Nonplanar Ion-acoustic Waves in a Relativistically Degenerate Quantum Plasma

Swarniv Chandra, Sibarjun Das, Agniv Chandra, Basudev Ghosh, Apratim Jash

Abstract—Using the quantum hydrodynamic (QHD) model the nonlinear properties of ion-acoustic waves in are lativistically degenerate quantum plasma is investigated by deriving a nonlinear Spherical Kadomtsev–Petviashvili (SKP) equation using the standard reductive perturbation method equation. It was found that the electron degeneracy parameter significantly affects the linear and nonlinear properties of ion-acoustic waves in quantum plasma.

I. INTRODUCTION

THE investigation of ultra-dense matter has been carried out quite extensively and intensively in the recent years. Such matter is found in metal nanostructures, neutron stars, white dwarfs and other astronomical bodies as well as in laser plasma interaction experiments. In such situations the average inter-fermionic distance is comparable or even less than the thermal de Broglie wavelength and as a result the quantum degeneracy becomes important. In such extreme conditions of density the thermal pressure of electrons may be negligible as compared to the Fermi degeneracy pressure which arises due to the Pauli Exclusion Principle. However in this case the quantum effect can't be neglected and proper mathematical modelling becomes necessary. Such quantum effects are generally studied with the help of two well-known formulations, the Wigner-poisson and the Schrodinger Poisson formulation. The former one studies the quantum kinetic behavior of plasmas while the latter describes the hydrodynamic behaviour of plasma waves. The quantum hydrodynamic (QHD) model is derived by taking the velocity space moments of the Wigner equations. The QHD model modifies the clasical fluid model for plasmas with the inclusion of a quantum correction term generally known as the Bohm potential. The model also incorporates the quantum

statistical effect through the equation of state. The model has been widely used to study quantum behaviour of plasma. A survey of the available literature [1]-[17] shows that most of the works done in quantum plasma in order to study the nonlinear behaviour of different plasma waves uses non relativistic case or with weakly relativistic approximation. The matter in some compact astrophysical objects (e.g. white dwarfs, neutron stars, magnetars etc.) exists in extreme conditions of density. In such situations the average inter-Fermionic distance is comparable to or less than the thermal de Broglie wavelength and hence quantum degeneracy effects become important. At extreme high densities the thermal pressure of electrons may be negligible as compared to the Fermi degeneracy pressure which arises due to implications of Pauli's exclusion principle. In such extreme conditions of density the electron Fermi energy $E_{Fe}[\ = \hbar^2 \left(3\pi^2 n_e\right)^{3/2}/2m_e\ ]$ may become comparable to the electron rest mass energy ($m_e c^2$) and the electron speed can approach the speed of light (c) in vacuum. So the plasma in the interior of such compact astrophysical objects is both degenerate and relativistic. Such a plasma is also likely to be produced in the next generation laser based plasma compression schemes. Under such conditions quantum and relativistic effects are unavoidable. Recent reviews of quantum plasma physics are carried out by [18] as well as by [19]. However regarding the ion-acoustic waves in degenerate quantum plasmas only a few works have been reported. Misra et al. [20] have investigated the modulational instability of EAWs in non-relativistic quantum plasma consisting of two distinct groups of electrons and immobile ions. The propagation of electron-acoustic solitary waves in a two-electron temperature quantum magnetoplasma has also been reported [21]. All these works use quantum hydrodynamic models and consider only the non-relativistic cases. But in extreme conditions of density such as in a typical white dwarf where the electron number density can be as high as 10^{28} cm^{-3} the degeneracy can be relativistic and both quantum and relativistic effects should be taken into account. Recent investigations indicate that such quantum-relativistic plasmas can support solitary structures at different length scale of excitation [22], [23]. The nonlinear propagation of ion-acoustic waves in relativistically degenerate quantum plasma has been studied by a few authors [24],[25]. Very recently we have investigated the solitary excitations of EAWs in a two electron populated relativistically degenerate super-dense plasma and shown that relativistic degeneracy significantly influences the conditions of formation and properties of solitary structures [26], [27]. To the best of our

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knowledge no investigation has been made for the non-planar wave propagation of ion-acoustic waves in degenerate quantum plasmas including relativistic effects. The motivation of the present paper is to investigate the solitary structures of IAWs in relativistically degenerate dense quantum plasma by applying Kadomtsev-Petviashvili equation.

II. Basic Formulation and Nonlinear Analysis

We consider a two-species quantum plasma system comprised of electrons and ions. We also first take the pressures for both electrons and ions via the so called fluid pressure equations. The governing equations are

$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_\alpha \vec{v}_\alpha) = 0, \tag{1}$$

$$\frac{\partial \vec{v}_\alpha}{\partial t} + \vec{v}_\alpha \cdot \nabla \vec{v}_\alpha = -\frac{\nabla p_\alpha}{m_\alpha n_\alpha} - \frac{q_\alpha}{m_\alpha}\nabla\phi + \frac{h^2}{2m_\alpha^2}\nabla\left(\frac{\nabla^2\sqrt{n_\alpha}}{\sqrt{n_\alpha}}\right), \tag{2}$$

$$\frac{\partial p_\alpha}{\partial t} + \vec{v}_\alpha \cdot \nabla p_\alpha + \gamma_\alpha p_\alpha(\nabla \cdot \vec{v}_\alpha) = 0, \tag{3}$$

$$\nabla^2\phi = -4\pi\sum q_\alpha n_\alpha \tag{4}$$

Here, p_α is the relativistic degeneracy pressure in dense plasma given by [28]

