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Nonlinear Large Deformation Analysis of Rotor

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***Abstract*—**Reliability assessment and risk analysis of rotating machine rotors in various overload and malfunction situations present challenge to engineers and operators. In this paper a new analytical method for evaluation of rotor under large deformation is addressed. Model is presented in general form to include also composite rotors. Presented simulation procedure is based on variational work method and has capability to account for geometric nonlinearity, large displacement, nonlinear support effect and rotor contacting other machine components. New shape functions are presented which capable to predict accurate nonlinear profile of rotor. The closed form solutions for various operating and malfunction situations are expressed. Analytical simulation results are discussed.

***Keywords*—**Large Deformation, Nonlinear, Rotor.

I. Introduction

DESIGN and safety assessment of rotating machines shall be involved complete simulations of all possible overload, malfunction and accident scenarios [1]–[3]. Otherwise it may end up with equipment that is not safe over the full operating ranges. Rotor large deformation can be result of overloading, over-speed, resonance, whirling, accident, component failure, surge, stall, off design operation, etc and may lead to stress exceeding the safe limit [4], failure of machine component, machine explosion and equipment coming apart. Each scenario can cause serous damages and injuries. For reliability and safety assessment of all possible scenarios of rotating machine malfunction, it is necessary to develop an analytical method to obtain details of forces, stress and strain values for rotor under large deformation.

Now a day composite rotors (multilayer rotors) are more considered in rotating machine design. But there are limited publications about this type of rotor especially in the field of deformation and stress analysis. It is necessary to develop nonlinear stress analysis approach for composite rotors.

There are many approximate and rough analytical methods for rotor or beam analysis. One term cosine are introduced and used in some publications for deflected rotors or beams. Comparison with refined Finite Element Method (F.E.M.) solution shows that one term cosine lead to unrealistic profile and stress, in some cases, 30% higher than refined numerical results. Some references recommended one term sine shape function for energy method which lead to simplified calculations but resulted in unrealistic high values for deformation, stress and strain. Two opposite quarter circle was also presented as shape function with geometrical conditions mainly with formulations other than energy method which is simple but inaccurate.

Large deflection of rotor entails large strain to sensitive multi-component structure and may impose excessive stress to each component of rotor which could lead to damage or even collapse. Repair processes of rotors are very expensive and time consuming. Also shut down results in lost of plant revenue for repair period. Therefore, operation of machine must be carried out with reasonable prediction and knowledge of stresses and deformations in rotor and forces to machine components.

In general, analysis of rotor under large deformation requires nonlinear complex behavior simulation, changes in stiffness due to the changes in rotor geometry, nonlinear restraint of support and considering of contact to the other machine components.

This paper offers an analytical method for assessing the effects of large deformation of rotor based on virtual work theory. It allows the investigation of numerous and different large deformation situations, provides parametric study and allows provision of curves or tables for allowable operating conditions of machine. These information and data are critical for operators for fast evaluation of rotating machine and application of new operating conditions or safety and reliability assessments. It is also very useful for designers and rotating machine engineers for risk management, HAZOP review and vendor drawing evaluations.

An alternative of present procedure is a Finite Element Method (F.E.M.). However, owing to the uncertainty involved in estimating support behavior and rotor embedment conditions and complex nonlinear behavior of rotor, numerical approach will need numerous sophisticated and advanced elements from several types for nonlinear effects and interactions of rotor. The appropriate use of nonlinear Finite Element Method (F.E.M.) for analysis of rotor, especially composite rotors, under large deformation requires specialized trainings and experiences, together with special software and various elements, considerable cost, time and engineering efforts. Aside from mentioned draw backs, the engineer may not gain inside the relationship between numerical results and nonlinear model. Also numerical results are not expansive and there is no provision for parametric study. Presented closed form equations in this paper are applicable for various situations such as free to deformed rotor and rotor contacting machine components. By solving these equations, unknowns

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can be determined and accurate rotor profile, stress and strain can be obtained. Present study was undertaken as important part of effort to develop analytical expressions and useful curves for assessing the effects of large rotor deformation for normal operation as well as accident, resonance, whirling, overloading and malfunction situations.

II. Model and Formulation

Fig. 1 presents model for nonlinear deformation of rotor under large deformation. This model accurately simulates rotor large deformation and includes all possible distributed and concentrated loads on rotor.

