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Nonlinear Integral-Type Sliding Surface for Synchronization of Chaotic Systems with Unknown Parameters

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Abstract**—**This paper presents a new nonlinear integral-type sliding surface for synchronizing two different chaotic systems with parametric uncertainty. On the basis of Lyapunov theorem and average dwelling time method, we obtain the control gains of controllers which are derived to achieve chaos synchronization. In order to reduce the gains, the error system is modeled as a switching system. We obtain the sufficient condition drawn for the robust stability of the error dynamics by stability analysis. Then we apply it to guide the design of the controllers. Finally, numerical examples are used to show the robustness and effectiveness of the proposed control strategy.

Keywords**—**Chaos synchronization, Nonlinear sliding surface, Control gains, Sliding mode control.

I. Introduction

SINCE the seminal work of Pecoora and Carrall [1], there has been an increasing interest in the study of chaos synchronization in physics, mathematics and engineering mechanics, etc. The idea of synchronization [2]-[4] and is to use the output of the drive system to control the response system so that the output of the response system follows the output of the drive system asymptotically. Various effective techniques and methods such as OGY method [5], linear or nonlinear feedback control method [6], active control method [7], [8], time-delay feedback control [9], [10], adaptive control [11], and sliding mode control method [12]-[14] have been proposed over the last decades to realize chaos synchronization.

The sliding mode control technique [15]-[17] is a discontinuous control strategy that involves, first, selecting a switching surface for the desired dynamics and, secondly, designing a discontinuous control law such that the system trajectory can reached the surface and then stays in it forever. However, most of researches mentioned above have concentrated on studying the active sliding mode control with linear sliding surface [18]. This method will divide out all the nonlinear parts of the chaotic systems and use linear sliding surface to control the system. In the practical applications, the method with linear reduction above cannot describe the nonlinear characteristics of the trajectories in some chaotic systems explicitly.

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Motivated by the above analysis, we consider the chaotic synchronization using a new nonlinear integral-type sliding surface which incorporates a virtual nonlinear nominal control to achieve prescribed specifications. Based on the nonlinear sliding mode control technique, the sufficient conditions are given to assure the complete synchronization occurs and then the stability analysis of the proposed sliding mode for the chaotic systems is obtained. The corresponding numerical simulations are provided to illustrate the effectiveness of these sliding mode controllers.

II. Problem Description

Let us define the following two uncertain chaotic systems as master and slave, respectively by:

$$\dot{x} = (A_1 + \Delta A_1)x + g_1(x) + \Delta g_1(x) \quad (1)$$

$$\dot{y} = (A_2 + \Delta A_2)y + g_2(y) + \Delta g_2(y) + u(t) \quad (2)$$

where $x(t) \in R^n$ and $y(t) \in R^n$ denote state vectors of the system. A_1 and $A_2 \in R^{n\times n}$ represent the linear parts of the system dynamic, $g_1 : R^n \to R^n$ and $g_2 : R^n \to R^n$ are the nonlinear parts of the system and $g_2(y)$ satisfies the following Lipchitz condition

$$\|g_2(y) - g_2(x)\| \le \mu \|y - x\|$$

for $\forall x, y \in R^n$. $\Delta A_1 \in R^{n\times n}$ and $\Delta A_2 \in R^{n\times n}$ are unknown linear parts of matrices. $\Delta g_1 : R^n \to R^n$ and $\Delta g_2 : R^n \to R^n$ are unknown nonlinear parts of the master and slave systems respectively. To synchronize the state $y(t)$ with the state of the master system $x(t)$, the controller $u(t) \in R^n$ has been added to the slave system.