$$P_\alpha = \left(\pi m_e^4 c^5/3h^3\right)\left[R_\alpha(2R_\alpha^2-3)\sqrt{1+R_\alpha^2}+3\sinh^{-1}R\right] \tag{5}$$

in which

$$R_\alpha = p_{F\alpha}/m_e c = \left[3h^3 n_\alpha/8\pi m_e^3 c^3\right]^{\frac{1}{3}} = R_{\alpha0}n_\alpha^{\frac{1}{3}} \tag{6}$$

where $R_{\alpha0} = (n_{\alpha0}/n_0)^{1/3}$ with $n_0 = 8\pi m_e^3 c^3/3h^3 \approx 5.9\times10^{29}cm^{-3}$, 'c' being the speed of light in vacuum. $p_{F\alpha}$ is the electron Fermi relativistic momentum. It is to be noted that in the limits of very small and very large values of relativity parameter R_α, we obtain:

$$P_\alpha = \frac{1}{20}\left(\frac{3}{\pi}\right)^{\frac{2}{3}}\frac{h^2}{m_e}n_\alpha^{\frac{5}{3}} \quad \text{(For } R_\alpha\to0) \tag{7}$$

$$P_\alpha = \frac{1}{8}\left(\frac{3}{\pi}\right)^{\frac{1}{3}}hcn_\alpha^{\frac{4}{3}} \quad \text{(For } R_\alpha\to\infty) \tag{8}$$

Here, $n_\alpha, \vec{v}_\alpha, q_\alpha$ are the density, mass, velocity and charge for the α-species particle respectively with α=e for electrons and i for ions $(q_e = -e, q_i = e)$. Also $p_{e(i)}$ is the electron (ion) pressure, $\hbar$ is the Reduced Plank's constant divided by 2π. Now using the following suitable normalizations, i.e.

$$(x,y,z)\to\frac{w_{pi}}{c_s}(x,y,z), \vec{v}_\alpha \to \frac{\vec{v}_\alpha}{c_s}, \theta\to\theta,$$

$$\phi\to\frac{e\phi}{k_B T_{Fe}}\ t\to w_{pi}\ t,\ p_\alpha\to\frac{p_\alpha}{n_0 k_B T_{F\alpha}}\ n_{e,i}\to\frac{n_{e,i}}{n_0} \tag{9}$$

in which $n_{e0} = n_{i0} = n_0$; $T_{F\alpha}$ is the α-particle Fermi temperature, φ is the electrostatic potential $c_s = \sqrt{k_B T_{Fe}/m_i}$ is the ion-sound speed and $\omega_{p\alpha} = \sqrt{4\pi n_0 e^2/m_\alpha}$ is the particle plasma frequency, and choosing the coordinates (r, θ, t), Now considering the fact that $\frac{1}{n_\alpha}\frac{\partial P_\alpha}{\partial x} = \frac{\partial\sqrt{1+R_\alpha^2}}{\partial x}$ the basic (1)-(4) can be written in the following normalized form:

$$\frac{\partial n_\alpha}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 n_\alpha u_\alpha) + \frac{1}{r}\frac{\partial}{\partial\theta}(n_\alpha u_\alpha) + \frac{n_\alpha u_\alpha}{r}\cot\theta = 0 \tag{10}$$

$$\frac{\partial u_e}{\partial t} + u_e\frac{\partial u_e}{\partial r} + \frac{v_e}{r}\frac{\partial u_e}{\partial\theta} - \frac{v_e^2}{r} = m\left(\frac{\partial\phi}{\partial r} - \frac{1}{n_e}\frac{\partial p_e}{\partial r} + \frac{H^2}{2}\frac{\partial}{\partial r}\left(\frac{\nabla^2\sqrt{n_e}}{\sqrt{n_e}}\right)\right) \tag{11}$$

$$\frac{\partial v_e}{\partial t} + u_e\frac{\partial v_e}{\partial r} + \frac{v_e}{r}\frac{\partial v_e}{\partial\theta} + \frac{u_e v_e}{r} =$$
$$m\left(\frac{1}{r}\frac{\partial\phi}{\partial\theta} - \frac{1}{n_e r}\frac{\partial p_e}{\partial\theta} + \frac{H^2}{2r}\frac{\partial}{\partial\theta}\left(\frac{\nabla^2\sqrt{n_e}}{\sqrt{n_e}}\right)\right) \tag{12}$$

$$\frac{\partial v_i}{\partial t} + u_i\frac{\partial v_i}{\partial r} + \frac{v_i}{r}\frac{\partial v_i}{\partial\theta} + \frac{u_i v_i}{r} =$$
$$-\frac{1}{r}\frac{\partial\phi}{\partial\theta} - \frac{\sigma}{n_i r}\frac{\partial p_i}{\partial\theta} + \frac{H^2}{2mr}\frac{\partial}{\partial\theta}\left(\frac{\nabla^2\sqrt{n_e}}{\sqrt{n_e}}\right) \tag{13}$$

$$\frac{\partial p_\alpha}{\partial t} + u_\alpha\frac{\partial p_\alpha}{\partial r} + \frac{v_\alpha}{r}\frac{\partial p_\alpha}{\partial\theta} +$$
$$\gamma_\alpha p_\alpha\left(\frac{\partial u_\alpha}{\partial r} + \frac{2u_\alpha}{r} + \frac{1}{r}\frac{\partial v_\alpha}{\partial\theta} + \frac{v_\alpha}{r}\cot\theta\right) = 0 \tag{14}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2} + \frac{\cot\theta}{r^2}\frac{\partial\phi}{\partial\theta} = n_e - n_i \tag{15}$$

where, u_α, v_α are the velocity components of α-species particles in the radial (r) and polar angle (θ) direction, $\sigma = T_{Fi}/T_{Fe}, m = m_i/m_e, H = \hbar w_{pe}/k_B T_{Fe}$ is the non-dimensional quantum parameter.

