Numerical Buckling of Composite Cylindrical Shells under Axial Compression Using Asymmetric Meshing Technique (AMT)

Zia R. Tahir, P. Mandal

Abstract—This paper presents the details of a numerical study of buckling and post buckling behaviour of laminated carbon fiber reinforced plastic (CFRP) thin-walled cylindrical shell under axial compression using asymmetric meshing technique (AMT) by ABAQUS. AMT is considered to be a new perturbation method to introduce disturbance without changing geometry, boundary conditions or loading conditions. Asymmetric meshing affects both predicted buckling load and buckling mode shapes. Cylindrical shell having lay-up orientation [0°/+45°/–45°/0°] with radius to thickness ratio (R/t) equal to 265 and length to radius ratio (L/R) equal to 1.5 is analysed numerically. A series of numerical simulations (experiments) are carried out with symmetric and asymmetric meshing to study the effect of asymmetric meshing on predicted buckling behaviour. Asymmetric meshing technique is employed in both axial direction and circumferential direction separately using two different methods, first by changing the shell element size and varying the total number elements, and second by varying the shell element size and keeping total number of elements constant. The results of linear analysis (Eigenvalue analysis) and non-linear analysis (Riks analysis) using symmetric meshing agree well with analytical results. The results of numerical analysis are presented in form of non-dimensional load factor, which is the ratio of buckling load using asymmetric meshing technique to buckling load using symmetric meshing technique. Using AMT, load factor has about 2% variation for linear eigenvalue analysis and about 2% variation for non-linear Riks analysis. The behaviour of load end-shortening curve for pre-buckling is same for both symmetric and asymmetric meshing but for asymmetric meshing curve behaviour in post-buckling becomes extraordinarily complex. The major conclusions are: different methods of AMT have small influence on predicted buckling load and significant influence on load displacement curve behaviour in post buckling; AMT in axial direction and AMT in circumferential direction have different influence on buckling load and load displacement curve in post-buckling.

Keywords—CFRP Composite Cylindrical Shell, Asymmetric Meshing Technique, Primary Buckling, Secondary Buckling, Linear Eigenvalue Analysis, Non-linear Riks Analysis.

I. INTRODUCTION

THE composite thin walled cylindrical shells are generally used as load carrying components in various structures due to their significant strength-to-weight ratio. These thin

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walled structures are usually subjected to compressive loads and buckling behaviour is the key design criteria. The composite cylindrical shells subjected to axial compression undergoes a primary buckling state from original equilibrium state. The primary buckling state is a stable state up to a certain point with constant load-deflection ratio. In post primary buckling region the shell can carry more load than previous region as value of displacement increases. After primary buckling the shell undergoes another equilibrium state with a different load-deflection ratio which is called secondary buckling and this phenomenon is secondary buckling of composite cylindrical shells. Finite element analysis of composite cylindrical shell subjected to axial compressive load provides an explanation of primary buckling, secondary buckling, post-primary buckling and post-secondary buckling. Asymmetric meshing technique (AMT) is used to evaluate primary and secondary buckling loads and buckling mode shapes by non-linear analysis from load-displacement curve.

Asymmetric meshing technique (AMT) was introduced by Wardle to solve a type of shell buckling benchmark problem [1]. The numerical solution proposed by [2] was supposed to be the benchmark solution for circular cylindrical shell section (panel) but Wardle showed it as incorrect by giving a new numerical solution using STAGS (STructural Analysis of General Shells). While solving the incorrect benchmark problem, the main concern was to traverse the bifurcation point and to calculate the associated correct post-bifurcation response by triggering bifurcation using various methods. Wardle [3] used traditional techniques (geometric or loading imperfection) and asymmetric meshing technique (AMT) for triggering bifurcation. The AMT was considered to be efficient method as complete solution for a given cylindrical shell problem was solved in a single finite element analysis whereas traditional techniques require 5-10 runs after interpreting the prior run's results; that is, the user must adjust amplitude of geometric imperfection so that it is large enough to cause bifurcation but not so large as to change the problem [4]. Wardle [5] calculated the large-deflection buckling behaviour of composite shell structures by the use of finite element method with asymmetric meshing technique (AMT) for the induction of bifurcation point. AMT does not impose bifurcation but allows evaluating the bifurcation point and post-buckling accurately and efficiently if it exists in the structural response. Zhang and Gu [6] used AMT to study the effect of asymmetric meshing on the predicted primary and secondary buckling behaviour of composite cylindrical shell

under axial compression. They concluded that different type of asymmetric meshing have a little influence on the predicted secondary buckling load and have no effect on the linear buckling behaviour of composite shell as geometry, boundary conditions or load has not been changed.

