \tag{23}$$

and

$$\varphi_{b_y}(x, y) = b_y \int_0^\infty \left[\left(\frac{A_y}{\xi^2} + \frac{B_y}{\xi} y \right) e^{-\xi y} + \left(\frac{C_y}{\xi^2} + \frac{D_y}{\xi} y \right) e^{\xi y} \right] \sin(\xi x)\, d\xi. \tag{24}$$

The corrective stress components can be obtained in terms of $\varphi(x, y)$, as follows

$$\sigma_{xx}^* = b_x \int_0^\infty \left[(A_x + (\xi y - 2)B_x)e^{-\xi y} + (C_x + (\xi y + 2)D_x)e^{\xi y} \right] \cos(\xi x)\, d\xi + b_y \int_0^\infty \left[(A_y + (\xi y - 2)B_y)e^{-\xi y} + (C_y + (\xi y + 2)D_y)e^{\xi y} \right] \sin(\xi x)\, d\xi, \tag{25}$$

$$\sigma_{yy}^* = -b_x \int_0^\infty \left[(A_x + \xi y B_x)e^{-\xi y} + (C_x + \xi y D_x)e^{\xi y} \right] \cos(\xi x)\, d\xi - b_y \int_0^\infty \left[(A_y + \xi y B_y)e^{-\xi y} + (C_y + \xi y D_y)e^{\xi y} \right] \sin(\xi x)\, d\xi, \tag{26}$$

$$\sigma_{xy}^* = b_x \int_0^\infty \left[(-A_x + (1 - \xi y)B_x)e^{-\xi y} + (C_x + (1 + \xi y)D_x)e^{\xi y} \right] \sin(\xi x)\, d\xi + -b_y \int_0^\infty \left[(-A_y + (1 - \xi y)B_y)e^{-\xi y} + (C_y + (1 + \xi y)D_y)e^{\xi y} \right] \cos(\xi x)\, d\xi. \tag{27}$$

The displacement components are given in gradient form in terms of $\varphi(x, y)$, as follows

$$\frac{\partial u^*}{\partial x} = \frac{b_x}{2\mu} \int_0^\infty \left[\left(-A_x + \left(1 - \xi y - \frac{\kappa + 1}{2} \right) B_x \right) e^{-\xi y} + \left(C_x + \left(1 + \xi y - \frac{\kappa + 1}{2} \right) D_x \right) e^{\xi y} \right] \sin(\xi x)\, d\xi - \frac{b_y}{2\mu} \int_0^\infty \left[\left(-A_y + \left(1 - \xi y - \frac{\kappa + 1}{2} \right) B_y \right) e^{-\xi y} + \left(C_y + \left(1 + \xi y - \frac{\kappa + 1}{2} \right) D_y \right) e^{\xi y} \right] \cos(\xi x)\, d\xi, \tag{28}$$

$$\frac{\partial v^*}{\partial x} = \frac{b_x}{2\mu} \int_0^\infty \left[\left(A_x + \left(\xi y - \frac{\kappa + 1}{2} \right) B_x \right) e^{-\xi y} + \left(C_x + \left(\xi y + \frac{\kappa + 1}{2} \right) D_x \right) e^{\xi y} \right] \cos(\xi x)\, d\xi + \frac{b_y}{2\mu} \int_0^\infty \left[\left(A_y + \left(\xi y - \frac{\kappa + 1}{2} \right) B_y \right) e^{-\xi y} + \left(C_y + \left(\xi y + \frac{\kappa + 1}{2} \right) D_y \right) e^{\xi y} \right] \sin(\xi x)\, d\xi. \tag{29}$$

B. Bimaterial Interface Conditions

The unknown constants A, B, C and D in the general solution for $\varphi(x, y)$ associated with each layer can be obtained by utilizing the conditions of displacement continuity and traction equilibrium, written as

$$u(x, y^+) + iv(x, y^+) = u(x, y^-) + iv(x, y^-), \tag{30}$$
$$\sigma_{yy}(x, y^+) + i\sigma_{xy}(x, y^+) = \sigma_{yy}(x, y^-) + i\sigma_{xy}(x, y^-). \tag{31}$$