III. Derivation of the Kadomtsev-Petviashvili Equation

In order to investigate the propagation of ion-acoustic (IA) waves in the quantum plasmas, we employ the standard reductive perturbation technique to obtain the Kadomtsev-Petviashvili equation (SKPE). The independent variables are stretched as $\xi = \varepsilon^{1/2}(r - v_0 t), \eta = \varepsilon^{-1/2}\theta, \tau = \varepsilon^{3/2}t$ and the dependent variables are expanded as:

$$n_\alpha = 1 + \varepsilon n_{\alpha1} + \varepsilon^2 n_{\alpha2} + \ldots\ldots\ldots\ldots \tag{16}$$

$$u_\alpha = 1 + \varepsilon u_{\alpha 1} + \varepsilon^2 u_{\alpha 2} + \ldots\ldots\ldots \quad (17)$$

$$v_\alpha = 1 + \varepsilon^{3/2} v_{\alpha 1} + \varepsilon^{5/2} v_{\alpha 2} + \ldots\ldots\ldots \quad (18)$$

$$\phi = 1 + \varepsilon \phi_1 + \varepsilon^2 \phi_2 + \varepsilon^3 \phi_3 \ldots\ldots\ldots \quad (19)$$

$$p_\alpha = 1 + \varepsilon p_{\alpha 1} + \varepsilon^2 p_{\alpha 2} + \ldots\ldots\ldots \quad (20)$$

where α=e for electrons and I for ions.

Now, substituting (20) in (10)-(15) and collecting the terms in different powers of ε, we obtain in the lowest order of ε as

$$n_{e1} = \frac{m}{m\gamma_e - v_0^2}\phi_1 = a_e \phi_1 \quad (21)$$

$$n_{i1} = \frac{1}{v_0^2 - \sigma\gamma_i}\phi_1 = a_i \phi_1 \quad (22)$$

$$u_{e1} = \frac{mv_0}{m\gamma_e - v_0^2}\phi_1 = b_e \phi_1 \quad (23)$$

$$u_{i1} = \frac{v_0}{v_0^2 - \sigma\gamma_i}\phi_1 = b_i \phi_1 \quad (24)$$

$$p_{e1} = \frac{Fm\gamma_e}{Fm\gamma_e - v_0^2}\phi_1 = c_e \phi_1 \quad (25)$$

$$p_{i1} = \frac{\gamma_i}{v_0^2 - \sigma\gamma_i}\phi_1 = c_i \phi_1 \quad (26)$$

$$\frac{\delta v_{e1}}{\delta \xi} = \frac{m}{(m\gamma_e - v_0^2)\tau}\frac{\delta \phi_1}{\delta \eta}, \quad (27)$$

$$\frac{\delta v_{i1}}{\delta \xi} = \frac{1}{(v_0^2 - \sigma\gamma_i)\tau}\frac{\delta \phi_1}{\delta \eta}, \quad (28)$$

and,

$$v_0 = \pm\sqrt{m(\gamma_e + \sigma\gamma_i)/(1+m)} \quad (29)$$

where $F_\alpha = (\chi/3)\left(R_{\alpha 0}^2 / \sqrt{1+R_{\alpha 0}^2}\right)$ is the term arising from relativistic pressure in weakly relativistic case, whereas for ultra-relativistic case $F_\alpha = \chi R_{\alpha 0}/3$ where $\chi = m_e c^2 / 2k_B T_{Fe}$. Equation (29) shows that the wave can propagate outward or inward depending on the consideration of the sign. For the next higher order in ε we obtain

$$\frac{\partial n_{e1}}{\partial \tau} - v_0 \frac{\partial n_{e2}}{\partial \xi} + \frac{\partial u_{e2}}{\partial \xi} + \frac{\partial}{\partial \xi}(n_{e1}u_{e1}) \quad (30)$$
$$- \frac{1}{v_0 \tau}\frac{\partial v_{e1}}{\partial \eta} + \frac{1}{v_0 \tau}(2u_{e1} + \frac{1}{\eta}v_{e1}) = 0,$$

$$\frac{\partial n_{i1}}{\partial \tau} - v_0 \frac{\partial n_{i2}}{\partial \xi} + \frac{\partial u_{i2}}{\partial \xi} + \frac{\partial}{\partial \xi}(n_{i1}u_{i1}) \quad (31)$$
$$+ \frac{1}{v_0 \tau}\frac{\partial v_{i1}}{\partial \eta} + \frac{1}{v_0 \tau}(2u_i + \frac{1}{\eta}v_{i1}) = 0,$$

$$\frac{\partial u_{e1}}{\partial \tau} - v_0 \frac{\partial u_{e2}}{\partial \xi} + u_{e1}\frac{\partial u_{e1}}{\partial \xi} = \quad (32)$$
$$m\frac{\partial \phi_2}{\partial \xi} + mF(n_{e1}\frac{\partial p_{e1}}{\partial \xi} - \frac{\partial p_{e2}}{\partial \xi}) + m\frac{H^2}{4}\frac{\partial^3 n_{e1}}{\partial \xi^3},$$