Bisagni [7] performed experiments on composite laminated cylindrical shells subjected to axial compressive load. Four shells made of carbon fabric with lay-up orientation of $[0^{o}/+45^{o}/–45^{o}/0^{o}]$ with R/t equal to 265 were tested. The ratio between experimental and analytical buckling load was in range from 0.63 to 0.72. Experimental results were presented in terms of plots of axial compressive load vs. axial displacement, post buckling pattern, and two-dimensional Fourier analysis of geometry of shells before and during testing. This experimental data is used for numerical analysis in this paper.

In this paper the buckling and post buckling behaviour of composite cylindrical shell subjected to axial compressive load with symmetric and asymmetric meshing are studied with emphasis on the following points:

- Two types of finite element analysis are utilized: linear analysis (Eigenvalue analysis) and non-linear analysis (Riks analysis).
- Two different types of asymmetric meshing techniques (AMT) are employed in both circumferential direction and axial direction of the shell and the results are presented in form of load factor.
- The load-displacement curve behaviour for symmetric meshing and asymmetric meshing is plotted and compared.

II. NUMERICAL MODEL

The numerical model of composite cylindrical shell considered in this study is a four-ply laminated shell used in the buckling experiments performed by [7]. The composite cylindrical shell was fabricated from carbon fiber reinforced plastic (CFRP) laminates having both internal diameter and overall length equal to 700 mm, with two tabs at bottom and top surfaces provided for attaching them to loading arrangements. The actual length of the shell is equal to 520 mm and the nominal thickness of shell wall is equal 1.32 mm. The radius to thickness ratio (R/t) is equal to 265, length to radius ratio (L/R) is equal to 1.5 and stacking sequence as $[0^{o}/45^{o}/-45^{o}/0^{o}]$. Mechanical properties of CFRP are presented in Table I.

The finite element analysis (FEA) [8] is used to analyse the buckling and post-buckling behaviour of cylindrical shell under axial compression and the computer program ABAQUS [9] is used for finite element analysis. The shell element S4R is selected, which is a four nodes reduced-integration shell element having six degrees of freedom at each node, three translational displacements in the nodal directions and three rotational displacements about the nodal axes. The boundary conditions are taken as: all three translational displacements and three rotational displacements are fixed at bottom of shell while only axial translational displacement is free at top of

shell. The axial compressive load is transferred to uniform nodal forces along the circumference of the shell.

TABLE I
MECHANICAL PROPERTIES OF THE MATERIAL

Parameter	Value
Elastic Modulus E_{11} (N/mm^2)	52000
Elastic Modulus E_{22} (N/mm^2)	52000
Shear Modulus G_{12} (N/mm^2)	2350
Poisson's Ratio v_{12}	0.302
Mass Density (kg/m^3)	1320
Thickness t (mm)	0.33

III. LINEAR EIGENVALUE ANALYSIS

The buckling phenomenon has been investigated by eigenvalue analysis. The eigenvalue analysis estimates theoretical buckling load (bifurcation load) of perfect linear elastic structure. The buckling (bifurcation) occurs when a maximum axial stress becomes equal to the buckling stress. The linear buckling analysis of a structure requires the solution of mathematical equation of eigenvalue problem in the form:

$$([K] + \lambda[S])\{\Phi\} = \{0\}$$

where $[K]$ is the linear stiffness matrix of the system, λ is the eigenvalue which determine the buckling load (or load factor), $[S]$ is the stress stiffness matrix, and $\{\Phi\}$ is the eigenvector. In terms of linear buckling analysis, the eigenvector $\{\Phi\}$ represents the buckling mode shape and the associated eigenvalue λ indicate the multiple of $[S]$ needed to make singular the equation which cause buckling. If the load applied to the structure is Q_N, the critical buckling load is λQ_N. The eigenvalue analysis gives both the buckling load and buckling mode shape of shell structure.