Since the general solution for $\varphi(x, y)$ is a Fourier integral, the unknown constants can be determined more readily if the bimaterial interface conditions are expressed in the Fourier domain. The traction equilibrium condition (31) yields the following bimaterial interface conditions

$$\tilde{\sigma}_{yy}^*(x, d_i^+) = \tilde{\sigma}_{yy}^*(x, d_i^-), \quad \tilde{\sigma}_{xy}^*(x, d_i^+) = \tilde{\sigma}_{xy}^*(x, d_i^-), \tag{32}$$

$$\tilde{\sigma}_{yy}^*\left(x, -c_j^+\right) = \tilde{\sigma}_{yy}^*\left(x, -c_j^-\right),$$
$$\tilde{\sigma}_{xy}^*\left(x, -c_j^+\right) = \tilde{\sigma}_{xy}^*\left(x, -c_j^-\right), \tag{33}$$

$$\tilde{\sigma}_{yy}^*(x, 0) = \tilde{\sigma}_{yy}^*(x, 0), \quad \tilde{\sigma}_{xy}^*(x, 0) = \tilde{\sigma}_{xy}^*(x, 0). \tag{34}$$

where $i = 1, \dots, n - 1$, $j = 1, \dots, m - 1$ and the tilde symbol represents the Fourier transform with respect to the variable x. The displacement continuity condition (30) requires that

$$\Delta \frac{\partial \tilde{u}^*}{\partial x}(x, d_i) = -\Delta \frac{\partial \tilde{u}}{\partial x}(x, d_i),$$
$$\Delta \frac{\partial \tilde{v}^*}{\partial x}(x, d_i) = -\Delta \frac{\partial \tilde{v}}{\partial x}(x, d_i), \tag{35}$$

$$\Delta \frac{\partial \tilde{u}^*}{\partial x}(x, c_j) = -\Delta \frac{\partial \tilde{u}}{\partial x}(x, c_j),$$
$$\Delta \frac{\partial \tilde{v}^*}{\partial x}(x, c_j) = -\Delta \frac{\partial \tilde{v}}{\partial x}(x, c_j), \tag{36}$$

$$\Delta \frac{\partial \tilde{u}^*}{\partial x}(x, 0) = 0, \quad \Delta \frac{\partial \tilde{v}^*}{\partial x}(x, 0) = 0. \tag{37}$$

Equations (35), (36) imply that the displacement gradient jump due to the correction field must be equal and opposite to the displacement gradient jump due to the dislocation field. The right hand sides of (35), (36) are obtained by taking appropriate Fourier transforms of (17)-(20).

C. Corrective Solution for Multi-Layer Medium

For an arbitrary medium composed of $m + n$ layers, a system of $4(m + n)$ linear equation can be obtained from (32)-(37). A global matrix must be assembled in an appropriate manner to find the $4(m + n)$ unknown constants. The algorithm for assembling the global matrix for a given multi-layered medium is presented in Appendix A. Once the unknown constants are determined, the corrective solution for stresses can be obtained by numerically integrating (25)-(27).

V. EXAMPLE: INTERFACIAL CRACK IN A PERIODIC BIMATERIAL MEDIUM

The edge dislocation solution obtained in the present work is applied to the plane strain problem of an interfacial crack in a composite medium comprising of alternating layers of two materials (Fig. 2). The distributed dislocation technique (DDT) is utilized to obtain the numerical solution to the interfacial crack problem. The details of the numerical procedure involved in solving the interfacial crack problem using DDT are covered in [38]. The numerical results for the normalized energy release rate, $\widehat{G}$, as a function of the layer thickness ratio, h_1/h_2 are shown in Fig. 3. The results are evaluated for Dundur's parameters, $\alpha = 0.8$ and $\beta = 0.2$ and $H_{min} = 1$.
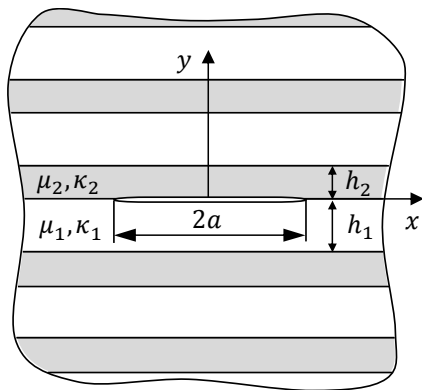