$$\frac{\partial u_{i1}}{\partial \tau} - v_0 \frac{\partial u_{i2}}{\partial \xi} + u_{i1}\frac{\partial u_{i1}}{\partial \xi} = \quad (33)$$
$$- \frac{\partial \phi_2}{\partial \xi} + \sigma(n_{i1}\frac{\partial p_{i1}}{\partial \xi} - \frac{\partial p_{i2}}{\partial \xi}),$$

$$F(\frac{\partial p_{e1}}{\partial \tau} - v_0 \frac{\partial p_{e2}}{\partial \xi} + u_{e1}\frac{\partial p_{e1}}{\partial \xi} + \gamma_e p_{e1}\frac{\partial u_{e1}}{\partial \xi}) \quad (34)$$
$$+ \gamma_e \frac{\partial u_{e2}}{\partial \xi} + \frac{2\gamma_e}{v_0 \tau}u_{e1} + \frac{\gamma_e}{v_0 \tau}\frac{\partial v_{e1}}{\partial \eta} + \frac{\gamma_e}{v_0 \tau\eta}v_{e1} = 0,$$

$$\frac{\partial p_{i1}}{\partial \tau} - v_0 \frac{\partial p_{i2}}{\partial \xi} + u_{i1}\frac{\partial p_{i1}}{\partial \xi} + \gamma_i p_{i1}\frac{\partial u_{i1}}{\partial \xi} \quad (35)$$
$$+ \gamma_i \frac{\partial u_{i2}}{\partial \xi} + \frac{2\gamma_i}{v_0 \tau}u_{i1} + \frac{\gamma_i}{v_0 \tau}\frac{\partial v_{i1}}{\partial \eta} + \frac{\gamma_i}{v_0 \tau\eta}v_{i1} = 0,$$

$$\frac{\partial^2 \phi}{\partial \xi^2} = n_{e2} - n_{e1} \quad (36)$$

Now we eliminate the second order quantities from (30)-(36) by means of (21)-(29)

$$\left.\begin{array}{l} a_e \dfrac{\partial \phi}{\partial \tau} + a_e b_e \dfrac{\partial \phi^2}{\partial \xi} + \dfrac{1}{v_0 \tau}\dfrac{\partial v_{e1}}{\partial \eta} + \dfrac{1}{v_0 \tau}(2b_e\phi + \dfrac{v_{e1}}{\eta}) \\[2mm] \qquad = v_0 \dfrac{\partial n_{e2}}{\partial \xi} - \dfrac{\partial u_{e2}}{\partial \xi} \\[2mm] i.e. \quad v_0 \dfrac{\partial n_{e2}}{\partial \xi} - \dfrac{\partial u_{e2}}{\partial \xi} = L_1^{/} - L_1^{//} \ (say) \\[2mm] \text{where } L_1^{/} = a_e \dfrac{\partial \phi}{\partial \tau} + a_e b_e \dfrac{\partial \phi^2}{\partial \xi} \\[2mm] \text{and } L_1^{//} = \dfrac{1}{v_0 \tau}\dfrac{\partial v_{e1}}{\partial \eta} + \dfrac{1}{v_0 \tau}(2b_e\phi + \dfrac{v_{e1}}{\eta}) \end{array}\right\} \quad (37)$$

$$\left.\begin{array}{l} a_i \dfrac{\partial \phi}{\partial \tau} + a_i b_i \dfrac{\partial \phi^2}{\partial \xi} + \dfrac{1}{v_0 \tau}\dfrac{\partial v_{i1}}{\partial \eta} + \dfrac{1}{v_0 \tau}(2b_i\phi + \dfrac{v_{i1}}{\eta}) \\[2mm] \qquad = v_0 \dfrac{\partial n_{i2}}{\partial \xi} - \dfrac{\partial u_{i2}}{\partial \xi} \\[2mm] i.e. \quad v_0 \dfrac{\partial n_{i2}}{\partial \xi} - \dfrac{\partial u_{i2}}{\partial \xi} = L_2^{/} - L_2^{//} \ (say) \\[2mm] \text{where } L_2^{/} = a_i \dfrac{\partial \phi}{\partial \tau} + a_i b_i \dfrac{\partial \phi^2}{\partial \xi} \\[2mm] \text{and } L_2^{//} = \dfrac{1}{v_0 \tau}\dfrac{\partial v_{i1}}{\partial \eta} + \dfrac{1}{v_0 \tau}(2b_i\phi + \dfrac{v_{i1}}{\eta}) \end{array}\right\} \quad (38)$$

$$
\left.\begin{aligned}
& b_e \frac{\partial \phi}{\partial \tau} + b_e^2 \phi \frac{\partial \phi}{\partial \xi} - mF a_e c_e \phi \frac{\partial \phi}{\partial \xi} - m\frac{H^2}{4} a_e \phi \frac{\partial^3 \phi}{\partial \xi^3} \\
& \qquad = m\frac{\partial \phi}{\partial \xi} - mF \frac{\partial p_{e2}}{\partial \xi} + v_0 \frac{\partial u_{e2}}{\partial \xi} \\
& i.e. \ \ m\frac{\partial \phi_2}{\partial \xi} - mF \frac{\partial p_{e2}}{\partial \xi} + v_0 \frac{\partial u_{e2}}{\partial \xi} = L_3 \\
& where \ \ L_3 = b_e \frac{\partial \phi}{\partial \tau} + b_e^2 \phi \frac{\partial \phi}{\partial \xi} \\
& \qquad\qquad - mF a_e c_e \phi \frac{\partial \phi}{\partial \xi} - m\frac{H^2}{4} a_e \phi \frac{\partial^3 \phi}{\partial \xi^3}
\end{aligned}\right\} \tag{39}
$$