The mesh sensitivity of finite element model has been initially studied using linear eigenvalue analysis. The results are verified by comparing with the analytical results given in [10]. The critical buckling load converges well when 52×220 elements (52 elements in the axial direction and 220 in the circumferential direction) are used and the difference with the analytical result is about 4%. The numerical buckling shape of the model with 52×220 elements exhibits a doubly periodic (diamond) shape with 7 axial half waves and 14 circumferential waves.

A. Symmetric Meshing Technique

Numerical model with 30×120 elements (30 elements in axial direction and 120 elements in circumferential direction) is analysed as it gives a reasonable agreement between CPU time and precise results. The numerical buckling shape of the model with 30×120 elements exhibits a doubly periodic (diamond) shape with 6 axial half waves and 14 circumferential waves, shown in Fig. 1. The results from eigenvalue analysis using symmetric meshing are taken as reference load (numerical buckling load using symmetric meshing technique) for further study and are presented in Table II.

TABLE II
COMPARISON AMONG BUCKLING LOADS

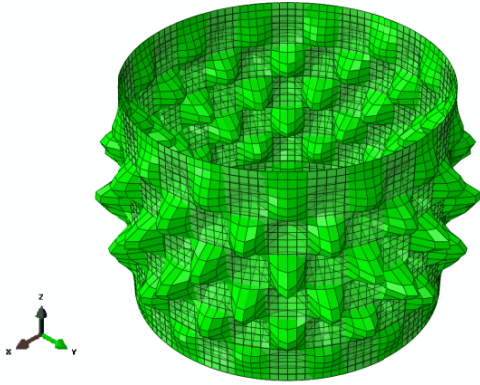| Solution Type	Buckling Load (kN)
Analytical Solution	240.00
Eigenvalue Analysis (52×220)	248.42
Eigenvalue Analysis (30×120)	274.25
Riks Analysis (52×220)	251.37
Riks Analysis (30×120)	277.51

Fig. 1 Numerical buckling shape of the model

B. Asymmetric Meshing Technique

AMT is considered to be a perturbation method to introduce disturbance without changing geometry, boundary conditions or loading conditions along with traditional perturbation methods to introduce external disturbance such as initial geometric imperfections, loading conditions or initial material imperfections. AMT is more or less similar to introducing geometric imperfections in a numerical model. In case of AMT the numerical effect of the asymmetric mesh is similar to the caused by the initial geometric imperfection, not by loading asymmetry. Asymmetric meshing affects the stiffness matrix **K** and does not affect the loading vector **F** in the relation **F** = **Kx**.

Asymmetric meshing technique (AMT) is primarily employed in circumferential direction and axial direction of the model separately to study its effect on buckling loads. For AMT in circumferential direction a partition is made by dividing shell into two halves as shown in Fig. 2 (a). For symmetric meshing the number of elements in left half and right half are equal to 60 for each and for AMT the number of elements in both halves may vary. For AMT in axial direction a partition is made by dividing the shell into two halves as shown in Fig. 2 (b). For symmetric meshing the number of elements in upper half and lower half are equal to 15 for each, and for AMT the number of elements in both halves may vary. Asymmetric meshing technique (AMT) in circumferential and axial direction is employed in two different ways: 'A' varying the number of elements in both directions and varying element size, and 'B' keeping the number of elements constant and varying element size.
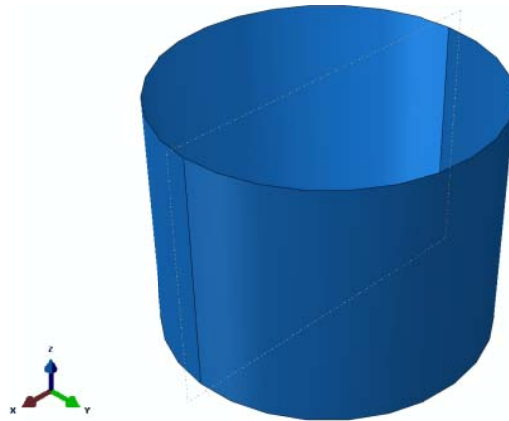