Fig. 2 An interfacial crack in a periodically layered bi-material medium

The normalized energy release rate, $\widehat{G}$ is defined as [29],

$$\widehat{G} = \frac{|K|}{(1 + 4\varepsilon^2)(\pi a)\left(\sigma_{yy}^{\infty}{}^2 + \sigma_{xy}^{\infty}{}^2\right)}, \qquad (38)$$

where K is the complex stress intensity factor, a is the crack half-length and ε is the oscillatory index and σ_{yy}^{∞} and σ_{xy}^{∞} are the applied normal and shear loading. The complex stress intensity factor is defined as [39]

$$K = K_1 + iK_2 = \lim_{r \to 0}\{r^{+i\varepsilon}\sqrt{2\pi r}\left[\sigma_{yy}(r) + i\sigma_{xy}(r)\right]\}, \qquad (39)$$

where r is the distance ahead of the crack tip. The oscillatory index ε is related to Dundur's parameter β according to [38]

$$\varepsilon = \frac{1}{2\pi}\log\left(\frac{1 + \beta}{1 - \beta}\right). \qquad (40)$$

Dundur's elastic mismatch parameters are defined by (8) and the dimensionless parameter H_{min} is defined as:

$$H_{min} = \min\left\{\frac{h_1}{2a}, \frac{h_2}{2a}\right\}. \qquad (41)$$

The results obtained using the present approach, are in excellent agreement with the results obtained from Fig. 5.4 of [29]. This serves as a direct validation of the present dislocation solution.
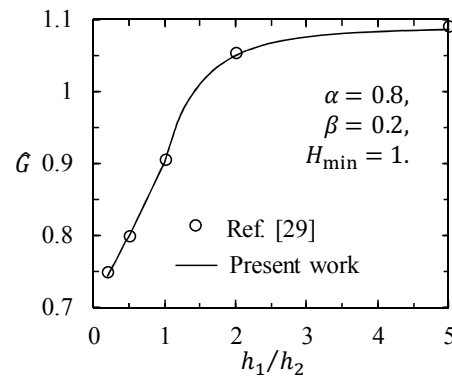

Fig. 3 Normalized ERR vs. layer thickness ratio

VI. CONCLUDING REMARKS

In this paper, we consider a general multi-layered composite, composed of perfectly bonded isotropic elastic layers, and present the solution for the elastic field induced by an interfacial edge dislocation. Finite composite structures can be modelled by setting shear modulus of elasticity of the top and bottom layers to zero, while the embedded dislocation corresponds to the case when $\mu_{-1} = \mu_1$ and $v_{-1} = v_1$.

The obtained solution for an interfacial edge dislocation in a multi-layered medium can be used in conjunction with the Distributed Dislocation Technique (DDT) to solve crack problems involving multiple interacting cracks in a multi-layered medium, with no restriction on the crack orientation, position and loading. A significant advance would be the extension of the present solution to anisotropic multi-layered structures, since modern composites are invariably composed of anisotropic laminae.