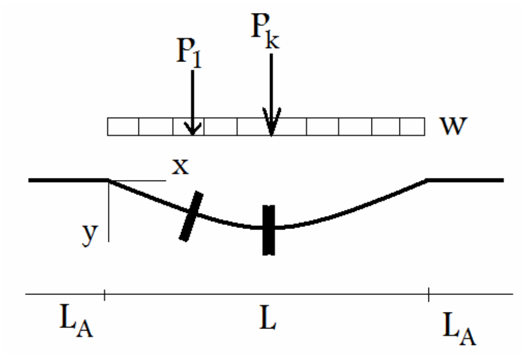

Fig. 1 Model for Nonlinear Deformation of Rotor under Large Deformation.

By considering the ′′Principle of Virtual Work′′ [5]–[8] if a body is in equilibrium, the total work of all forces acting on the body in any virtual displacement vanishes. The profile of the deformed rotor is assumed as (1). Variable and functions "x", "q_i" and "f_i" are coordinate along the original rotor axis, undetermined (unknown) coefficients and shape functions. With reference to physical model (Fig. 1), shape functions ("$f_i(x)$") are assumed smooth.

$$y(x)=\sum_{i=1}^{n} q_i f_i(x) \tag{1}$$

Stated mathematically, the equilibrium condition is as (2). Where "U" is the strain work and "W" is the load potential.

$$\frac{\partial U}{\partial q_j}-\frac{\partial W}{\partial q_j}=0 \ , \quad j=1 \ to \ n \tag{2}$$

Equations (2) can be solved to yield the values of "q_i" that best approximate equilibrium condition. It should be noted that more terms of shape functions assumed in profile (1), closely the assumed shape resembles the true state of deformed rotor. As result, the more accurate is the approximation to the equilibrium position and the resulting internal forces and moments in machine.

Total length of rotor between the points of effective anchorage supports is denoted ($L'=L+2L_A$) and represents the portion of rotor that undergoes axial elongation as a result of loading. However, only the laterally displaced portion of rotor (L) undergoes bending deformation. With respect to this model, (2) can be re-written as expression (3). Where "U_a", "U_b" and "W_e" are the axial deformation strain work, bending deformation strain work and externally applied forces potential, respectively.

$$\frac{\partial U_a}{\partial q_j}+\frac{\partial U_b}{\partial q_j}-\frac{\partial W_e}{\partial q_j}=0 \ , \quad j=1 \ to \ n \tag{3}$$

For elastic material behavior, the variation axial deformation strain work term can be expressed as (4). Parameters and variables are as (5) to (7). Parameter "E" and "A" are rotor elastic modulus and rotor cross section area respectively.

$$\frac{\partial U_a}{\partial q_j}=ke\frac{\partial e}{\partial q_j} \tag{4}$$

$$k=\frac{E\ A}{L'} \ , \quad L'=L+2L_A \tag{5}$$

$$e=\widetilde{L}-L \tag{6}$$

$$\widetilde{L}=\int_0^L\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx \tag{7}$$

Variation of bending deformation strain work is expressed as (8). Parameter "K" is as (9). Parameter "I_{eq}" is moment of inertia of composite rotor.

$$\frac{\partial U_b}{\partial q_j}=\int_0^l K\frac{d^2y}{dx^2}f_j''\,dx \tag{8}$$

$$K=EI_{eq} \tag{9}$$

The laterally applied forces can be distributed or concentrated. The k-th concentrated lateral load "P_k" is assumed to be located at "$x=x_k$". Distributed lateral load "$w(x)$" is assumed to extend over rotor length "L". Distributed load "$P(x)$" and parameter "a" and "b" are distributed load on rotor, initial and final locations of this load ("$P(x)$"), respectively. The variation of externally applied forces potential is given by (10).

$$\frac{\partial W_e}{\partial q_j}=\int_0^L w(x)f_j(x)dx+\sum_{k=1}^{m}f_j(x_k)P_k + \int_a^b P(x)f_j(x)dx \tag{10}$$

With reference to (5) and Fig. 1, in order to determine suitable value for the length of rotor undergoing axial straining (L'), certain aspects of axial effective anchor lengths (parameter (L_A) which determined by axial support rotor interaction) shall be considered.

A rotor experiencing large lateral displacements will develop axial tension forces. It will behave like a cable. To satisfy equilibrium, this tension must be primarily balanced by the axial restraint of the support or other rotating machine or equipment shaft or rotor coupled to simulated rotor. It results in tension in rotor and coupling or shaft extension coupled to rotor. In present paper this effect has been modeled by providing for additional length of rotor (as parameter "L_A") that extend to points of effective anchorage or fix support.