$$
\left.\begin{aligned}
& b_i \frac{\partial \phi}{\partial \tau} + b_i^2 \phi \frac{\partial \phi}{\partial \xi} - \sigma a_i c_i \phi \frac{\partial \phi}{\partial \xi} = \\
& \qquad -\frac{\partial \phi_2}{\partial \xi} - \sigma \frac{\partial p_{i2}}{\partial \xi} + v_0 \frac{\partial u_{i2}}{\partial \xi} \\
& i.e. \ -\frac{\partial \phi_2}{\partial \xi} - \sigma \frac{\partial p_{i2}}{\partial \xi} + v_0 \frac{\partial u_{i2}}{\partial \xi} = L_4 \\
& where \ \ L_4 = b_i \frac{\partial \phi}{\partial \tau} + b_i^2 \phi \frac{\partial \phi}{\partial \xi} - \sigma a_i c_i \phi \frac{\partial \phi}{\partial \xi}
\end{aligned}\right\} \tag{40}
$$

$$
\left.\begin{aligned}
& c_e F \frac{\partial \phi}{\partial \tau} + F b_e c_e \phi \frac{\partial \phi}{\partial \xi} + F \gamma_e b_e c_e \phi \frac{\partial \phi}{\partial \xi} + \frac{2\gamma_e b_e \phi}{v_0 \tau} \\
& \qquad + \frac{\gamma_e}{v_0 \tau} \frac{\partial v_{e1}}{\partial \eta} + \frac{\gamma_e v_{e1}}{v_0 \tau \eta} = F v_0 \frac{\partial p_{e2}}{\partial \xi} - \gamma_e \frac{\partial u_{e2}}{\partial \xi} \\
& i.e. \ \ F v_0 \frac{\partial p_{e2}}{\partial \xi} - \gamma_e \frac{\partial u_{e2}}{\partial \xi} = L_5' + L_5'' \\
& where \ \ L_5' = c_e F \frac{\partial \phi}{\partial \tau} + F b_e c_e \phi \frac{\partial \phi}{\partial \xi} + F \gamma_e b_e c_e \phi \frac{\partial \phi}{\partial \xi} \\
& and \ \ L_5'' = \frac{2\gamma_e b_e \phi}{v_0 \tau} + \frac{\gamma_e}{v_0 \tau} \frac{\partial v_{e1}}{\partial \eta} + \frac{\gamma_e v_{e1}}{v_0 \tau \eta}
\end{aligned}\right\} \tag{41}
$$

$$
\left.\begin{aligned}
& c_i \frac{\partial \phi}{\partial \tau} + b_i c_i \phi \frac{\partial \phi}{\partial \xi} + \gamma_i b_i c_i \phi \frac{\partial \phi}{\partial \xi} + \frac{2\gamma_i b_i \phi}{v_0 \tau} \\
& \qquad + \frac{\gamma_i}{v_0 \tau} \frac{\partial v_{i1}}{\partial \eta} + \frac{\gamma_i v_{i1}}{v_0 \tau \eta} = v_0 \frac{\partial p_{i2}}{\partial \xi} - \gamma_i \frac{\partial u_{i2}}{\partial \xi} \\
& i.e. \ \ v_0 \frac{\partial p_{i2}}{\partial \xi} - \gamma_i \frac{\partial u_{i2}}{\partial \xi} = L_6' + L_6'' \\
& where \ \ L_6' = c_i \frac{\partial \phi}{\partial \tau} + b_i c_i \phi \frac{\partial \phi}{\partial \xi} + \gamma_i b_i c_i \phi \frac{\partial \phi}{\partial \xi} \\
& and \ \ L_6'' = \frac{2\gamma_i b_i \phi}{v_0 \tau} + \frac{\gamma_i}{v_0 \tau} \frac{\partial v_{i1}}{\partial \eta} + \frac{\gamma_i v_{i1}}{v_0 \tau \eta}
\end{aligned}\right\} \tag{42}
$$

$$
\frac{\partial^2 \phi}{\partial \xi^2} = n_{e2} - n_{i2} \tag{43}
$$

$$
\frac{\partial^3 \phi}{\partial \xi^3} = \frac{\partial n_{e2}}{\partial \xi} - \frac{\partial n_{ei2}}{\partial \xi} \tag{44}
$$

From (37) $\times \gamma_e - (41)$

$$
\gamma_e L_1' - L_5' = \gamma_e (v_0 \frac{\partial n_{e2}}{\partial \xi} - \frac{\partial u_{e2}}{\partial \xi}) - F v_0 \frac{\partial p_{e2}}{\partial \xi} + \gamma_e \frac{\partial u_{e2}}{\partial \xi}
$$

$$
\gamma_e L_1' - L_5' = \gamma_e v_0 \frac{\partial n_{e2}}{\partial \xi} - F v_0 \frac{\partial p_{e2}}{\partial \xi} \tag{45}
$$

From (38) $\times \gamma_i - (42)$

$$
\gamma_i L_2' - L_6' = \gamma_i \left(v_0 \frac{\partial n_{i2}}{\partial \xi} - \frac{\partial u_{i2}}{\partial \xi} \right) - v_0 \frac{\partial p_{i2}}{\partial \xi} + \gamma_i \frac{\partial u_{i2}}{\partial \xi}
$$

$$
\gamma_i L_2' - L_6' = \gamma_i v_0 \frac{\partial n_{i2}}{\partial \xi} - v_0 \frac{\partial p_{i2}}{\partial \xi} \tag{46}
$$