APPENDIX A: MATLAB PROGRAM TO EVALUATE UNKNOWN
CONSTANTS ASSOCIATED WITH THE CORRECTIVE SOLUTION

```
function [Nx,Ny] = MultilayerConstants(xi,L_index,mu,ka,D)
%--------------------------------------------------------
% Description: Find the unknown constants associated with
% the solution to the strip problem (Eqs. 22-24). For the
% problem geometry, see Fig. 1.

% Input parameters:
% xi: transformed variable in Fourier domain
% L_index: Array storing values of layer indices
% (-m:L_index:n)
% mu: Array storing the corresponding values of the shear
% moduli of layers
% ka: Array storing the corresponding values of Kolosov's
% constant
% D: Array storing the distance of the interfaces from
% origin

% Output parameters:
% Nx: Constants associated with the x-component of the
% Burger's vector, bx. Nx is an array of size m+n (rows)
% x 4 (columns). The ith row contains the constants
% {Ax, Bx, Cx, Dx} associated with the ith layer (-m:i:n)
%
% Ny: Constants associated with the y-component of the
% Burger's vector, by. Ny is an array of size m+n (rows)
% x 4 (columns). The ith row contains the constants
% {Ay, By, Cy, Dy} associated with the ith layer (-m:i:n)

 warning('off', 'MATLAB:nearlySingularMatrix')
%--------------------------------------------------------
% STEP 1: Evaluate bimaterial parameters

m = abs(min(L_index)); % No. of layers below the origin
n = max(L_index); % No. of layers above the origin

% Elastic properties of layers immediately above and below
% the dislocation
mu_1 = mu(L_index == 1); ka_1 = ka(L_index == 1);
mu_n1 = mu(L_index == -1); ka_n1 = ka(L_index == -1);

% Bi-material constants (Eq.7)
Ga_1 = (mu_1*mu_n1)/(mu_1+mu_n1*ka_1);
Ga_n1 = (mu_1*mu_n1)/(mu_1*ka_n1+mu_n1);
%--------------------------------------------------------
% STEP 2: Assemble global matrix

% Pre-allocate memory for system of equations
M = zeros(4*(m+n),4*(m+n));
V = zeros(4*(m+n),1); W = zeros(4*(m+n),1);
for i = 1:m+n-1
    if i <= m-1
% Assemble equations for interfaces below the origin
        % Material properties above and below an interface
        mu_top = mu(i+1); ka_top = ka(i+1);
        mu_bottom = mu(i); ka_bottom = ka(i);
% distance of interface from origin
        c = abs(D(i+1));
        [mi,vi,wi] = ...
localMatrix1(mu_top,ka_top,mu_bottom,ka_bottom,Ga_1,Ga_n1,c
,xi);
        M(4*(i-1)+1:4*(i-1)+4,4*(i-1)+1:4*(i-1)+8) = mi;
        V(4*(i-1)+1:4*(i-1)+4) = vi;
        W(4*(i-1)+1:4*(i-1)+4) = wi;
% Assemble equations for interface at the origin
    else if i == m
            mi = localMatrix3(mu_1,ka_1,mu_n1,ka_n1);
            M(4*(i-1)+1:4*(i-1)+4,4*(i-1)+1:4*(i-1)+8) =
mi;
% Assemble equations for interfaces above the origin
        else
            mu_top = mu(i+1); ka_top = ka(i+1);
