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Nonlinear Evolution of Electron Density Under High-Energy-Density Conditions

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***Abstract*—**Evolution of one-dimensional electron system under high-energy-density (HED) conditions is investigated, using the principle of least-action and variational method. In a single-mode modulation model, the amplitude and spatial wavelength of the modulation are chosen to be general coordinates. Equations of motion are derived by considering energy conservation and force balance. Numerical results show that under HED conditions, electron density modulation could exist. Time dependences of amplitude and wavelength are both positively related to the rate of energy input. Besides, initial loading speed has a significant effect on modulation amplitude, while wavelength relies more on loading duration.

***Keywords*—**Electron density modulation, HED, nonlinear evolution, plasmas.

I. Introduction

HIGH energy density physics (HEDP) is a fast-developing field of research, promoted mainly by endeavors in ICF [1], astrophysics [2], intense laser-matter interaction [3], and so on. Under HED conditions, plasmas are the most common form of matter. However, traditional theories encounter some problems in dealing with HED plasmas, mainly due to their dominant collective behaviors [4]. Examples of such collective, nonlinear motions range from laser-based acceleration of charged particles [5], to highly-collimated coherent electromagnetic radiations from compact celestial entities like neutron stars or pulsars [6]. In order to understand the nature of these complex phenomena, knowledge of basic physical processes that plasmas undertake in extreme conditions is critical.

Although motions in HEDP system bear great diversities, one common characteristic is that they all involve density variations or modulations, either those of electrons [7] or ions [8], or both. A widely used method of studying them is based on fluid mechanics [9]. On the other hand, the system can be regarded as an integrated entity, with a set of eigen-states which correspond to different modulations of density, analogous to wavefunctions in quantum mechanics. Adopting the principle of least-action, combined with some other conditions, it is possible find out how the system might evolve. This method serves as a complement to fluid method, and may shed some light on complex motions of HEDP systems.

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II. The Principle of Least-Action

Generally, the action of a system is defined by:

$$S = \int_{t_1}^{t_2} (T - V)\, dt, \tag{1}$$

where T is the total energy of various internal motions, and V the external driving energy. Any realistic evolution that the system follows makes S a minimum, that is δS=0. Therefore, given the forms of external as well as internal energies, the equations of motion of the system could be obtained by variational approach.

III. External Driving Energy

In HEDP experiments, intense lasers and Z-pinch facilities are two major means of delivering energies into targets. Despite differences in photon energy between these two approaches, there are common characteristics: driving intensities can be described by two factors, namely the peak values I_0 and the temporal profiles $I(t)/I_0$:

$$I(t) = I_0 f(t). \tag{2}$$

For different driving conditions, parameters and profiles vary accordingly.

A. Gaussian Profile

In most of nowadays ultrashort-ultraintense laser experiments, incident laser beams are well-described by Gaussian profile, which is given by:

$$I(t) = I_0 \exp[-(t - 0.5)^2]. \tag{3}$$

Throughout this work, the units of I_0 and t are $W \cdot m^{-2}$ and ns, respectively. Gaussian driving could be the fastest way of delivering huge amount of energy into targets. Input intensity raises rapidly from zero but also declines quickly. The duration is usually very short, typically tens of femtoseconds.

B. Sinusoidal Profile

Another common way of performing laser matter interaction experiments is to use longer-duration lasers with relatively lower intensities. In these circumstances, plane-waves are better approximations for incident laser beams, with sinusoidally oscillating electric fields. Thus, the corresponding sinusoidal intensity profile is also considered as follows:

$$I(t) = 2 + I_0 \sin(\pi t / 12), \tag{4}$$

where constant 2 on the right hand side is to avoid singularity when solving the equation of motion, and it is negligible compared to I_0. Obviously, sinusoidal driving inputs energy on targets much slower than Gaussian driving.

C. Power Profile

In power profiles, driving intensity keeps increasing, which means that energy is being injected into a system at a raising speed. Since experimentally such kind of energy-loading is not yet feasible, theoretical studies are needed to estimate evolution of a system under such conditions. Three different power profiles are discussed, namely linear, quadratic, and cubic, given as follows:

$$I(t) = 1 + I_0 t / 10, \tag{5}$$

$$I(t) = 1 + I_0 t^2 / 100, \tag{6}$$

$$I(t) = 1 + I_0 t^3 / 1000. \tag{7}$$

The constants, 10, 100, and 1000 make sure at the end of calculation time (10ns), they all reach the same maximum I_0.

IV. Internal Energy

Under HED conditions, matters usually exist in the form of plasmas. Since in the most experiments, driving duration is short (e.g. in laser experiments less than 1 ps), and positively charged ions are way too heavier than electrons, they can be treated as a positive charge background, and it is a good approximation to deal with electrons only. In this paper, one dimensional electron system is considered.