From (39) $\times v_0 - m \times$ (45)

$$
m v_0 \frac{\partial \phi_2}{\partial \xi} - m\gamma_e v_0 \frac{\partial n_{e2}}{\partial \xi} + v_0^2 \frac{\partial u_{e2}}{\partial \xi} = v_0 L_3 - m\gamma_e L_1' + m L_5' \tag{47}
$$

From (40) $\times v_0 - \sigma \times (46)$

$$
-v_0 \frac{\partial \phi_2}{\partial \xi} + v_0^2 \frac{\partial u_{i2}}{\partial \xi} - \sigma \gamma_i v_0 \frac{\partial n_{i2}}{\partial \xi} = v_0 L_4 - \sigma \left(\gamma_i L_2' - L_6' \right) \tag{48}
$$

From (38)&(47)

$$
\left.\begin{aligned}
& v_0^2 \left(v_0 \frac{\partial n_{e2}}{\partial \xi} - L_1' - L_1'' \right) - m\gamma_e v_0 \frac{\partial n_{e2}}{\partial \xi} = v_0 L_3 - m\gamma_e L_1' + m L_5' \\
& (v_0^3 - m\gamma_e v_0) \frac{\partial n_{e2}}{\partial \xi} = v_0 L_3 - m\gamma_e L_1' + m L_5' + v_0^2 L_1' + v_0^2 L_1''
\end{aligned}\right\} \tag{49}
$$

From (37) & (47)

$$
\left.\begin{aligned}
& v_0^2 \left(v_0 \frac{\partial n_{i2}}{\partial \xi} - L_2' - L_2'' \right) - \sigma \gamma_i v_0 \frac{\partial n_{i2}}{\partial \xi} - v_0 L_4 - \sigma \left(\gamma_i L_2' - L_6' \right) \\
& \left(v_0^3 - \sigma \gamma_i v_0 \right) \frac{\partial n_{i2}}{\partial \xi} = v_0 L_4 - \sigma \left(\gamma_i L_2' - L_6' \right) + v_0 L_2' + v_0^2 L_2''
\end{aligned}\right\} \tag{50}
$$

From (42), (48), and (49)

$$
\begin{aligned}
\frac{\partial^3 \phi}{\partial \xi^3} = & \left\{ \frac{v_0 L_3 - m\gamma_e L_1' + m L_5' + v_0^2 L_1' + v_0^2 L_1''}{(v_0^3 - m\gamma_e v_0)} \right\} \\
& - \left\{ \frac{v_0 L_4 - \sigma \gamma_i L_2' + \sigma L_6' + v_0 L_2' + v_0^2 L_2''}{\left(v_0^3 - \sigma \gamma_i v_0 \right)} \right\}
\end{aligned} \tag{51}
$$

$$\frac{\partial^3 \phi}{\partial \xi^3}\left[1+\frac{v_0 ma_e \frac{H^2}{4}}{(v_0^3-m\gamma_e v_0)}\right]$$

$$+\frac{\partial \phi}{\partial \tau}\left[\frac{(v_0 b_i+\sigma c_i-\sigma\gamma_i a_i+v_0^2 a_i)}{(v_0^3-\sigma\gamma_i v_0)}-\frac{(v_0 b_e-m\gamma_e a_e+mc_e F+v_0^2 a_e)}{(v_0^3-m\gamma_e v_0)}\right]$$

$$+\phi\frac{\partial \phi}{\partial \xi}\left[\frac{\left(\begin{array}{c}v_0 b_i^2-\sigma a_i c_i v_0-2\sigma\gamma_i a_i b_i\\+\sigma b_i c_i+\sigma\gamma_i b_i c_i+2v_0^2 a_i b_i\end{array}\right)}{(v_0^3-\sigma\gamma_i v_0)}-\frac{\left(\begin{array}{c}v_0 b_e^2-v_0 mFa_e c_e-2m\gamma_e a_e b_e\\+mFb_e c_e+mF\gamma_e b_e c_e+2v_0^2 a_e c_e\end{array}\right)}{(v_0^3-m\gamma_e v_0)}\right]$$

$$+\frac{\phi}{\tau}\left[\frac{2v_0 b_i}{(v_0^3-\sigma\gamma_i v_0)}-\frac{2v_0 b_e}{(v_0^3-m\gamma_e v_0)}\right]$$

$$+\frac{v_0^2}{v_0\tau}\left[\frac{\frac{\partial v_{i1}}{\partial \eta}}{(v_0^3-\sigma\gamma_i v_0)}-\frac{\frac{\partial v_{e1}}{\partial \eta}}{(v_0^3-m\gamma_e v_0)}\right]$$

$$+\frac{v_0^2}{v_0\tau\eta}\left[\frac{v_{i1}}{(v_0^3-\sigma\gamma_i v_0)}-\frac{v_{e1}}{(v_0^3-m\gamma_e v_0)}\right]=0 \tag{52}$$