Remark 1: If $A_1 = A_2$, $g_1(x) = g_2(x)$, then x, y are the states of two different chaotic systems due to the existence of $\Delta A_1, \Delta A_2, \Delta g_1(x), \Delta g_2(y)$ x, y are the states of two identical chaotic systems if and only if $A_1 = A_2$, $g_1(x) = g_2(x)$, $\Delta A_1 = \Delta A_2$, $\Delta g_1(x) = \Delta g_2(x)$

Define $e(t) = y(t) - x(t)$, the dynamics of synchronization error can be expressed as

$$\dot{e}(t) = (A_2 + \Delta A_2)e(t) + G(x,e) + H(x) + M(x,y) + u(t) \quad (3)$$

where
$G(x,e) = g_2(y) - g_2(x)$,
$H(x) = g_2(x) - g_1(x) + (A_2 - A_1)x$,
$M(x,y) = (\Delta A_2 - \Delta A_1)x + \Delta g_2(y) - \Delta g_1(x)$.

The synchronization problem is to design the controller $u(t)$ which synchronizes the states of the slave with that of the master. So the synchronization goal is as follows:

$$\lim_{t\to\infty}\|e(t)\| = \lim_{t\to\infty}\|x(t) - y(t)\| = 0$$

where $\|\cdot\|$ is the Euclidean norm (2-norm) of the vector.

III. Methodology of Sliding Mode Control Design

In general, sliding mode control design methodology comprises two steps. First, the sliding surface is designed, so that not only the sliding mode can occur in the surface, but also the controlled system will yield the desired dynamic performance. The second phase is to design the sliding mode controller such that the trajectory of the system arrives at the sliding surface and remains on the sliding surface for all subsequent time.

A. Sliding Surface and Equivalent Control Law Design

In this subsection, our main aim is to design an appropriate nonlinear integral sliding surface for the error system (3). That is designing an integral sliding mode controller such that the sliding motion is asymptotically stable and the state trajectory of the error system (3) is driven onto the specified sliding surface and maintained there for all subsequent time.

An appropriate nonlinear integral-type sliding surface can be constructed as follows

$$s(t) = e(t) - e(0) - \int_0^t [(A_2 + K)e(s) + G(x,e)]ds \quad (4)$$

where $K \in R^{n\times n}$ is to be chosen suitally.

Remark 2: The terms of $e(0)$ achieves the nice property that $s(0) = 0$ such that the reaching phase is eliminated. Since the sliding mode exists from the beginning, the system is more robust against perturbations than other sliding mode control systems with reaching phase.

According to the sliding mode control theory, it is true that $s(t) = 0$ and $\dot{s}(t) = 0$ as the state trajectory of the error system (3) enter into the sliding mode. An equivalent control law can be designed as

$$u_{eq} = Ke(t) - H(x) - M(x,y) - \Delta A_2 e(t) \quad (5)$$

Substituting (5) into (3), the sliding mode dynamics can be obtained as

$$\dot{e}(t) = (A_2 + K)e(t) + G(x,e) \quad (6)$$

B. Stability Analysis of the Sliding Motion

In this subsection, the stability analysis of the sliding mode dynamics is investigated.

Note that the function $G(x,e)$ is a nonlinear function and satisfies $\|G(x,e)\| \le \bar{\mu}\|e\|$. If the constant $\bar{\mu}$ is large, then the feedback gain matrix will be large enough. In order to reduce the gain matrix as small as possible, we model the system (6) as a switch system.

Define the following sets

$$\Omega_1 = \{t : \|G(x,e)\| \le \mu_1\|e\|\},$$
$$\Omega_2 = \{t : \mu_1\|e\| \le \|G(x,e)\| \le \bar{\mu}\ \|e\|\}.$$

where $0 < \mu_1 < \bar{\mu}$

The system (6) is rewritten as follows

$$\dot{e}(t) = \begin{cases} (A_2 + K)e(t) + G_1(x,e), & t \in \Omega_1 \\ (A_2 + K)e(t) + G_2(x,e), & t \in \Omega_2 \end{cases} \quad (7)$$

where
$G_1(x,e) = G(x,e), t \in \Omega_1, \quad G_2(x,e) = G(x,e), t \in \Omega_2$,
then

$$\|G_1(x,e)\| \le \mu_1\|e\|, \quad \mu_1\|e\| < \|G_2(x,e)\| \le \bar{\mu}\ \|e\|. \quad (8)$$

