

# Non-Linear Control Based on State Estimation for the Convoy of Autonomous Vehicles

M-M. Mohamed Ahmed, Nacer K. M'Sirdi, Aziz Naamane

**Abstract**—In this paper, a longitudinal and lateral control approach based on a nonlinear observer is proposed for a convoy of autonomous vehicles to follow a desired trajectory. To authors best knowledge, this topic has not yet been sufficiently addressed in the literature for the control of multi vehicles. The modeling of the convoy of the vehicles is revisited using a robotic method for simulation purposes and control design. With these models, a sliding mode observer is proposed to estimate the states of each vehicle in the convoy from the available sensors, then a sliding mode control based on this observer is used to control the longitudinal and lateral movement. The validation and performance evaluation are done using the well-known driving simulator Scanner-Studio. The results are presented for different maneuvers of 5 vehicles.

**Keywords**—Autonomous vehicles, convoy, nonlinear control, nonlinear observer, sliding mode.

## I. INTRODUCTION

**I**N recent decades, research on the convoy of autonomous vehicles has received particular attention and is of growing interest to laboratories, researchers, vehicle manufacturers, and suppliers. Indeed, several research projects, both national and international, have been set up to respond to the problems linked to the evolution of means of transport in convoys, their control approaches and their safety. Examples include projects AutoNet2030, SARTE and Chauffeur [1], [2].

Several research works have developed models and controls to present the movement and also to control the convoy to follow a vehicle that represents the leader, which is driven by a driver or by an automatic pilot. The linear double integrator model is the most widely used model for longitudinal control of the convoy. It neglects many system parameters; the vehicle is represented by a double integrator [3]. All vehicles are considered independent and are coupled by the control system using information from the previous vehicle. Another model in [4], the vehicles are modeled longitudinally by a second-order system (double integrator). They move in a straight line, and their (neighboring) environment reacts, on each side, by applying a damping force and a stiffening force.

A proposal in [5] that aims to transform the kinematic and dynamic model coupled with a chain transformation; that transforms this non-linear model to a double integrator linear model. Several laws of lateral and longitudinal control of the convoy based on a bicycle model are proposed in [6]. A control approach based on the kinematic model proposed in [7], [8]; the vehicle motion is controlled by a distribution algorithm based on the relative error of the preceding vehicle and the position.

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In the literature, most of the leader-follower control approaches belong to local control category [1], [9], with the vehicle being controlled by the data of its predecessor. This control approach is easy to implement and requires very little exchange of information between the different vehicles in the convoy. For this approach, the stability of the convoy's movements is defeated by the accumulation of errors throughout the convoy chain, causing oscillations due to accumulated errors [2], [10]. Another global control architecture uses information from all vehicles in the convoy or part of it, for example, the control referenced to the preceding vehicle and the leader [11]. This control approach is divided into two categories, using either a centralized or decentralized architecture [5]. For the centralized architecture, the control law applied to each vehicle of the fleet is based on the information of all the vehicles of the convoy [12]. On the other hand, the decentralized architecture [13] is based on the data of a part of the convoy, to minimize the number of sensors used [14]. In the domain of non-linear observation, we can cite the work of [15], [16] who have estimated the contact forces for a vehicle using the sliding mode observer. These theoretical results have been validated experimentally with demonstrations proving its robustness.

In this work, we propose a control approach for the convoy of autonomous vehicles that solves the problem of tracking trajectories for the longitudinal and lateral movements of the fleet. This approach couples the longitudinal and lateral control and is based on the states estimated by a non-linear observer considering that not all states are accessible in real-time. We will first define the model used to estimate the states and control the convoy. This model represents the longitudinal and lateral movement and the yaw angle. In the second part, we will propose a local observer using a sliding mode approach to estimate the states of each vehicle of the convoy. Finally, we use a sliding mode control based on the state estimates by the previous observer to compute the control laws for each vehicle. The objective of the leader control law is to follow a reference trajectory, and the convoy control aims to follow the leader's trajectory and ensure a safe distance between vehicles to avoid collisions.

The validation of our proposed is done with the driving simulator 'Scanner Studio' [17] coupled with Matlab Simulink.

The paper is organized as follows. Section II represents the dynamic and kinematic model of a vehicle and a convoy. The Sliding Mode Observer and the study of its convergence are presented in Section III. Section IV presents the longitudinal and lateral control of the leader in order to

follow a desired trajectory, then the control of the convoy to follow the trajectory of the leader and the study of the convergence and the attractiveness of the sliding surface. The results of simulations and conduction are presented in Sections V and VI.

## II. MODELING

Dynamic description of the fleet is considered in this part. The vehicle fleet model to be used is related to the control approach to be applied. The control approach may require some models, with specific features.