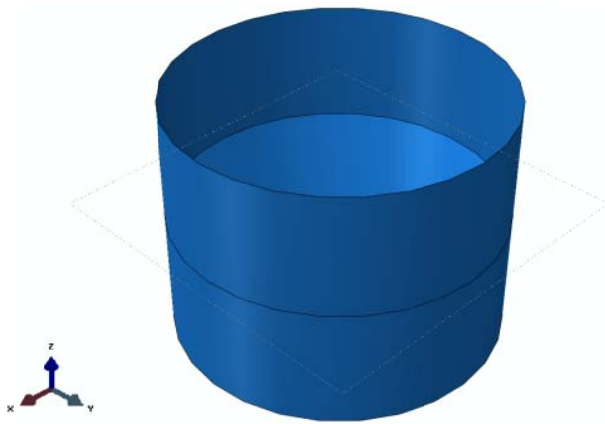Fig. 2 (a)   Partition of the model in circumferential direction

Fig. 2 (b)   Partition of the model in axial direction

C. Asymmetric Meshing Technique A

Circumferential Direction A1: For this technique the number of elements in axial direction are kept constant equal to 30 and the number of elements in circumferential direction are varied. The model in circumferential direction is divided into two halves, in one half the number of elements are kept constant equal to 60 and in other half the number of elements are varied from 61 to 65. For numerical experiments, the model is designated as 'A' with suffix 1, 2, 3, 4, 5 represents serial number of numerical experiments as A1-1, A1-2, A1-3, A1-4, A1-5 and for symmetric meshing numerical model is designated as A1-0. In above notations, 'A' represents the type of technique, 1 and 2 after 'A' represents asymmetric meshing technique in circumferential and axial direction respectively, and suffix from 1 to 5 as serial number of numerical experiment.

Axial Direction A2: For this technique the number of elements in circumferential direction are kept constant equal to 120 and the number of elements in axial direction are varied. The model is divided into two halves, in one half the number of elements are kept constant equal to 15 and in other half the number of elements are varied from 16 to 20. For numerical experiments, the model is designated as A2-1, A2-

2, A2-3, A2-4, A2-5 and for symmetric meshing the model is designated as A2-0.

The results of eigenvalue analysis in the form of load factor for different numerical experiments are presented in Fig. 3. The load factor is the ratio of load from asymmetric meshing technique to reference load. AMT in axial direction has almost no effect on buckling load while AMT in circumferential direction has a variation of about 2%.

D. Asymmetric Meshing Technique B

Circumferential Direction B1: For this technique the number of elements in axial direction are kept constant equal to 30 and the number of elements in circumferential direction are also kept constant equal to 120. The model in circumferential direction is divided into two halves, in one half the number of elements are varied from 61 to 65 and in other half the number of elements are varied from 59 to 55 to keep total number of elements equal to 120. For numerical experiments, the model is designated as B1-1, B1-2, B1-3, B1-4, B1-5 and for symmetric meshing the model is designated as B1-0.

Axial Direction B2: For this technique the number of elements in axial direction are kept constant equal to 30 and the number of elements in circumferential direction are also kept constant equal to 120. The model in axial direction is divided into two halves, in one half the number of elements are varied from 16 to 20 and in other half the number of elements are varied from 14 to 11 to keep total number of elements equal to 30. For numerical experiments, the model is designated as B2-1, B2-2, B2-3, B2-4, B2-5 and for symmetric meshing the model is designated as B2-0.