In order to express skimpily, we introduce the following notations. $T^-(0,t)$ $T^+(0,t)$ are denoted as the total length of $t \in \Omega_1, t \in \Omega_2$ over the time interval $[0,t)$, respectively. $N_\sigma(0,t)$ is denoted as the switching number of the system in time interval $[0,t)$. $T_a = \dfrac{t}{N_\sigma(0,t)}$ is the average dwell time of the system. Since for all t, $t \in \Omega_1$ or $t \in \Omega_2$, without loss of general, we assume $[t_0, t_1) \subset \Omega_1$ and

$$\Omega_1 = \bigcup[t_{2j}, t_{2j+1}), \Omega_2 = \bigcup[t_{2j+1}, t_{2(j+1)}).$$

Next, we will investigate the asymptotically properties of the system

$$\dot{e} = (A_2 + K)e(t) + G_1(x,e), \quad (9)$$

and

$$\dot{e} = (A_2 + K)e(t) + G_2(x,e), \quad (10)$$

respectively.

Lemma 1 The system (9) is exponentially asymptotically stable if there exist positive definite matrix P_1, such that

$$(A_2 + K)^T P_1 + P_1(A_2 + K) = -I$$

and

$$a_1 = (1 - 2\mu_1 \|P_1\|)\lambda_{\min}(P_1^{-1}) > 0 . \quad (11)$$

Furthermore,

$$\|e(t)\|^2 \le b_1 e^{-a_1 t}\|e(0)\|^2, \; b_1 = \frac{\lambda_{\max}(P_1)}{\lambda_{\min}(P_1)} \quad (12)$$

Proof: Consider the following candidate Lyapunov function

$$V_1(t) = e^T(t)P_1 e(t) \quad (13)$$

Taking the derivative of $V_1(t)$ along the solution of (9), we get

$$\begin{aligned}\dot{V}_1(t) &= e^T(t)[(A_2+K)^T P_1 + P_1(A_2+K)]e(t) + 2e^T(t)P_1G(x,e)\\ &\le -e^T(t)e(t) + 2\mu_1\|P_1\|\|e(t)\|^2\\ &= -(1-2\mu_1\|P_1\|)e^T(t)e(t)\\ &\le -(1-2\mu_1\|P_1\|)\lambda_{\min}(P_1^{-1})V(t) = -a_1V_1(t)\end{aligned}$$

where $a_1 = (1-2\mu_1\|P_1\|)\lambda_{\max}(P_1^{-1})$.

Then, we have that

$$\lambda_{\min}(P_1)\|e(t)\|^2 \le V_1(t) \le e^{-a_1t}V_1(0) \le e^{-a_1t}\lambda_{\max}(P_1)\|e(0)\|^2 \quad (14)$$

which means the system (9) is exponentially asymptotically stable. That completes the proof.

Considering the system (10) and choose the following "so called" candidate Lyapunov function

$$V_2 = e^T(t)P_2e(t) \quad (15)$$

Along the trajectory of (10), it is obtained that

$$\begin{aligned}\dot{V}_2(t) &= e^T(t)[(A_2+K)^T P_2 + P_2(A_2+K)]e(t)\\ &\quad + 2e^T(t)P_2G_2(x,e)\\ &\le e^T(t)Qe(t) + 2\overline{\mu}\;\|P_2\|\|e(t)\|^2\\ &\le (c+2\overline{\mu}\|P_2\|)e^T(t)e(t)\\ &\le (c+2\overline{\mu})\|P_2\|)\lambda_{\max}(P_2^{-1})V(t) = a_2V_2(t)\end{aligned}$$

where

$$Q = (A_2+K)^T P_2 + P_2(A_2+K),$$

$$a_2 = (c+2\overline{\mu})\|P_2\|)\lambda_{\max}(P_2^{-1}), \quad c = \lambda_{\max}(Q).$$