### A. Dynamic Model

The vehicle model can be determined by the fundamental principle of dynamics or the Lagrangian method. This leads to a set of dynamic equations to describe the vehicle motion [8], [18]. The vehicle is represented in Fig. 1 with the following variables in  $(G, x, y)$  the vehicle reference frame. G is the gravity center.

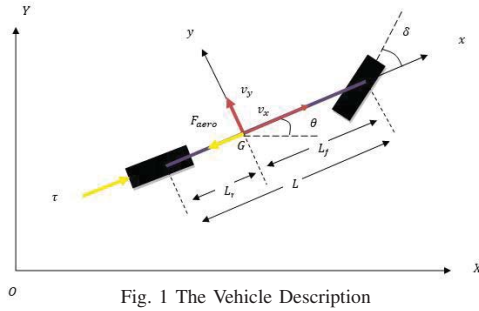


Fig. 1 The Vehicle Description

$L_f$  is distance from the front wheel to G;  $L_r$ : is the distance from the rear wheel center to G; wheels;  $\tau$ : driving/braking wheels torque.  $\delta$ : steering wheel angle;  $F_{aero} = \frac{1}{2}\rho c s \dot{x}^2$ : aerodynamic force, where  $\rho, s$  and  $c$  are the air density, the vehicle frontal surface and the aerodynamic constant;  $R_t$ : radius of the tire and  $E$ : Vehicle's track;  $L_3, I_3$ : the interconnection between the different bodies composing the vehicle.

Before presenting the dynamic model, we have the following assumptions:

**Assumption 1.** The vehicle considered to be a rear engine (only the rear wheels are motorized).

**Assumption 2.** The steering angle of the rear wheels is assumed to be equal, and the steering angles of two front wheels are considered equal.

**Assumption 3.** A linear model estimates the ground/pneumatic contact forces.

**Assumption 4.** The longitudinal slip rate is assumed to be negligible.

The dynamic model of a vehicle is presented as [19]:

$$\begin{cases} m_e \ddot{x} - m \dot{y} \dot{\theta} + L_3 \dot{\theta}^2 + \delta (2C_{\alpha f} \delta - 2C_{\alpha f} \frac{\dot{x}(\dot{y} + L_f \dot{\theta})}{\dot{x}^2 - (\dot{\theta} E/2)^2}) + \\ F_{aero} = \frac{\tau}{R_t} \\ m \ddot{y} - L_3 \ddot{\theta} + m \dot{x} \dot{\theta} + 2C_{\alpha f} \frac{\dot{x}(\dot{y} + L_f \dot{\theta})}{\dot{x}^2 - (\dot{\theta} E/2)^2} + 2C_{\alpha r} \frac{\dot{x}(\dot{y} - L_r \dot{\theta})}{\dot{x}^2 - (\dot{\theta} E/2)^2} \\ = (2C_{\alpha f} - 2\frac{L_{w_i}}{R_t^2} \ddot{x}) \delta \\ I_3 \ddot{\theta} - L_3 \ddot{y} + 2L_f C_{\alpha f} \frac{\dot{x}(\dot{y} + L_f \dot{\theta})}{\dot{x}^2 - (\dot{\theta} E/2)^2} - 2L_r C_{\alpha r} \frac{\dot{x}(\dot{y} - L_r \dot{\theta})}{\dot{x}^2 - (\dot{\theta} E/2)^2} \\ - L_3 \dot{x} \dot{\theta} = L_f (2C_{\alpha f} - 2\frac{L_{w_i}}{R_t^2} \ddot{x}) \delta - (\frac{E}{2} C_{\alpha f} \frac{E \dot{\theta} (\dot{y} + L_f \dot{\theta})}{\dot{x}^2 - (\dot{\theta} E/2)^2}) \delta \end{cases} \quad (1)$$

where  $m_e = m + 4\frac{L_{w_i}}{R_t^2}$ ,  $L_3 = 2m_w(L_r - L_f)$  and  $I_3 = I_z + m_w E^2$ .

The model represents the longitudinal and lateral movement of the fleet and the movement of the yaw angle in the vehicle frame  $(G, x, y)$ . In the following, we will write this model in the robotic form. Take position vector  $q_i = [q_{1i}, q_{2i}, q_{3i}]^T = [x_i, y_i, \theta_i]^T$ , this model can be written for the  $i$ th vehicle in the following form:

$$M_i(q_i) \cdot \ddot{q}_i + H_i(\dot{q}_i, q_i) = U_i \quad (2)$$

where the inertia Matrix  $M_i(q_i)$  is:

$$M_i(q_i) = \begin{pmatrix} m_{e_i} & 0 & 0 \\ 0 & m_i & -L_{3i} \\ 0 & -L_{3i} & I_{3i} \end{pmatrix}$$

and the vector  $H_i(\dot{q}_i, q_i)$  is equal to:

$$\begin{pmatrix} -m_i \dot{q}_{2i} \dot{q}_{3i} + L_{3i} \dot{q}_{3i}^2 + \delta_i (2C_{\alpha f i} \delta_i - 2C_{\alpha f i} \frac{\dot{q}_{1i}(\dot{q}_{2i} + L_{f i} \dot{q}_{3i})}{\dot{q}_{1i}^2 - (\dot{q}_{3i} E_i/2)^2}) + F_{aero i} \\ m_i \dot{q}_{1i} \dot{q}_{3i} + 2C_{\alpha f i} \frac{\dot{q}_{1i}(\dot{q}_{2i} + L_{f i} \dot{q}_{3i})}{\dot{q}_{1i}^2 - (\dot{q}_{3i} E_i/2)^2} + 2C_{\alpha r i} \frac{\dot{q}_{1i}(\dot{q}_{2i} - L_{r i} \dot{q}_{3i})}{\dot{q}_{1i}^2 - (\dot{q}_{3i} E_i/2)^2} \\ 2L_{f i} C_{\alpha f i} \frac{\dot{q}_{1i}(\dot{q}_{2i} + L_{f i} \dot{q}_{3i})}{\dot{q}_{1i}^2 - (\dot{q}_{3i} E_i/2)^2} - 2L_{r i} C_{\alpha r i} \frac{\dot{q}_{1i}(\dot{q}_{2i} - L_{r i} \dot{q}_{3i})}{\dot{q}_{1i}^2 - (\dot{q}_{3i} E_i/2)^2} - L_{3i} \dot{q}_{1i} \dot{q}_{3i} \end{pmatrix}$$

and the input vector  $U_i = (u_{1i}, u_{2i}, u_{3i})^T$ :

$$U_i = \begin{pmatrix} \frac{\tau_i}{R_{t_i}} \\ (2C_{\alpha f i} - 2\frac{L_{w_i}}{R_{t_i}^2} \ddot{q}_{1i}) \delta_i \\ L_{f i} u_{2i} - (\frac{E_i}{2} C_{\alpha f i} \frac{E_i \dot{q}_{3i} (\dot{q}_{2i} + L_{f i} \dot{q}_{3i})}{\dot{q}_{1i}^2 - (\dot{q}_{3i} E_i/2)^2}) \delta_i \end{pmatrix}$$

The input  $U_i$  controls the longitudinal movement of the fleet by the driving/braking wheels torque ( $u_{1i} = \frac{\tau_i}{R_{t_i}}$ ) and the lateral movement by the steering wheel angle ( $u_{2i} = (2C_{\alpha f i} - 2\frac{L_{w_i}}{R_{t_i}^2} \ddot{q}_{1i}) \delta_i$ ).

### B. Kinematic Equations

The transformation of the velocity, from the absolute vehicle frame  $(G, x, y)$  to the velocity in the reference frame  $R(0, X, Y)$  is defined by:

$$\begin{bmatrix} \dot{X}_i \\ \dot{Y}_i \\ \dot{\theta}_i \end{bmatrix} = \begin{pmatrix} \cos \theta_i & -\sin \theta_i & 0 \\ \sin \theta_i & \cos \theta_i & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\theta}_i \end{bmatrix} \quad (3)$$

Such as we get the kinematics of the  $i$ th vehicle:

$$\begin{cases} \dot{X}_i = \dot{x}_i \cos \theta_i - \dot{y}_i \sin \theta_i \\ \dot{Y}_i = \dot{x}_i \sin \theta_i + \dot{y}_i \cos \theta_i \end{cases} \quad (4)$$

### C. Convoy Motion in the Geometric Model

The curvilinear abscissa of the  $i$ -th vehicle (G is the gravity center of each vehicle) and of the preceding vehicle is defined by  $S_i$  and  $S_{i-1}$ , with  $S_i$ : the curvilinear abscissa. This abscissa located with respect to the desired trajectory, Fig. 2. Let us note  $l_{d_i}$ , the desired curvilinear distance between 2 vehicles, and therefore  $e_{s_i}$  is a curvilinear spacing error or difference between 2 vehicles.  $e_{s_i} = S_{i-1} - S_i - l_{d_i}$ .

$$e_{s_i} = \int_0^T (\dot{x}_{i-1}^2 + \dot{y}_{i-1}^2)^{\frac{1}{2}} dt - \int_0^T (\dot{x}_i^2 + \dot{y}_i^2)^{\frac{1}{2}} dt - l_{d_i} \quad (5)$$