The results of eigenvalue analysis in the form of load factor for different numerical experiments are presented in Fig. 3. AMT in axial direction has almost no effect on buckling load while AMT in circumferential direction has a variation of about 2%.
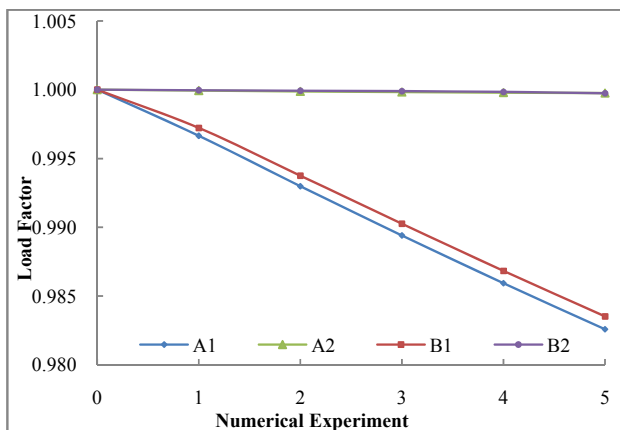

Fig. 3 Load factor of various experiments using eigenvalue analysis

IV. Non-Linear Riks Analysis

The buckling phenomenon has also been investigated by non-linear analysis using Riks method [11]. In this analysis using ABAQUS, the numerical experiments are performed with different load increment from 1% to 0.1% with five intervals. Fine load increments are used during numerical simulations as the load becomes equal to the expected critical load estimated by eigenvalue analysis. The approach used in this analysis is to constantly reduce the load increments until the solution starts to converge. The load-deflection curve behaviour changes slightly near buckling in some cases by varying load increments.

A. Symmetric Meshing Technique

The Riks analysis is performed for symmetric meshing with 52×220 elements and 30×120 elements and the results are presented in Table II along with analytical results. The critical load for symmetric meshing is determined by increasing the applied load gradually to find the limit point at which the load end-shortening curve reaches a maximum a maximum value of load. The results presented in Table II are for the buckling load obtained from load end-shorting curve when load reaches its maximum value. The load-displacement (magnitude of axial displacement) curve for the model is shown in Fig. 4 (a). The same curve is plotted in Fig. 4 (b) to explorer buckling phenomena. The curve in Fig. 4 (a) near the buckling is smooth so that it is difficult to predict critical load, primary buckling load or secondary buckling load. For symmetric meshing this prediction can be done by magnifying the buckling region of the curve and by comparing the curves obtained from asymmetric meshing of the same model.

In Fig. 4 (b) the point 'P' is the primary bifurcation point, beyond this point there is an approximate linear relationship between load and axial displacement, after this point the change in slope occurs. The buckling phenomena from point 'P' to 'S' may refer as post-primary buckling. The point 'S' is secondary bifurcation point, after this point the curve become stable and then its slope become negative, the buckling phenomena after this point may referred as post-secondary buckling. The load at point 'S' is termed as secondary buckling load and the maximum load is termed as critical buckling load. In the present study Riks analysis is performed for symmetric meshing with different load increments and the behaviour of load displacement curve is almost same.
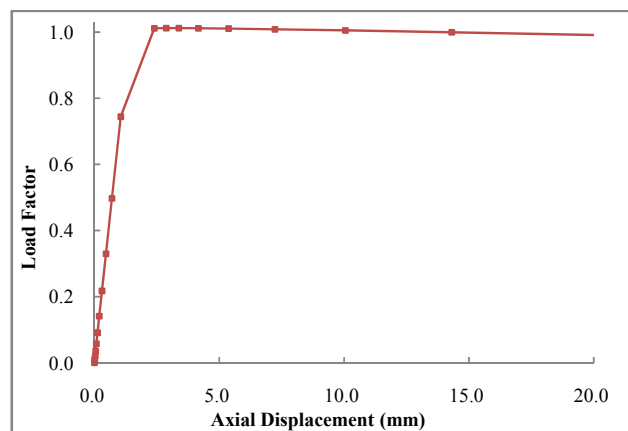