Hence, we get that

$$V_2(t) \le e^{a_2t}V_2(0) .$$

Now, we are in the position to investigate the stability of the system (7)

Theorem 1 If the following conditions hold,

1、$\dfrac{T^+(0,t)}{T^-(0,t)} \le \dfrac{a_1 - a^*}{a_2 + a^*}, \quad a^* \in (0, a_1)$

2、$T_a \ge \dfrac{\ln\mu}{a}, \; a \in (0, a^*), \; \mu \ge 1$

3、$P_1 \le \mu P_2, \; P_2 \le \mu P_1$

The system (7) is exponentially stable.

Proof: Construct the piecewise Lyapunov function candidate as follows

$$V_\sigma(t) = \begin{cases} V_1(t), t \in \Omega_1 = \bigcup[t_{2j}, t_{2j+1}) \\ V_2(t), t \in \Omega_2 = \bigcup[t_{2j+1}, t_{2(j+1)}) \end{cases} \quad (16)$$

where $V_1(t) = e^T(t)P_1e(t)$, $V_2 = e^T(t)P_2e(t)$.

According to the condition 3, we have

$$V_1(t) \le \mu V_2(t), V_2(t) \le \mu V_1(t) \; .$$

Assume that $t \in \Omega_2$, then

$$\begin{aligned}V_\sigma(t) &= V_2(t) \le V_2(t_{2j+1})e^{a_2(t-t_{2j+1})} \le \mu V_1(t_{2j+1})e^{a_2(t-t_{2j+1})}\\ &\le \mu V_1(t_{2j})e^{a_2(t-t_{2j+1})-a_1(t_{2j+1}-t_{2j})}\\ &\le \mu^2 V_2(t_{2j})e^{a_2(t-t_{2j+1})-a_1(t_{2j+1}-t_{2j})} \le \cdots\\ &\le \mu^{N_\sigma(0,t)}V_\sigma(0)e^{a_2T^+(0,t)-a_1T^-(0,t)}\\ &= V_1(0)e^{a_2T^+(0,t)-a_1T^-(0,t)+N_\sigma(0,t)\ln\mu}\\ &= V_1(0)e^{a_2T^+(0,t)-a_1T^-(0,t)+t\ln\mu/T_a}\end{aligned}$$

where

$a_1 = (1-2\mu_1\|P_1\|)\lambda_{\min}(P_1^{-1})$,.

$a_2 = (c+2\bar{\mu}\|P_2\|)\lambda_{\max}(P_2^{-1})$

According to the condition 2, we have that $\frac{\ln\mu}{T_a} \le a$. Hence

$$a_2T^+(0,t) - a_1T^-(0,t) + t\ln\mu/T_a \le (a_2+a)T^+(0,t) - (a_1-a)T^-(0,t)$$

According to the condition 1, we get that

$$a_2T^+(0,t) - a_1T^-(0,t) + t\ln\mu/T_a \le (a_2+a)T^+(0,t) - (a_1-a)T^-(0,t) \le -(a^*-a)t$$

Therefore, $V_\sigma(t) \le V_1(0)e^{-(a^*-a)t}$. Noticing that $V_\sigma(t) \ge c_1\|e\|$, where $c_1 = \min\left(\lambda_{\min}(P_1), \lambda_{\min}(P_2)\right)$, then $\|e\| \le \sqrt{\frac{V_1(0)}{c_1}}e^{-0.5(a^*-a)t}$

C. Reachability Analysis and Sliding Mode Controller Design

In this subsection, we are in a position to synthesize a sliding mode controller to drive the system trajectories onto the predefined sliding surface $s(t)=0$ in (4) and have the following results.