To model composite rotors, analytical expressions for the equivalent composite rotor parameters (for example composite rotor moment of inertia as parameter "I_{eq}") shall be used in presented method. The flexural rigidity of the composite section in the elastic region can be evaluated based on the assumption that the all rotor components resist the bending as a unit with no slippage between components.

III. Free to Deform Rotor

After selecting assumed shape functions ("$f_i(x)$" in (1)), variational expressions given by (3) are substituted, which are then solved by iteration to determine coefficients and to obtain equilibrium. Upon solution of the equilibrium equations, the profile of deformation, strain and stress of rotor are known. For "Free to Deform Rotor" the shape of deformed profile of rotor in the rotor deformed region (length "L") was assumed to be series of cosine curves as (11).

$$y(x)=\sum_{i=1}^{n} q_i \frac{1}{2}\left(1-\cos\left(\frac{2(2i-1)\pi x}{L}\right)\right) \tag{11}$$

Equations are presented for a simple loading as uniform distributed load. Equilibrium equations can be obtained for complex and sophisticated loadings in the same manner. The equilibrium equations as (3) can be expressed as (12) for uniform distributed load assumption. Coefficients "q_i" can be obtained by solving "n" equations (j=1 to n) as per expression (12). Function (y') is calculated as per (13).

$$k\left(\frac{(2j-1)\pi}{L}\right)\left(\left(\int_0^L \sqrt{1+(y')^2}\,dx\right)-L\right)\left(\int_0^L \frac{\sin\left(\frac{2(2j-1)\pi x}{L}\right)y'dx}{\sqrt{1+(y')^2}}\right)$$
$$+2K(2j-1)^4\frac{\pi^4}{L^3}q_j-\frac{1}{2}Lw=0 \tag{12}$$

$$y'=\frac{\pi}{L}\sum_{i=1}^{n}(2i-1)q_i\sin\left(\frac{2(2i-1)\pi x}{L}\right) \tag{13}$$

Solution of (12) is for free to deform rotor under the uniform distributed load "w". The load "w" is the assumed equivalent uniform load of the rotor and its components. The rotor is considered to be free to deform. For an actual rotor large deformation, the rotor may not be free to deform and assumed equivalent uniform load may not be correct rotor loading situation. For some loading scenarios, depending on loading details or rotor geometry, uniform load assumption may result in error. For these cases proper concentrated forces and locally distributed loads shall be assigned and general solution ($w(x)$, P_k and $P(x)$ in Fig. 1 and (10)) shall be used. Solution of (12) can be used to check a rough clearance needed to avoid contact and accident in machine. The clearance shall be more than the calculated displacement for defined loadings. For API rotating machines at least 25% safety margin shall also be respected for calculated maximum displacement (for all possible loading from zero to trip speed) to obtain clearance.

This solution (Equations (12)) can also be used to keep rotor under controlled stress/strain conditions. The problem can be solved by formulation ((12) and (13)) and iterating to find the maximum loadings of rotor to given allowable stress/strain. Normally the calculated stress shall be below or equal to 75% of allowable stress limit of rotor. In case of rotor contacting casing or other components, the shape function as (11) is not valid and solution for rotor contacting machine components (next section) shall be used.

IV. Rotor Contacting Machine Component

For some assessment purposes, it is necessary to consider the effect of the rotor contacting the machine component (such as machine casing or housing). Generally contact will happen in the central portion of the rotor where rotor has maximum deflection. This solution is applicable for some severe malfunction situations including whirling, resonance, sever machine internal accidents, severe overloading and tight machine clearances. For this case assumed rotor shape to be modified to include a contact in central portion of rotor. This problem is solved by considering the half of model due to symmetric assumption. New compatible shape function shall be written and equilibrium equation as (3) for this new shape profile shall be obtained. Parameter "a" (initial location of contacting load) and other unknown coefficients ("q_i") can be found by iteration. New shape function for rotor contacting machine component is assumed as (14).

$$y(x)=\begin{cases}\sum_{i=1}^{n}\frac{1}{2}q_i\left(1-\cos\left(\frac{(2i-1)\pi x}{a}\right)\right) & 0<x\le a\\ D_m & a<x\le \frac{L}{2}\end{cases} \tag{14}$$