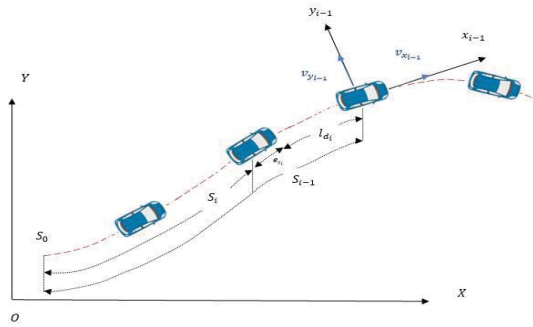


Fig. 2 Geometric description of the convoy motion

## III. THE VEHICLES STATE OBSERVATION

### A. Vehicle State Space Model

We have as a state vector, the position and speed of each vehicle:

$$z_i = \begin{pmatrix} z_{1i} \\ z_{2i} \end{pmatrix} \quad (6)$$

with positions:  $z_{1i} = [x_i, y_i, \theta_i]^T$  and velocities:  $z_{2i} = [\dot{x}_i, \dot{y}_i, \dot{\theta}_i]^T$

The dynamic model of a  $i$ -th vehicle of the convoy is represented in canonical forms, assuming that all the vehicle positions  $z_{1i}$  are measurable. The dynamic model of the fleet in (2) can be written in the following form:

$$\begin{cases} \dot{z}_{1i} = z_{2i} \\ \dot{z}_{2i} = f(z_{1i}, z_{2i}) + g(z_{1i})U_i \end{cases} \quad (7)$$

where:  $f(z_{1i}, z_{2i}) = -M^{-1}(z_{1i})H_i(z_{1i}, z_{2i})$  and  $g(z_{1i}) = M^{-1}(z_{1i})$ . This canonical representation of the model will be used in the following for the observation and the control of a vehicle then the global control of the convoy.

### B. First Order Sliding Mode Observer

To define the observer and estimate the states of each vehicle with the available information, we have the following assumptions:

**Assumption 5.** The positions ( $z_{1i} = [x_i, y_i, \theta_i]^T$ ) are available in real-time.

**Assumption 6.** Model parameters  $f(z_{1i}, z_{2i})$  and  $g(z_{1i})$  are measurable.

The observed state noted  $\hat{z}_i$  is estimated by the following block (Fig. 3), where we assume that vehicle positions  $z_{1i}$  are measurable. The observation error of the state of the system is noted  $\tilde{z}_i$ .

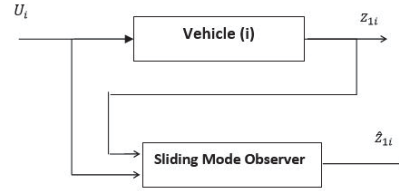


Fig. 3 Principal Diagram of an observer

The observer model is presented as:

$$\begin{cases} \dot{\hat{z}}_{1i} = \hat{z}_{2i} - \lambda_{1i} \text{sign}(\hat{z}_{1i} - z_{1i}) - \Lambda_{1i}(\hat{z}_{1i} - z_{1i}) \\ \dot{\hat{z}}_{2i} = f(z_{1i}, \hat{z}_{2i}) + g(z_{1i})U_i - \lambda_{2i} \text{sign}(\hat{z}_{1i} - z_{1i}) \\ \quad - \Lambda_{2i}(\hat{z}_{1i} - z_{1i}) \end{cases} \quad (8)$$

where  $f(z_{1i}, \hat{z}_{2i}) = -M^{-1}(z_{1i})H(z_{1i}, \hat{z}_{2i})$ .  $\lambda_{1i}, \lambda_{2i}, \Lambda_{1i}$  and  $\Lambda_{2i}$  are positive definite diagonal gain matrices.

1) *Observer Convergence Analysis:* To study the convergence of this observer, let us write the dynamic error equations taking as position and speed error:  $\tilde{z}_{1i} = \hat{z}_{1i} - z_{1i}$  and  $\tilde{z}_{2i} = \hat{z}_{2i} - z_{2i}$ . The dynamic error equations is defined:

$$\begin{cases} \dot{\tilde{z}}_{1i} = \tilde{z}_{2i} - \lambda_{1i} \text{sign}(\tilde{z}_{1i}) - \Lambda_{1i}\tilde{z}_{1i} \\ \dot{\tilde{z}}_{2i} = \Delta f_i - \lambda_{2i} \text{sign}(\tilde{z}_{1i}) - \Lambda_{2i}\tilde{z}_{1i} \end{cases} \quad (9)$$

with:  $\Delta f_i = f(z_{1i}, \hat{z}_{2i}) - f(z_{1i}, z_{2i})$

To study the convergence of this observer in finite time, we choose as Lyapunov function:

$$V_i = V_{1i} + V_{2i}$$

The first part of this function ( $V_{1i}$ ) is to converge the first equation of the error ( $\dot{\tilde{z}}_{1i} = 0$ ).  $V_{1i}$  is defined as:

$$V_{1i} = \frac{1}{2} \tilde{z}_{1i}^T \tilde{z}_{1i}$$

Therefore the sliding surface is attractive if the derivative of this function is negative:

$$\dot{V}_{1i} = \tilde{z}_{1i}^T (\tilde{z}_{2i} - \lambda_{1i} \text{sign}(\tilde{z}_{1i}) - \Lambda_{1i}\tilde{z}_{1i})$$