Assume that the unknown nonlinear parts $\Delta g_1(x), \Delta g_2(y)$ are Lipchitz, then there exist constants L_1, L_2 such that $\|\Delta g_i(x)\| \le L_i\|x\|, i=1,2$. $\Delta A_1, \Delta A_2$ are bounded. Hence, we have that

$$\begin{aligned}\|M(x,y)\| &\le (\|\Delta A_2 - \Delta A_1\| + L_1 + L_2)\|x(t)\| \\ &+ (\|\Delta A_2\| + L_2)\|e(t)\|\end{aligned}$$

Theorem 2 Suppose the condition of Theorem 1 hold and the sliding surface is given by (4). Then the state of the system (3) can enter the sliding surface in finite time, and it subsequently remains on it by employing the following sliding mode controller

$$u(t) = Ke(t) + H(x) - \rho(t)sign(s(t)) - rs(t) \quad (17)$$

where $r>0$, $\rho(t) = q + \alpha\|x(t)\| + \beta\|e(t)\|, q>0$ $\alpha \ge \|\Delta A_2 - \Delta A_1\| + L_1 + L_2, \beta \ge \|\Delta A_2\| + L_2$.

Proof: Let us consider the following candidate Lyapunov function

$$V(t) = \frac{1}{2}s^T(t)s(t) \quad (18)$$

Its derivative along the trajectory of (3) is

$$\begin{aligned}\dot{V}(t) &= s^T(t)[\dot{e}(t) - (A_2+K)e(t) - G(x,e)] \\ &= s^T(t)[\Delta A_2 e(t) + H(x) + M(x,y) + u(t) - Ke(t)] \\ &\le s^T(t)[u(t) - Ke(t) + H(x)] + \|s(t)\|[\|\Delta A_2\|\|e(t)\| \\ &+ \|M(x,y)\|]\end{aligned} \quad (19)$$

Note that

$$\begin{aligned}\|M(x,y)\| &= \|\Delta g_2(y) - \Delta g_1(x) + (\Delta A_2 - \Delta A_1)x\| \\ &\le (L_1 + L_2 + \|\Delta A_2 - \Delta A_1\|)\|x(t)\| + L_2\|e(t)\|\end{aligned}$$

Applying the variable structure controllers (17) to (19) results in

$$\dot{V}(t) \le -r\|s(t)\|^2 < 0 \quad (20)$$

Hence the state of the system (3) will reach the sliding surface (4) in finite time and subsequently remain on it. This completes the proof.

Remark 3: If the system (1), (2) are two identical chaotic systems, then the controller in (17) has the form $u(t) = Ke(t) - qsign(s(t)) - rs(t)$. We can find that only a linear feedback can stabilize the system (3) on the sliding surface $s(t)=0$. Note that the controller must eliminate effect of the nonlinear part. Hence, the controller gains maybe larger. However, in [19], nonlinear feedback is needed in order to stabilize the system (3) although on the sliding surface $s(t)=0$.

If $\Delta A_1 = \Delta A_2 = \Delta g_1(x) = \Delta g_2(y) = 0$, i.e. there are no uncertainties or disturbance, then the controller (17) has the form

$$u(t) = Ke(t) + H(x) - qsign(s(t)) - rs(t) \quad (21)$$

Remark 4: In [3], a modified active sliding mode control is used to realize the synchronization of chaotic system, where a robust controller is designed in order to stabilize the error system. No information on the uncertainties or disturbance is used when the controller is designed. In fact, the controller design using the information on the uncertainties or disturbance is more robust when the information on the uncertainties or disturbance can be used. A feasible substitute method is adaptive control.

IV. Numerical Simulation

In this section, we present numerical simulation examples to illustrate the effectiveness of the developed control design strategy. A fourth order Runge-Kutta solver with time step size of 0.001 s is performed to solve the set of differential equations, concerning the master and slave system.