Now in order to solve the equation we transform the set of equations into a simple looking structure. Let us consider

$$X=\frac{\partial^3 \phi}{\partial \xi^3}\left[1+\frac{v_0 ma_e \frac{H^2}{4}}{(v_0^3-m\gamma_e v_0)}\right]$$

$$+\frac{\partial \phi}{\partial \tau}\left[\frac{(v_0 b_i+\sigma c_i-\sigma\gamma_i a_i+v_0^2 a_i)}{(v_0^3-\sigma\gamma_i v_0)}-\frac{(v_0 b_e-m\gamma_e a_e+mc_e F+v_0^2 a_e)}{(v_0^3-m\gamma_e v_0)}\right]$$

$$+\phi\frac{\partial \phi}{\partial \xi}\left[\frac{\left(\begin{array}{c}v_0 b_i^2-\sigma a_i c_i v_0-2\sigma\gamma_i a_i b_i\\+\sigma b_i c_i+\sigma\gamma_i b_i c_i+2v_0^2 a_i b_i)\end{array}\right)}{(v_0^3-\sigma\gamma_i v_0)}-\frac{\left(\begin{array}{c}v_0 b_e^2-v_0 mFa_e c_e-2m\gamma_e a_e b_e\\+mFb_e c_e+mF\gamma_e b_e c_e+2v_0^2 a_e c_e)\end{array}\right)}{(v_0^3-m\gamma_e v_0)}\right]$$

$$+\frac{\phi}{\tau}\left[\frac{2v_0 b_i}{(v_0^3-\sigma\gamma_i v_0)}-\frac{2v_0 b_e}{(v_0^3-m\gamma_e v_0)}\right] \tag{53}$$

$$Y=\frac{v_0^2}{v_0\tau}\left[\frac{\frac{\partial v_{i1}}{\partial \eta}}{(v_0^3-\sigma\gamma_i v_0)}-\frac{\frac{\partial v_{e1}}{\partial \eta}}{(v_0^3-m\gamma_e v_0)}\right]$$

$$+\frac{v_0^2}{v_0\tau\eta}\left[\frac{v_{i1}}{(v_0^3-\sigma\gamma_i v_0)}-\frac{v_{e1}}{(v_0^3-m\gamma_e v_0)}\right] \tag{54}$$

$$\frac{\partial}{\partial \xi}[X]+\frac{\partial}{\partial \xi}[Y]=0 \tag{55}$$

$$\frac{\partial X}{\partial \xi}+\frac{1}{v_0\tau}\frac{\partial Y}{\partial \eta}=0 \tag{56}$$

$$\frac{\partial X}{\partial \xi}+\frac{1}{v_0\tau}\left[\frac{v_0^2}{v_0\tau}\right]\left[\frac{\frac{\partial^2 v_{i1}}{\partial \eta^2}}{(v_0^3-\sigma\gamma_i v_0)}-\frac{\frac{\partial^2 v_{e1}}{\partial \eta^2}}{(v_0^3-m\gamma_e v_0)}\right]+$$

$$\frac{1}{v_0\tau}\left[\frac{v_0^2}{v_0\tau}\right]\left[\frac{1}{\eta}\left\{\frac{v_{i1}}{(v_0^3-\sigma\gamma_i v_0)}-\frac{v_{e1}}{(v_0^3-m\gamma_e v_0)}\right\}\right]=0 \tag{57}$$

$$\frac{\partial X}{\partial \xi}+\left[p\frac{\partial}{\partial \eta}\left(\frac{\partial v_{i1}}{\partial \eta}\right)-q\frac{\partial}{\partial \eta}\left(\frac{\partial v_{e1}}{\partial \eta}\right)\right]$$

$$+\left[\frac{p}{\eta}\frac{\partial v_{i1}}{\partial \xi}\left(\frac{\partial \xi}{\partial \eta}\right)-\frac{q}{\eta}\frac{\partial v_{e1}}{\partial \xi}\left(\frac{\partial \xi}{\partial \eta}\right)\right]=0 \tag{58}$$

where

$$p=\frac{1}{v_0\tau}\left[\frac{v_0^2}{v_0\tau}\right]\frac{1}{(v_0^3-\sigma\gamma_i v_0)} \tag{59}$$

$$q=\frac{1}{v_0\tau}\left[\frac{v_0^2}{v_0\tau}\right]\frac{1}{(v_0^3-m\gamma_e v_0)} \tag{60}$$

$$\frac{\partial X}{\partial \xi}+\left[pd_i v_0\tau\left(\frac{\partial^2 \phi}{\partial \eta^2}\right)-qd_e v_0\tau\left(\frac{\partial^2 \phi}{\partial \eta^2}\right)\right]+$$

$$\left[\frac{p}{\eta}d_i v_0\tau\left(\frac{\partial \phi}{\partial \eta}\right)-\frac{q}{\eta}d_e v_0\tau\left(\frac{\partial \phi}{\partial \eta}\right)\right]=0 \tag{61}$$

i.e. $\frac{\partial X}{\partial \xi}+v_0\tau\left[p_1\left(\frac{\partial^2 \phi}{\partial \eta^2}\right)+\frac{p_2}{\eta}\left(\frac{\partial \phi}{\partial \eta}\right)\right]=0$

where $p_1=\left[pd_i v_0\tau-qd_e v_0\tau\right]$

and $p_2=\left[\frac{p}{\eta}d_i v_0\tau-\frac{q}{\eta}d_e v_0\tau\right]$

Next eliminating the second order quantities from (30)–(36) by means of (21)-(29) we obtain the variable coefficient SKPE as:

$$\frac{\delta}{\delta \xi}\left[\frac{\partial \phi}{\partial \tau}+A\phi\frac{\partial \phi}{\partial \xi}+B\frac{\partial^3 \phi}{\partial \xi^3}+\frac{\phi}{\tau}\right]+v_0\tau\left[p_1\left(\frac{\partial^2 \phi}{\partial \eta^2}\right)+\frac{p_2}{\eta}\left(\frac{\partial \phi}{\partial \eta}\right)\right]=0 \tag{62}$$