Take it  $\lambda_{1i} > |\tilde{z}_{2i} - \Lambda_{1i}\tilde{z}_{1i}|$ . We can calculate the average sign function ( $\text{sign}_e(\tilde{z}_{1i})$ ) such as:

$$\text{sign}_e(\tilde{z}_{1i}) = \lambda_{1i}^{-1}(\tilde{z}_{2i} - \Lambda_{1i}\tilde{z}_{1i})$$

By replacing the sign function ( $\text{sign}_e(\tilde{z}_{1i})$ ) with its expression, (9) becomes:

$$\begin{cases} \dot{\tilde{z}}_{1i} = \tilde{z}_{2i} - \lambda_{1i} \text{sign}_e(\tilde{z}_{1i}) - \Lambda_{1i}\tilde{z}_{1i} = 0 \\ \dot{\tilde{z}}_{2i} = \Delta f_i - \lambda_{2i} \lambda_{1i}^{-1}(\tilde{z}_{2i} - \Lambda_{1i}\tilde{z}_{1i}) - \Lambda_{2i}\tilde{z}_{1i} \end{cases} \quad (10)$$

The second part of the function ( $V_{2i}$ ) is defined as:

$$V_{2i} = \frac{1}{2} \tilde{z}_{2i}^T \tilde{z}_{2i}$$

This function is intended to converge the second error to zero ( $\dot{\tilde{z}}_{2i} = 0$ ). By calculating the derivative of this function:

$$\dot{V}_{2i} = \tilde{z}_{2i}^T [\Delta f_i - \lambda_{2i} \lambda_{1i}^{-1} \tilde{z}_{2i} + (\lambda_{2i} \lambda_{1i}^{-1} \Lambda_{1i} - \Lambda_{2i}) \tilde{z}_{1i}]$$

This function is negative when we choose  $\Lambda_{2i} = \lambda_{2i} \lambda_{1i}^{-1} \Lambda_{1i}$  and  $\lambda_{2i} > |\Delta f_i \lambda_{1i}|$  with  $|\Delta f_i| < \varepsilon_0$ .

With these convergence conditions for the two parts of the function  $V_i$ , the convergence of this observer in finite time is guaranteed.

#### IV. LONGITUDINAL AND LATERAL CONTROL

The objective of the control is:

- Control the leader to follow the desired path.
- Control the convoy with the decentralized global approach to follow the trajectory of the leader with the available information.
- Ensure a safe distance between vehicles to avoid a collision.

In this section, the model used to calculate the control laws of the leader and the convoy is defined in (7). To simplify the writing, we define the error position of the fleet as defined in the dynamic model:

$$e_i = (e_{1i}, e_{2i}, e_{3i})^T = (e_{x_i}, e_{y_i}, e_{\theta_i})^T$$

where  $e_{x_i}$ ,  $e_{y_i}$  and  $e_{\theta_i}$  represents the longitudinal, lateral and yaw errors positions of each vehicle of the convoy. The sliding mode control is used to control the movement of the leader and the convoy. This nonlinear control allows fast and robust convergence of the system state. This control ensures robust stability [15], and based on a switching function in the sliding surface.

##### A. Control of the Leader

In this paper, leader is controlled to follow a desired trajectory. The sliding surface chosen for leader is defined as:

$$s_0 = \dot{e}_0 + K_1 e_0 \quad (11)$$

where  $s_0$  represents the sliding surface and  $e_0$  represents the tracking error such as:

$$\begin{cases} \dot{e}_{x_0} = \dot{x}_0 - \dot{x}_{0d} \\ \dot{e}_{y_0} = a_{y_0} - a_{y_{0d}} \end{cases} \quad (12)$$

$\dot{x}_{0d}$ : the longitudinal speed desired for the leader;  $a_{y_{0d}}$ : the desired lateral acceleration for the leader;  $\dot{e}_{x_0}$ : the longitudinal velocity error and  $\dot{e}_{y_0}$ : the lateral acceleration error.  $K_1$  and  $K_2$  are positive definite diagonal gain matrices. The desired lateral acceleration is chosen according to the longitudinal velocity and the radius of the desired trajectory. That is to say, in our approach, the longitudinal and lateral control are coupled. The desired lateral acceleration is defined as:

$$a_{y_{0d}} = \dot{x}_{0d}^2 / R_0$$

where  $R_0$ : the radius curvature of the trajectory desired of the leader.