A. Synchronization of Different Chaotic Systems with the Perfect Model

When the drive system and response system are non-identical chaotic systems, the Lorenz system and Chen system are considered as drive system and response system, respectively. They are described as follows:

$$\begin{cases} \dot{x}_1 = -10x_1 + 10x_2 \\ \dot{x}_2 = 28x_1 - x_2 - x_1x_3 \\ \dot{x}_3 = (-8/3)x_3 + x_1x_2 \end{cases} \tag{22}$$

and

$$\begin{cases} \dot{y}_1 = -35y_1 + 35y_2 + u_1 \\ \dot{y}_2 = -7y_1 + 28y_2 - y_1y_3 + u_2 \\ \dot{y}_3 = -3y_3 + y_1y_2 + u_3 \end{cases} \tag{23}$$

where

$$A_1 = \begin{bmatrix} -10 & 10 & 0 \\ 28 & -1 & 0 \\ 0 & 0 & -\frac{8}{3} \end{bmatrix}, A_2 = \begin{bmatrix} -35 & 35 & 0 \\ -7 & 28 & 0 \\ 0 & 0 & -3 \end{bmatrix},$$

$g_1(x) = [0 \quad -x_1x_3 \quad x_1x_2]^T$, $g_2(y) = [0 \quad -y_1y_3 \quad y_1y_2]^T$.

At first, choose the expected poles for the system (6) as -45, -20, -30. The corresponding gain matrix is $K = \begin{bmatrix} -10 & -35 & 0 \\ 7 & -48 & 0 \\ 0 & 0 & -27 \end{bmatrix}$. Solving the Lyapunov equation in Lemma 1, we get

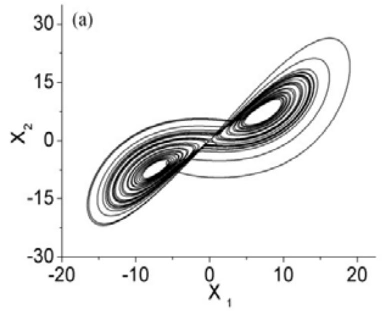
$$P_1 = diag(0.0111 \quad 0.025 \quad 0.0385).$$

For simplicity, we choose $P_2 = P_1$, then $\mu = 1$ which means there are no restrictions on the average dwell time, i.e. arbitrarily switching. Choose $\mu_1 = 12$, $\bar{\mu} = 85$, then, $a_1 = 1.976$, $a_2 = 499.05$. Choose $a^* = 0.976$. According to the condition of Theorem 1, we get, if

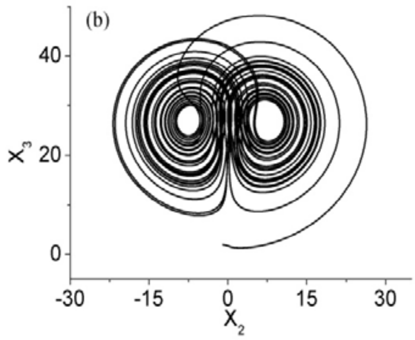
$$T^+(0,t)/T^-(0,t) = 1/500.026,$$

the system is exponentially stable and the biggest decay rate is $a^* = 0.976/2$.

The Lorenz system and Chen system without control of $u(t)$ exhibit chaotic behaviors, as shown in Figs. 1 and 2 with initial values $X(0) = (1 \quad -1 \quad 2)^T$ and $Y(0) = (2 \quad 3 \quad 1)^T$. Then the error state trajectories are depicted in Fig. 3.