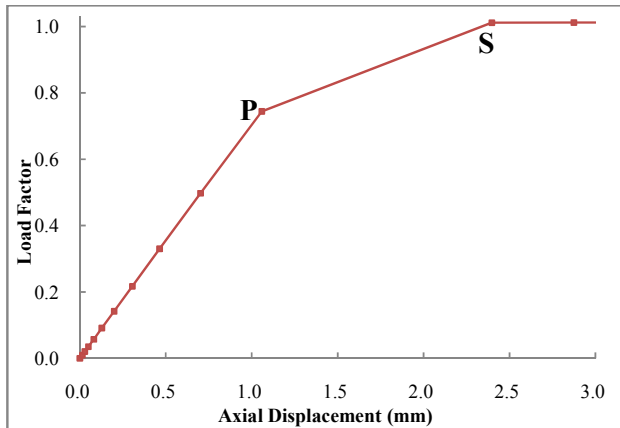Fig. 4 (a) Load-displacement curve for symmetric meshing

Fig. 4 (b) Load-displacement curve near bifurcation for symmetric meshing

B. Asymmetric Meshing Technique A

The results of non-linear Riks analysis for asymmetric meshing technique A are presented in the form of load factor in Fig. 5 for critical buckling load (the maximum load in load-displacement curve) and in Fig. 6 for secondary buckling load (the load corresponding to secondary bifurcation point). The load factor for critical buckling load and secondary buckling load is plotted for different numerical experiments and is shown in Figs. 5 and 6 respectively. AMT in circumferential direction has a variation of about 1% in secondary buckling load and 2% in critical buckling load while AMT in axial direction has a variation of about 1% for both critical and secondary buckling load.

Using AMT in axial direction the curve behaviour in post-primary and post-secondary regions is similar to that for symmetric meshing. Using AMT in circumferential direction the curve behaviour in post-primary region is similar to that for symmetric meshing, while post-secondary buckling behaviour of curve is different from symmetric meshing. The load-displacement curve behaviour near bifurcation is magnified and shown in Fig. 7.
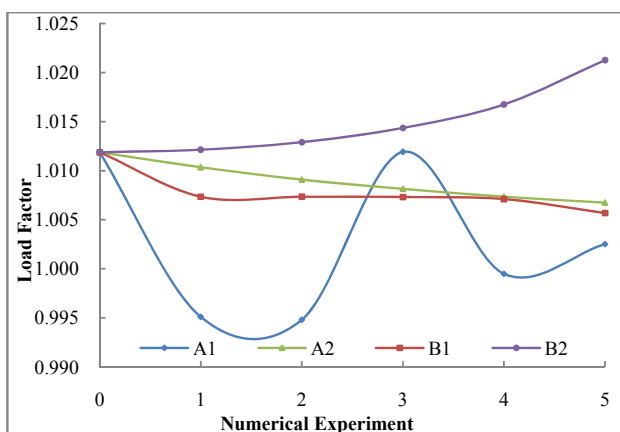

Fig. 5 Load factor for critical buckling of various experiments using Riks Analysis
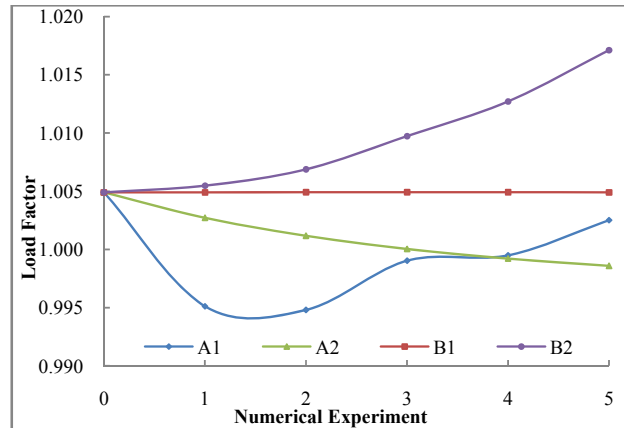