We recall that :  $a_{y_0} = \ddot{y}_0 + \dot{x}_0 \dot{\theta}_0$ , we can write  $\ddot{e}_{y_0} = \ddot{y}_0 - \ddot{y}_{0d}$ , with  $\ddot{y}_{0d} = -\dot{x}_0 \dot{\theta}_0 + \dot{x}_0^2 / R_0$ . The equivalent control brings back the state of the vehicle to the desired state, it is calculated from the derivative of the sliding surface ( $\dot{s} = 0$ ) such as:

$$\dot{s}_0 = \ddot{e}_0 + K_1 \dot{e}_0 \quad (13)$$

According to (13), and using the state model defined in (7), the equivalent control is defined as:

$$u_{eq_0} = g(\hat{z}_{1_0})^{-1} (\dot{z}_{2d} - k_1 (\hat{z}_{2_0} - z_{2d}) - f(\hat{z}_{1_0}, \hat{z}_{2_0}))$$

The global control applied to the leader represents the sum of the two controls (equivalent control and robust control):

$$U_0 = g(\hat{z}_{1_0})^{-1} (\dot{z}_{2d} - k_1 (\hat{z}_{2_0} - z_{2d}) \dots - f(\hat{z}_{1_0}, \hat{z}_{2_0})) - K_2 \text{sign}(s_0) \quad (14)$$

where the robust control:  $u_{rob_0} = -k_2 \text{sign}(s_0)$ . This control law (14) controls the longitudinal and lateral movement and allows to follow the desired trajectory.

To study the existence of the surface, we can choose a Lyapunov function  $V_0 = \frac{1}{2} s_0^T s_0$ . The sliding surface is attractive if the derivative of this function is negative:

$$\dot{V}_0 = s_0^T \dot{s}_0 = s_0^T [\ddot{e} + K_1 \dot{e}] = s_0^T [\dot{z}_{2_0} - \dot{z}_{2d} + K_1 \dot{e}]$$

Replacing  $\dot{z}_{2_0}$  with its expression:

$$\dot{V}_0 = s_0^T [f(z_{1_0}, z_{2_0}) + g(z_{1_0}) U_0 - \dot{z}_{2d} + K_1 \dot{e}]$$

Replacing the expression of the  $U_0$  control in the previous equation:

$$\dot{V}_0 = s_0^T [f(z_{1_0}, z_{2_0}) - g(z_{1_0}) K_2 \text{sign}(s_0) + \dot{z}_{2d} - K_1 (\hat{z}_{2_0} - z_{2d}) - f(\hat{z}_{1_0}, \hat{z}_{2_0}) - \dot{z}_{2d} + K_1 \dot{e}] \quad (15)$$

After the calculation we find:

$$\dot{V}_0 = s_0^T [-\Delta f_0 - M^{-1}(z_{1_0}) K_2 \text{sign}(s_0)] \quad (16)$$

The matrix  $M(z_{1_0})$  is positive definite, and reversible. To ensure the convergence and the existence of this sliding surface, we can increase the gains of the matrix  $K_2$  in the following:  $|\Delta f_0| < \varepsilon_0 < K_2$ . The derivative of (16) is negative:

$$\dot{V}_0 < -\eta_0 |s_0| < 0$$

where  $\varepsilon_0$  and  $\eta_0$ : positive constants. By respecting the conditions of the convergence of the sliding surface, the stability of the path tracking for the leader is ensured.

### B. Control of the Convoy

The decentralized global approach is considered in this paper. That is, the control applied to each vehicle of the convoy is based on the information of the previous vehicle and the leader.

The longitudinal control is intended to impose a reference speed on the convoy and ensure a safe distance between vehicles to avoid collisions (Fig. 4), and the lateral control makes it possible to follow the leader trajectory and to minimize the error of the lateral deviation with respect to the desired path by the steering angle (Fig. 5) [20]. Both controls are coupled with longitudinal velocity.

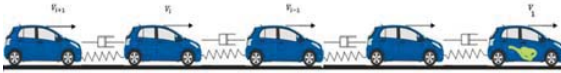


Fig. 4 The longitudinal movement of the convoy

The following assumptions are essential for calculating the longitudinal and lateral control laws for the convoy: **Assumption 7.** The information of the leader and the previous vehicle are available for each vehicle. **Assumption 8.** The characteristic of the leader's trajectory is available for the convoy (the radius or curvature of the path).

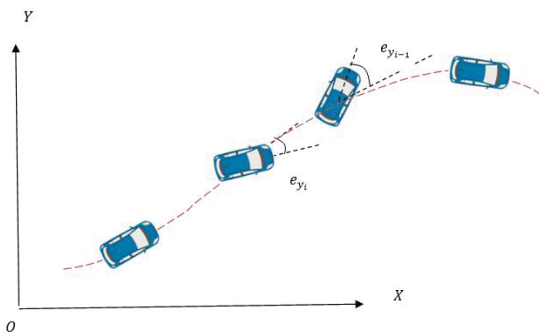


Fig. 5 Lateral deviation of the convoy

Let the sliding surface defined for the control applied to  $i$ th vehicle based on the leading vehicle and the previous vehicle information as:

$$s_i = \dot{e}_i + K_3 \dot{e}_{i,0} + K_4 e_i \quad (17)$$

where  $s_i$  represents the sliding surface for the control of the  $i$ th vehicle,  $\dot{e}_{i,0}$  the speed error between the  $i$ th vehicle and leader,  $e_i$  represents the position errors between the  $i$ th vehicle and the previous ( $i-1$ )th vehicle.  $K_3$ ,  $K_4$  and  $K_5$  are positive definite diagonal gain matrices.