            mu_bottom = mu(i); ka_bottom = ka(i);
            d = D(i); % distance of interface from origin
            [mi,vi,wi] = ...
localMatrix2(mu_top,ka_top,mu_bottom,ka_bottom,Ga_1,Ga_n1,d
,xi);
            M(4*(i-1)+1:4*(i-1)+4,4*(i-1)+1:4*(i-1)+8) =
mi;
            V(4*(i-1)+1:4*(i-1)+4) = vi;
            W(4*(i-1)+1:4*(i-1)+4) = wi;
        end
    end
end
```

```
% Assemble equations for far - field
M(4*(m+n-1)+1,1) = 1; M(4*(m+n-1)+2,2) = 1;
M(4*(m+n-1)+3,4*(m+n-1)+3) = 1;
M(4*(m+n-1)+4,4*(m+n-1)+4) = 1;
%--------------------------------------------------------
% STEP 3: Solve system of 4*(m+n) equations and find
unknown constants

X_sol = M\V;
Y_sol = M\W;

% Constants for all layers
Nx = reshape(X_sol,4,[])'; Ny = reshape(Y_sol,4,[])';

% Constants for layer just above the dislocation
Nx_1 = Nx(L_index == 1,:)'; Ny_1 = Ny(L_index == 1,:)';

% Constants for layer just below the dislocation
Nx_n1 = Nx(L_index == -1,:)'; Ny_n1 = Ny(L_index == -1,:)';
%--------------------------------------------------------
% Function to assemble equations for interfaces below the
origin

function [m1,v1,w1] = ...
    localMatrix1(mu_t,ka_t,mu_b,ka_b,Ga_1,Ga_n1,c,xi)
EC = exp(-2*xi*c); LC = xi*c;
K_b = (ka_b+1)/2; K_t = (ka_t+1)/2;
m1 = zeros(4,8); v1 = zeros(4,1); w1 = zeros(4,1);

% Equations of stress compatibility
m1(1,1) =  1; m1(1,2) = -LC;      m1(1,3) =  EC; m1(1,4) = -
LC*EC;
m1(1,5) = -1; m1(1,6) =  LC;      m1(1,7) = -EC; m1(1,8) =
LC*EC;
m1(2,1) =  1; m1(2,2) = -(1+LC); m1(2,3) = -EC; m1(2,4) = -
(1-LC)*EC;
m1(2,5) = -1; m1(2,6) =  (1+LC); m1(2,7) =  EC; m1(2,8) =
(1-LC)*EC;

% Equations of displacement compatibility
m1(3,1) = -1/(2*mu_b);      m1(3,2) = -1/(2*mu_b)*(-LC-K_b);
m1(3,3) = -1/(2*mu_b)*EC; m1(3,4) = -1/(2*mu_b)*(-
LC+K_b)*EC;
m1(3,5) =  1/(2*mu_t);      m1(3,6) =  1/(2*mu_t)*(-LC-K_t);
m1(3,7) =  1/(2*mu_t)*EC; m1(3,8) =  1/(2*mu_t)*(-
LC+K_t)*EC;
m1(4,1) =  1/(2*mu_b);      m1(4,2) = -1/(2*mu_b)*(1+LC-K_b);
m1(4,3) = -1/(2*mu_b)*EC; m1(4,4) = -1/(2*mu_b)*(1-LC-
K_b)*EC;
m1(4,5) = -1/(2*mu_t);      m1(4,6) =  1/(2*mu_t)*(1+LC-K_t);
m1(4,7) =  1/(2*mu_t)*EC; m1(4,8) =  1/(2*mu_t)*(1-LC-
K_t)*EC;

% Displacement jumps (for soln. of B_x = 1, B_y = 0)
v1(3) = (EC/pi)*(-(((ka_t*Ga_n1+Ga_1)/(2*mu_t))-...
    ((ka_b*Ga_n1+Ga_1)/(2*mu_b)))+Ga_n1*LC*((1/mu_t)-
(1/mu_b)));
v1(4) = (EC/pi)*(+(((ka_t*Ga_n1-Ga_1)/(2*mu_t))-...
    ((ka_b*Ga_n1-Ga_1)/(2*mu_b)))+Ga_n1*LC*((1/mu_t)-
(1/mu_b)));

% Displacement jumps (for soln. of B_x = 0, B_y = 1)
w1(3) = (EC/pi)*(-(((ka_t*Ga_n1-Ga_1)/(2*mu_t))-...
    ((ka_b*Ga_n1-Ga_1)/(2*mu_b)))+Ga_n1*LC*((1/mu_t)-
(1/mu_b)));
w1(4) = (EC/pi)*(+(((ka_t*Ga_n1+Ga_1)/(2*mu_t))-...
    ((ka_b*Ga_n1+Ga_1)/(2*mu_b)))+Ga_n1*LC*((1/mu_t)-
(1/mu_b)));
end