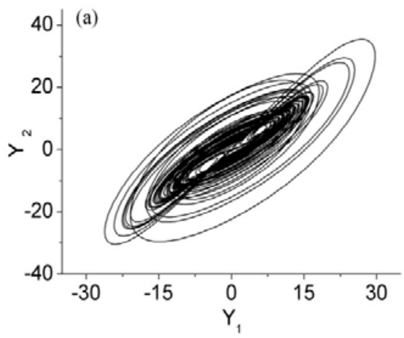


(a) Phase portrait on plane of (x_1, x_2)

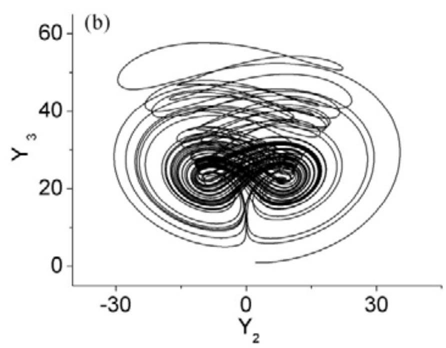


(b) Phase portrait on plane of (x_2, x_3)

Fig. 1 Chaotic attractor of Lorenz system



(a) Phase portrait on plane of (y_1, y_2)



(b) Phase portrait on plane of (y_2, y_3)

Fig. 2 Chaotic attractor of Chen system

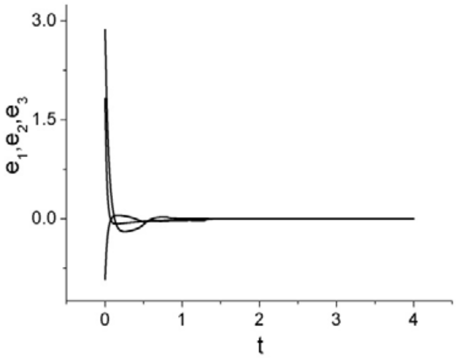


Fig. 3 Dynamics of the variables e_1, e_2 and e_3 in error system with time t

B. Synchronization of Different Chaotic Systems with Uncertain Parameters

Similar to the above part, the master and slave systems are Lorenz and Chen respectively. Uncertainties will be considered in both the system dynamics in matrices of form, described as:

$$\Delta A_1 = diag\left(0.09\delta(t) \quad 0.06\delta(t) \quad 0.11\delta(t)\right),$$
$$\Delta A_2 = diag\left(-0.02\delta(t) \quad 0.05\delta(t) \quad 0.01\delta(t)\right),$$
$$\Delta f_1(x) = \left(0.1\sin x_1 \quad 0.2\cos x_2 \quad 0.15\sin x_1\right)^T,$$
$$\Delta f_2(x) = \left(0.1\cos x_1 \quad 0.2\sin x_2 \quad 0.15\cos x_3\right)^T.$$

Fig. 4 shows the simulation results. During the simulation, the control parameters are chosen the same as in the case of A. As seen, results are similar to that of Fig.3. The errors still finally converges to zero.

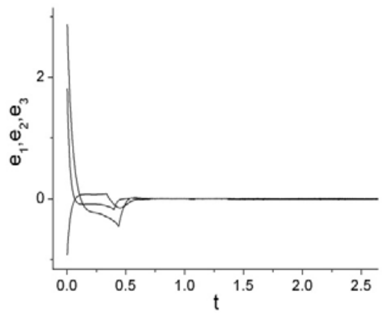


Fig. 4 Dynamics of the variables e_1, e_2 and e_3 of error system with time t

V. Conclusion

In this paper, a new method of designing a nonlinear integral sliding surface has been applied for synchronization of two different chaotic systems with parameters uncertainty. The nonlinear systems are highly unstable and the proposed algorithm of nonlinear sliding surface will have more effectiveness for solving the nonlinear systems. According to the boundedness of two different chaotic systems, we have derived a stabilization criterion for chaos sychronization and guaranteed exponential stability by using Lyapunov theorem. Furthermore, appropriate balanced coupling coefficients based on average dwelling time method are derived. Finally, the numerical simulations are given to demonstrate the effectiveness of the proposed nonlinear integral surface mode controllers although the chaotic systems involve the parameters uncertainty.

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