We define the curvilinear spacing error between the vehicles of the convoy:

$$e_{x_i} = S_i - S_{i-1} + l_d$$

where:  $l_d$ : the safety distance.  $S_i$ : the curvilinear abscissa. The error of the longitudinal velocity is defined as follows:

$$\dot{e}_{x_i} = \dot{x}_i - \dot{x}_{i-1}$$

The speed error between the  $i$ th vehicle and the leader is:

$$\dot{e}_{x_{i,0}} = \dot{x}_i - \dot{x}_0$$

where  $\dot{x}_0$ ,  $\dot{x}_i$  and  $\dot{x}_{i-1}$  are longitudinal speed of leader,  $i$ th vehicle and ( $i-1$ )th vehicle. The lateral movement of the fleet (Fig. 5) is controlled based on the lateral acceleration. The objective of this control is to follow the reference trajectory of the leader, and to cancel the lateral deviation, that is to say  $e_{y_i} = 0$ . The lateral acceleration error is defined as:

$$\ddot{e}_{y_i} = a_{y_i} - a_{y_{i,d}} \quad (18)$$

where  $a_{y_i}$ : the lateral acceleration and  $a_{y_{i,d}}$ : the desired lateral acceleration. The desired lateral acceleration is calculated according to the desired trajectory and the longitudinal speed:

$$a_{y_{i,d}} = \dot{x}_i^2 / R_i$$

where  $R_i$ : the radius curvature of the trajectory desired of the  $i$ th vehicle. In order to calculate the law of the control, we replace  $a_{y_{i,d}}$  by its expression:  $a_{y_i} = \ddot{y}_i + \dot{x}_i \dot{\theta}_i$ . The lateral acceleration error defined in (18) can be written in the form:

$$\ddot{e}_{y_i} = \ddot{y}_i - \ddot{y}_{i,d}$$

where  $\ddot{y}_{i,d} = -\dot{x}_i \dot{\theta}_i + \dot{x}_i^2 / R_i$ .

To calculate the equivalent control, we derive the equation of the sliding surface (17) and canceling ( $\dot{s}_i = 0$ ) we find:  $u_{eq_i} = g(\hat{z}_{1_i})^{-1} [(I + K_3)^{-1} (\hat{z}_{2_{i-1}} + K_3 \hat{z}_{2_0} - K_4 \dot{e}_i) - f(\hat{z}_{1_i}, \hat{z}_{2_i})]$

The global control applied to the  $i$ -th vehicle of the convoy is defined as:

$$U_i = -K_5 \text{sign}(s_i) + g(\hat{z}_{1_i})^{-1} [(I + K_3)^{-1} (\hat{z}_{2_{i-1}} + K_3 \hat{z}_{2_0} \dots - K_4 \dot{e}_i) - f(\hat{z}_{1_i}, \hat{z}_{2_i})] \quad (19)$$

This control (19) makes it possible to control the longitudinal and lateral movement of the convoy and to follow the path of the leader. The longitudinal movement is controlled by the driving/braking wheels torque, and the lateral movement controlled by the steering wheel angle. To study the convergence of the sliding surface defined to control the convoy, we choose the Lyapunov function:

$$V_i = \frac{1}{2} s_i^T s_i \quad (20)$$

The sliding surface is attractive if the derivative of this function is negative. We derive (20) and replace  $\dot{s}_i$  by its expression:

$$\dot{V}_i = s_i^T (\ddot{e}_i + K_3 \ddot{e}_{i,0} + K_4 \dot{e}_i)$$

We replace the acceleration error ( $\ddot{e}_i, \ddot{e}_{i,0}$ ) by its expression in (7):

$$\dot{V}_i = s_i^T [(I + K_3)(f(z_{1_i}, z_{2_i}) + g(z_{1_i})U_i) - \dot{z}_{2_{i-1}} - K_3 \dot{z}_{2_0} + K_4 \dot{e}_i]$$

Replacing  $U_i$  by its expression (19):

$$\dot{V}_i = s_i^T [- (I + K_3)(\Delta f_i + g(z_{1_i})K_5 \text{sign}(s_i) - \dot{z}_{2_{i-1}} - K_3 \dot{z}_{2_0} + K_4 \dot{e}_i + \hat{z}_{2_{i-1}} + K_3 \dot{z}_{2_0} - K_4 \dot{e}_i)] \quad (21)$$

After the calculation:

$$\dot{V}_i = s_i^T [-(I + K_3)(\Delta f_i + g(z_{1_i})K_5 \text{sign}(s_i))]$$

The inertia matrix ( $g(z_{1_i}) = M^{-1}(z_{1_i})$ ) is defined as strictly positive and reversible. If the parameters and the states are well estimated,  $\Delta f_i$  converges quickly to zero ( $\Delta f_i \Rightarrow 0$ ). We have  $I$  represents the identity vector, let  $K_3$  be a matrix of positive defined gains, so  $(I + K_3) > 0$ .