%--------------------------------------------------------
% Function to assemble equations for interfaces above the
% origin

function [m2,v2,w2] = ...
    localMatrix2(mu_t,ka_t,mu_b,ka_b,Ga_1,Ga_n1,d,xi)
ED = exp(-2*xi*d); LD = xi*d;
K_b = (ka_b+1)/2; K_t = (ka_t+1)/2;
m2 = zeros(4,8); v2 = zeros(4,1); w2 = zeros(4,1);

% Equations of stress compatibility
m2(1,1) =  ED; m2(1,2) =  LD*ED; m2(1,3) =  1; m2(1,4) =
LD;
m2(1,5) = -ED; m2(1,6) = -LD*ED; m2(1,7) = -1; m2(1,8) = -
LD;
m2(2,1) =  ED; m2(2,2) = -(1-LD)*ED; m2(2,3) = -1; m2(2,4)
= -(1+LD);
```

```
m2(2,5) = -ED; m2(2,6) =  (1-LD)*ED; m2(2,7) =  1; m2(2,8)
=  (1+LD);

% Equations of displacement compatibility
m2(3,1) = -1/(2*mu_b)*ED; m2(3,2) = -1/(2*mu_b)*(LD-
K_b)*ED;
m2(3,3) = -1/(2*mu_b);    m2(3,4) = -1/(2*mu_b)*(LD+K_b);
m2(3,5) =  1/(2*mu_t)*ED; m2(3,6) =  1/(2*mu_t)*(LD-
K_t)*ED;
m2(3,7) =  1/(2*mu_t);    m2(3,8) =  1/(2*mu_t)*(LD+K_t);
m2(4,1) =  1/(2*mu_b)*ED; m2(4,2) = -1/(2*mu_b)*(1-LD-
K_b)*ED;
m2(4,3) = -1/(2*mu_b);    m2(4,4) = -1/(2*mu_b)*(1+LD-K_b);
m2(4,5) = -1/(2*mu_t)*ED; m2(4,6) =  1/(2*mu_t)*(1-LD-
K_t)*ED;
m2(4,7) =  1/(2*mu_t);    m2(4,8) =  1/(2*mu_t)*(1+LD-K_t);

% Displacement jumps (for soln. of B_x = 1, B_y = 0)
v2(3) = -(ED/pi)*(-(((ka_t*Ga_1+Ga_n1)/(2*mu_t))-...
    ((ka_b*Ga_1+Ga_n1)/(2*mu_b)))+Ga_1*LD*((1/mu_t)-
(1/mu_b)));
v2(4) =  (ED/pi)*(+(((ka_t*Ga_1-Ga_n1)/(2*mu_t))-...
    ((ka_b*Ga_1-Ga_n1)/(2*mu_b)))+Ga_1*LD*((1/mu_t)-
(1/mu_b)));

% Displacement jumps (for soln. of B_x = 0, B_y = 1)
w2(3) = (ED/pi)*(-(((ka_t*Ga_1-Ga_n1)/(2*mu_t))-...
    ((ka_b*Ga_1-Ga_n1)/(2*mu_b)))+Ga_1*LD*((1/mu_t)-
(1/mu_b)));
w2(4) = -(ED/pi)*(+(((ka_t*Ga_1+Ga_n1)/(2*mu_t))-...
    ((ka_b*Ga_1+Ga_n1)/(2*mu_b)))+Ga_1*LD*((1/mu_t)-
(1/mu_b)));
end
%---------------------------------------------------------
% Function to assemble equations for interface along the
% origin

function m3 = localMatrix3(mu_t,ka_t,mu_b,ka_b)
K_b = (ka_b+1)/2; K_t = (ka_t+1)/2;

% Equations of stress compatibility
m3(1,1) =  1; m3(1,2) = 0; m3(1,3) =  1; m3(1,4) = 0;
m3(1,5) = -1; m3(1,6) = 0; m3(1,7) = -1; m3(1,8) = 0;
m3(2,1) =  1; m3(2,2) = -1; m3(2,3) = -1; m3(2,4) = -1;
m3(2,5) = -1; m3(2,6) =  1; m3(2,7) =  1; m3(2,8) =  1;

% Equations of displacement compatibility
m3(3,1) = -1/(2*mu_b); m3(3,2) = -1/(2*mu_b)*(-K_b);
m3(3,3) = -1/(2*mu_b); m3(3,4) = -1/(2*mu_b)*K_b;
m3(3,5) =  1/(2*mu_t); m3(3,6) =  1/(2*mu_t)*(-K_t);
m3(3,7) =  1/(2*mu_t); m3(3,8) =  1/(2*mu_t)*K_t;
m3(4,1) =  1/(2*mu_b); m3(4,2) = -1/(2*mu_b)*(1-K_b);
m3(4,3) = -1/(2*mu_b); m3(4,4) = -1/(2*mu_b)*(1-K_b);
m3(4,5) = -1/(2*mu_t); m3(4,6) =  1/(2*mu_t)*(1-K_t);
m3(4,7) =  1/(2*mu_t); m3(4,8) =  1/(2*mu_t)*(1-K_t);
end
end
```

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