Internal motions of the system consist of two types: single-particle motions and collective ones. Random thermal motion is the most important single-particle motion in our discussion. All kinds of waves and excitations in plasma are examples of collective motions. The focus of this work is on the motion of electron density modulation and its resulting radiation.

A. Electrostatic Energy of Density Modulation

In one dimension, a single-mode density modulation can be regarded as:

$$n_e(t) = n_1(t) \sin[2\pi x / \lambda(t)], \tag{8}$$

where $n_1(t)$ is the amplitude and $\lambda(t)$ the spatial wavelength of the modulation. Since the evolution of the amplitude and the wavelength fully determines the behavior of the system, they are chosen as the general coordinates to describe the action. The reason for the choice of studying single-mode modulation is that most of waves and turbulences can be thought of as superposition of and interaction among single-modes. Thus, investigations of single-mode behavior set up basis for researches on more complex phenomena.

Let L to be the length of the system, and the cross section is a d^2 square, the total electrostatic energy is:

$$U_L = e^2 n_1^2 \lambda^2 L d^2 / 8\pi^2 \varepsilon_0, \tag{9}$$

where e is the electron charge and ε_0 the vacuum permittivity. Electrostatic energy is proportional to λ^2, which means larger scale modulation could store more energy.

B. Coherent Radiation by Density Modulation

As electron density varies with time, coherent radiation could be emitted, which is approximated here by electric dipole radiation. The power of radiation reads [10]:

$$P_{\text{dipole}} = 4\pi^3 c^2 Z_0 e^2 n_1^2 L^2 d^4 / 3\lambda^2, \tag{10}$$

where c is the light velocity in vacuum and Z_0 the vacuum impedance.

C. Kinetic Energy of Density Modulation

Density modulation varies with time because of collective motions of electrons, which bear kinetic energy as follows:

$$T_{\text{kinetic}} = \hbar^2 \pi^2 n_1 L d^2 / 2m\lambda^2, \tag{11}$$

where $\hbar$ the reduced Plank constant and m the rest electron mass.

D. Thermal Energy of Electrons

The three types of energy mentioned above are all due to collective effects of electrons motions. On the other hand, an important single-particle motion, random thermal motion, should also be considered. Given the temperature T of the system, the total thermal energy is:

$$T_{\text{ther}} = n_1 k_B T L d^2, \tag{12}$$

with k_B the Boltzmann constant.

V. Equations of Motion

Conservation of energy gives the first equation of motion, that is, the energy injected into the system per unit time should be equal to the increase of internal energy during that time interval. The external power is:

$$W(t) = I(t) L d. \tag{13}$$

Then the first equation reads:

$$W = dU_L / dt + P_{\text{dipole}} + dT_{\text{kinetic}} / dt + dT_{\text{ther}} / dt. \tag{14}$$

The second equation comes from the balance of forces exerted on electrons. Density modulation creates an electrostatic potential which tends to pull electrons back to their original positions, weakening the amplitude of modulation. However, external energy keeps pushing electrons away from

the bottom of the Coulomb potential well, enlarging the amplitude of modulation. Assuming these two factors are more or less in balance leads to the second equation:

$$I = c^2 e^2 n_1^2 \lambda^2 / 2\pi\varepsilon_0. \quad (15)$$

Combining these two equations of motion, solutions for $n_1(t)$ and $\lambda(t)$ are ready to be found.

VI. Numerical Results and Discussions

The equations of motion obtained above can be written in a more convenient form for calculation:

$$a_1 dI/dt + a_2\sqrt{a_4}\, d(\sqrt{I}/\lambda^3)/dt + a_3 d(I/\lambda^4)/dt = a_4 I L d\tau, \quad (16)$$
$$n_1^2 = I/(a_4\lambda^2), \quad (17)$$

where a_1 to a_4 are coefficients after some algebra, and τ the characteristic time. Numerical values used in calculation are listed in Table I. The total calculation time is 10 ns, using software *Mathematica*.

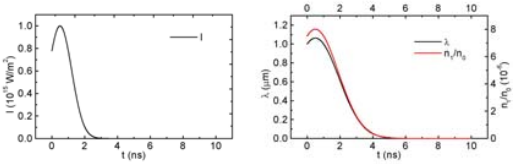
In this work, five different driving forms are adopted: Gaussian, sinusoidal, linear, quadratic, and cubic (Fig.1, left column). To focus on the effects of temporal profiles of intensities, the peak values in all these five cases are chosen to be identical of I_0=10^{15} W · cm^{-2}. Evolutions of spatial wavelength λ and amplitude n_1/n_0 of electron density modulation are obtained (Fig.1, right column).

Trends of evolution of λ and n_1/n_0 are similar. As driving intensity increases, λ becomes larger and n_1/n_0 becomes greater, which is shown clearly in Fig.1(c) to (e) and in early time in Fig.1(a) and (b). When driving intensity decreases, spatial wavelength and amplitude reduce to smaller values, as can be seen in late time in Fig.1(a) and (b).