Fig. 6 Load factor for secondary buckling of various experiments using Riks analysis
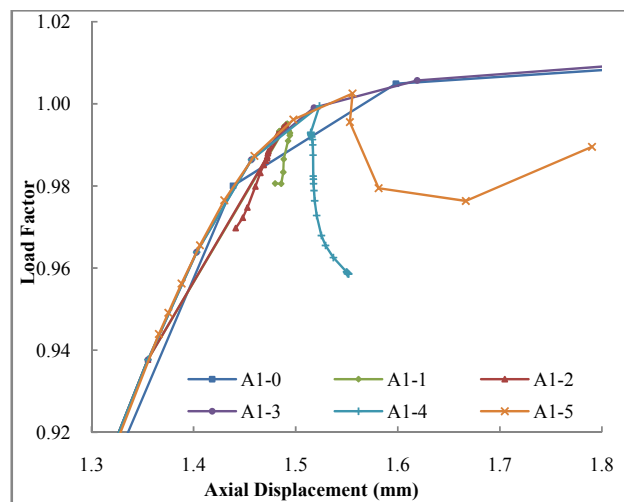

Fig. 7 Load-displacement curve near bifurcation for AMT A

C. Asymmetric Meshing Technique B

The results of Riks analysis for asymmetric meshing technique B are presented in the form of load factor in Figs. 5 and 6. The load factor for critical buckling load and secondary buckling load is plotted for different numerical experiments and is shown in Figs. 5 and 6.

AMT in circumferential direction has almost no effect on secondary buckling load but there is a variation of about 1% in critical buckling load while AMT in axial direction has a variation of about 1% for both critical and secondary buckling load.

Using AMT in axial direction the curve behaviour in post-primary and post-secondary regions is similar to that for symmetric meshing. Using AMT in circumferential direction the curve behaviour in post-primary region is similar to that for symmetric meshing, while post-secondary buckling behaviour of curve is different from symmetric meshing. The load-displacement curve behaviour near bifurcation is magnified and shown in Fig. 8.
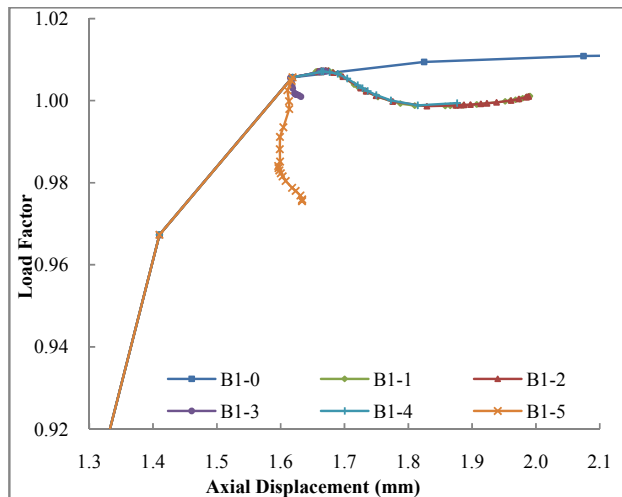
Fig. 8 Load-displacement curve near bifurcation for AMT B

V. CONCLUSIONS

In this paper, the effect of AMT on the buckling and post-buckling behaviour of composite circular cylindrical shell under axial compression is presented. The buckling of shell is numerically analysed by using two methods; linear analysis (Eigenvalue analysis) and non-linear analysis (Riks analysis). AMT in the numerical model is employed in two different ways in axial and circumferential direction by changing number of elements and elements size in different combinations to generate asymmetric meshing. The main purpose of this numerical investigation is to study the effect of asymmetric meshing on the buckling load of shell. Initially shell are analysed for symmetric meshing and then for asymmetric mesh. The ratio of buckling load for asymmetric meshing to buckling load for symmetric meshing is taken as load factor. For linear eigenvalue analysis, asymmetric meshing affects both buckling loads and buckling mode shapes and there is a variation of 2% in buckling load which increase further by increasing asymmetry in meshing. For non-linear Riks analysis, asymmetric meshing affects both buckling loads and load-deflection curve behaviour in post-buckling region and there is a variation of about 2% in buckling load which may increase further by increasing asymmetry in meshing. Finally, it is concluded that: different methods of AMT have some influence on predicted buckling load and significant influence on load displacement curve in post-buckling; AMT in axial direction and AMT in circumferential direction has different influence on buckling load and load displacement curve in post-buckling.

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