To have a smooth profile equation " $y(a) = D_m$ " shall be governed. This equation can be expressed as (15). Parameter "D_m" is initial clearance between rotor and contacted component.

$$\sum_{i=1}^{n} q_i = D_m \tag{15}$$

Assumed profile (14) is substituted to variational expression (as (3)) for uniform distributed load. Equilibrium equations are obtained as (16). Function (y') is calculated as per (17). Variation of externally applied loads from contacted component to rotor is zero. In the other words, this load can not affect the profile of deformed rotor.

$$k\left(\frac{(2j-1)\pi}{a}\right)\left(\left(\int_0^{a/2}\sqrt{1+(y')^2}\,dx\right)-\frac{a}{2}\right)\left(\int_0^{a/2}\frac{\sin\left(\frac{(2j-1)\pi x}{a}\right)y'dx}{\sqrt{1+(y')^2}}\right) \tag{16}$$
$$+K(2j-1)^4\frac{\pi^4}{a^3}q_j-\frac{1}{4}aw=0$$

$$y' = \frac{\pi}{a}\sum_{i=1}^{n}(2i-1)q_i \sin\left(\frac{(2i-1)\pi x}{a}\right) \tag{17}$$
$$For\ 0 < x \leq \frac{a}{2}$$

Equations (15) and (16) are "*n+1*" equations. By solving these equation system, "*n+1*" unknown including "*a*", "q_1", "q_2",…, "q_n" will be determined..

V. Results

Analytical simulation results are presented for a heavy duty special purpose centrifugal compressor rotor in refinery service. Rotor length, span, mass, diameter at coupling hub and impeller are 1880 mm, 1450 mm, 550 KG, 110 mm and 160 mm respectively. It is a rotor with five impellers (five mechanical stage compressor). Compressor nominal operating speed is 9500 rpm. Machine operating speed range is from 67% to 105% of nominal operating speed with trip at 115% of nominal operating speed. First and second critical speeds are at 57% and 200% of machine nominal operating speed.

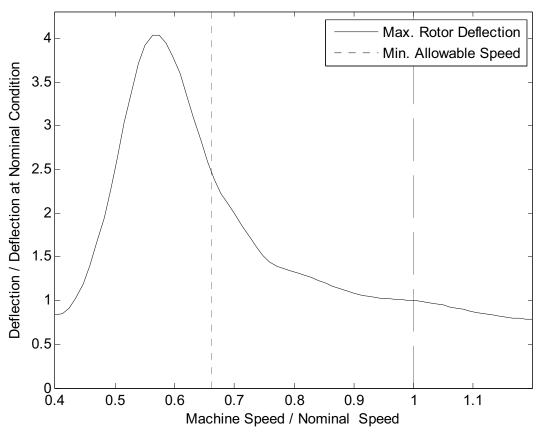

Fig. 2 Rotor Deflection vs. Machine Speed for Centrifugal Compressor Rotor.

This rotor is simulated free to deform to find rotor performance and clearance orders for working near first critical speed (because rotor is flexible and operating speed range is above first critical speed). Since large deformation of rotor near first critical speed is investigated, distributed variable load is respected. Rotor is modeled by equivalent uniformly distributed mass and variable distributed dynamic loading.

Fig. 2 presents rotor dynamic deflection vs. machine speed. This plot shows rotor deflection for machine speed from 40% to 120% of nominal operating speed (including first critical speed at around 57% of nominal operating speed). Based on this plot rotor dynamic deflection in first critical speed is more than four times of rotor deflection in nominal operating condition. Presented results show importance of controlling operating speed range in variable speed rotating machine. Clearances shall be selected with respect to maximum deflection at operating speed range. For flexible rotors (rotors which work above first critical speed) start up and speed up shall be implemented very carefully to avoid machine damage. In addition operating speed range, control system, clearances and condition monitoring shall be selected properly.

VI. Conclusion

Reliability and safety assessment of rotating machines need nonlinear analysis of rotor under large deformation. The theoretical basis and analytical solution for assessing the effects of large deformation of rotor has been presented. The procedure has the capability to account nonlinear effects, large displacement geometry effects, complicated effects of composite rotors and effects of rotor contacting the casing and other machine component. This study develops analytical procedure for fast and effective assessing the consequence of large rotor deformation in case of machine overloading, over-speed, whirling, resonance and other malfunctions or accidents. It is an effective tool for risk assessment, machine HAZOP review, operation reliability and vendor document evaluation.

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