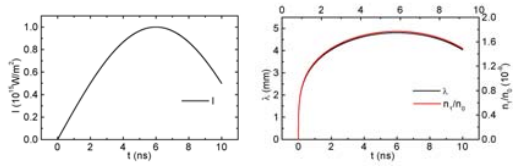
However, considerable differences exist among these cases. For Gaussian driving, the input intensity grows rapidly to its maximum in about 1 ns, then decays quickly to nearly zero in another 1 ns. The resulting modulation occurs on a very small scale, roughly several micrometers. In the other four cases, growing stages last for longer time, leading to millimeter-scale modulations, orders of magnitudes larger than in Gaussian case. Therefore, the duration of intensity growth has an important effect on the spatial scale of electron density modulation. In order to create large scale ripples, an increasingly intense energy input is needed. This effect is understandable. Spatial modulation of electron density of the system reflects collective motions of electrons, which is driven by the energy input to the system. The collectiveness of such motions needs certain interval of time to be built up. If the external driving force ceases before correlations between electrons far away from one another are established, no large-wavelength modulations can occur. Besides, random motions, such as thermal fluctuations, might destroy collective motions if the later ones are vulnerable. Thus, there exists competition between factors that to set up and to break out collective motions. When the incident intensity increases continuously, collective motions are powered strongly for long enough time to grow larger in space, while vice verse, only short range correlation could be maintained.

TABLE I
Units for Magnetic Properties

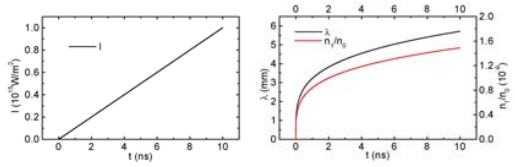
Quantity	Unit	Value
L	m	1
d	m	10^{-3}
n_0	m^{-3}	10^{28}
τ	s	10^{-9}
a_1	J	3.66×10^{21}
a_2	J	5.44×10^{-16}
a_3	J	3.59×10^{27}
a_4	J·m^{-2}·s^{-1}	1.38×10^{37}



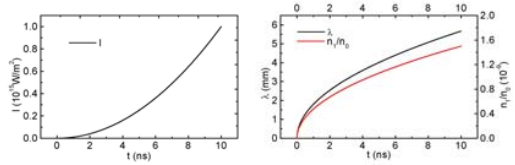
(a) Gaussian driving pulse and results



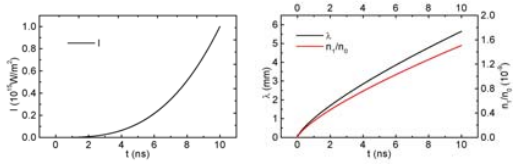
(b) Sinusoidal driving pulse and results



(c) Linear driving pulse and results



(d) Quadratic driving pulse and results



(e) Cubic driving pulse and results

Fig. 1 Numerical results of different driving pulses and electron density modulation evolutions. Figures in the left column show temporal profiles of five different driving pulses: (a) Gaussian, (b) sinusoidal, (c) linear, (d) quadratic, and (e) cubic. Figures in the right column illustrate evolutions of spatial wavelength λ (black) and amplitudes n_1/n_0 (red) of charge density modulation.

The amplitude of electron density modulation seems to rely more significantly on the initial growth rate of driving intensity. In Gaussian case, where the initial growth rate is the fastest, n_1/n_0 increases to an order of magnitude of 10^{-6}. However, in the other four cases, where initial growth rates are all zero, n_1/n_0 remains less than 2×10^{-9}, even when the intensities keep increasing. One possible interpretation is to consider the huge Coulomb interactions among electrons. Initially, the plasma is in thermal equilibrium, in which state electron density is generally homogeneous and electrostatic potential is equal to zero. At this state, electrons are relatively easy to move away from their balance positions, resulting in density modulation. However, once there is modulation, there is nonzero Coulomb potential that tends to restore the separation of charges. Thus later on, forcing more electrons to join in the modulation would be much harder. Bearing these pictures in mind, it is straightforward to understand the dependence of n_1/n_0 on initial growth rate of input intensity.

VII. Conclusions

In this paper, least-action principle and variational method are used to analyze the evolution of one-dimensional electron system under HED conditions. Electron density modulation is the basic form of motion, of which amplitude and spatial wavelength are two general coordinates determining the behavior of the system. Equations of motion are derived from conservation law of energy and force balance. Numerical calculations show that in HEDP regime, such electron density modulation states can exist. As input of energy continues, the spatial wavelength of modulation tends to increase, while the amplitude of density variation relies more significantly on initial injecting speed.

The growth of modulation wavelength might shed light on some non-thermal radiation phenomena in astrophysical circumstances, such as pulsar radio radiations. However, more detailed theoretical work as well as experimental studies are needed in order to get a complete understanding of such HEDP phenomena.

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