where $\phi=\phi_1, \mu=1/m$ and

$$A=\frac{\left[\begin{array}{c}(v_0^3-m\gamma_e v_0)\left(\begin{array}{c}v_0 b_i^2-\sigma a_i c_i v_0-2\sigma\gamma_i a_i b_i\\+\sigma b_i c_i+\sigma\gamma_i b_i c_i+2v_0^2 a_i b_i\end{array}\right)\\-(v_0^3-\sigma\gamma_i v_0)\left(\begin{array}{c}v_0 b_e^2-v_0 mFa_e c_e-2m\gamma_e a_e b_e\\+mFb_e c_e+mF\gamma_e b_e c_e+2v_0^2 a_e c_e\end{array}\right)\end{array}\right]}{\left[\begin{array}{c}(v_0^3-m\gamma_e v_0)(v_0 b_i+\sigma c_i-\sigma\gamma_i a_i+v_0^2 a_i)\\-(v_0^3-\sigma\gamma_i v_0)(v_0 b_e-m\gamma_e a_e+mc_e F+v_0^2 a_e)\end{array}\right]} \tag{63}$$

$$B=\frac{(v_0^3-m\gamma_e v_0+v_0 ma_e\frac{H^2}{4})(v_0^3-\sigma\gamma_i v_0)}{\left[\begin{array}{c}(v_0^3-m\gamma_e v_0)(v_0 b_i+\sigma c_i-\sigma\gamma_i a_i+v_0^2 a_i)\\-(v_0^3-\sigma\gamma_i v_0)(v_0 b_e+m\gamma_e a_e+mc_e F+v_0^2 a_e)\end{array}\right]} \tag{64}$$

Now if the angular dependence can be neglected, the SKPE (62) reduces to usual KdV equation. If we assume the similarity solution, viz.

$$\zeta \equiv \xi - \frac{1}{2} v_0 \eta^2 \tau, \; \phi = \phi(\zeta, \tau) \tag{65}$$

Then (62) reduces under the suitable boundary conditions (66) to the following standard Korteweg-de Vries equation (67)with solution as (68):

$$\phi \to 0, \; \frac{d^2\phi}{d\xi^2} \to 0 \;\; \text{as} \;\; \xi \to \pm\infty \tag{66}$$

$$\frac{\partial \phi}{\partial \tau} + A\phi \frac{\partial \phi}{\partial \zeta} + B \frac{\partial^3 \phi}{\partial \zeta^3} = 0 \tag{67}$$

$$\phi = \frac{3U_0}{A} \operatorname{sech}^2 \left[\sqrt{\frac{U_0}{4B}} \left(\zeta - U_0 \tau \right) \right] \tag{68}$$

In order to study the effects of relativistic degeneracy parameter F (which depends on R), Quantum diffraction parameter (H) and ion temperature (σ) we have made plots in Figs. 1-3.
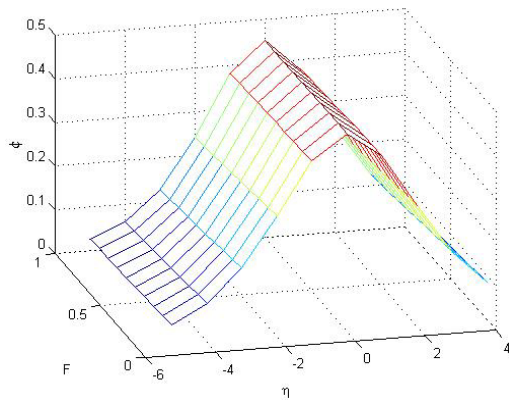

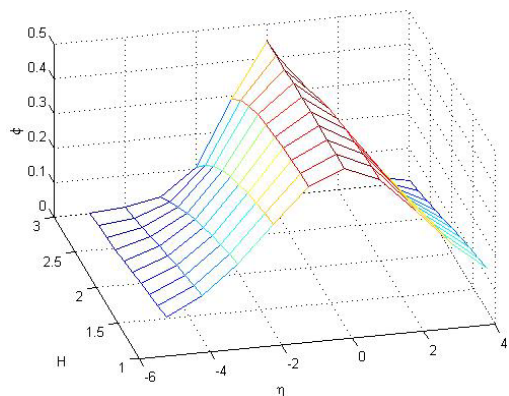Fig. 1 Solitary profiles for different value of F
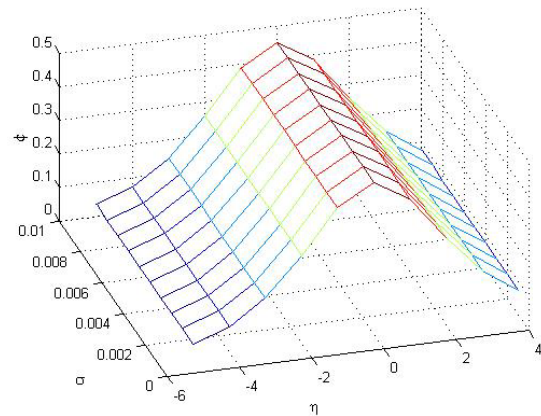

Fig. 2 Solitary profiles for different value of H

Fig. 3 Solitary profiles for different value of σ

IV. DISCUSSION AND CONCLUSION

As the relativistic degeneracy parameter F determines the transition from ultrarelativistic to non relativistic cases it is important to know how it affects the structure and properties of ion-acoustic waves in a quantum plasma.It has been found that relativistic degeneracy effects have little effect in the properties of solitary profiles. On the other hand as quantum diffraction parameter increases the solitary structures become more narrow but the amplitude remains almost constant [26]. Ion temperature has very minute effect on solitary profiles in plasmas. Finally we would like to point out that the investigation presented here may be helpful in the understanding of the basic features of long wavelength electron plasma waves in dense and hot plasmas such as can be found in white dwarfs, neutron stars and intense laser-solid plasma experiments.

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