Let  $K_5$  defined as follows:  $|\Delta f_i| < \varepsilon_i < K_5$ . With these criteria, the derivative of the function (20):

$$\dot{V}_i < -\eta_i |s_i| < 0$$

where  $\varepsilon_i$  and  $\eta_i$ : positive constants. The convergence and attractiveness of the sliding surface defined in (17) are proven. The stability of the convoy is always checked and ensured provided that the conditions of convergence of the sliding surface are respected.

## V. SIMULATIONS

To validate the results, we used two software; MATLAB Simulink and the SCANer Studio driving platform. The five vehicles of the convoy, Fig. 6, are controlled to follow the imposed trajectory represented in Fig. 7. The leading vehicle moves on this trajectory, and the convoy follows it with a safety distance between vehicles to avoid a collision. We choose the parameters of a vehicle in SCANer Studio:

TABLE I  
VEHICLE PARAMETER VALUES SCANER-STUDIO

Parameter	Value	Parameter	Value
$m$	1500 kg	$m_w$	23.2kg
$I_z$	1652.7kg.m <sup>2</sup>	$I_w$	2kg.m <sup>2</sup>
$C_{\alpha_f}$	67689N/rad	$C_{\alpha_r}$	69253N/rad
$L_r$	1.441m	$L_f$	1.099m
$s$	2m <sup>2</sup>	$E$	1.5m
$c$	0.3	$\rho$	1.3

This trajectory (Fig. 7) has been chosen to validate the longitudinal and lateral control of the fleet. First, we assume that we have the data (position, velocity, and acceleration) of a vehicle moving in the SCANer studio to validate the observer defined in Section III.



Fig. 6 Convoy 5-vehicle SCANer Studio

We present the Leader's estimation results to compare the Leader's actual states provided by SCANer Studio with the



Fig. 7 Trajectory-SCANer Studio

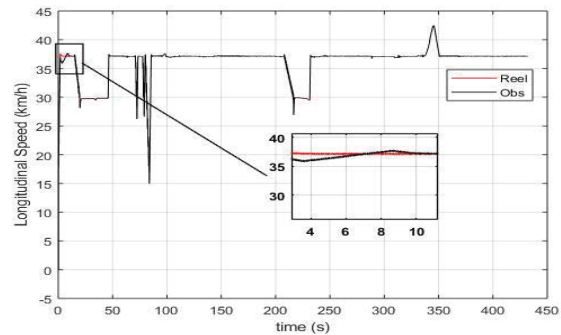


Fig. 8 Velocity  $v_x$  of the leader real and observer

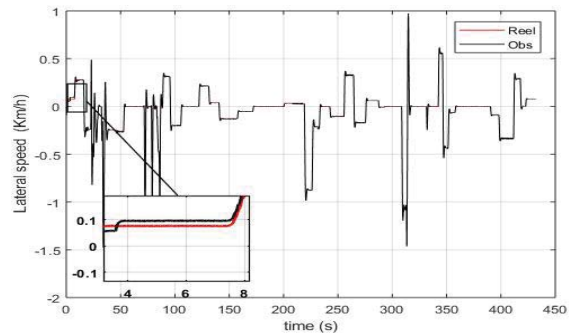


Fig. 9 Velocity  $v_y$  of the leader real and observer

states estimated by the observer. As defined in assumption 5, we consider that all positions ( $x_i$ ,  $y_i$  and  $\theta_i$ ) are available in real-time, the objective being to estimate the remaining states (speed and acceleration). This observer has been used for all the vehicles in the fleet to estimate their real state, and it represents a local observer so that each car estimates its state from the available sensors (position) to calculate its control law. To avoid the problem of reluctance, we have replaced the *sign* function by the *sat* function for the observer and control. To test the robustness of this observer, we give the initial conditions for the positions:  $\hat{x}_{i_0} = 2m$ ,  $\hat{y}_{i_0} = 2m$  and  $\hat{\theta}_{i_0} = 1rad/s$ .

Fig. 8 shows the real longitudinal velocity and the velocity estimated by the observer. It can be seen that after  $t = 9s$ , the estimated states converge with the actual states. The convergence time depends on the observer's gain and the initial conditions chosen for the positions. For the non-linear observer and the control, the gains are chosen among several tests while respecting the stability conditions defined in the convergence study. The estimated lateral acceleration is calculated according to the states estimated by the observer ( $\hat{a}_{y_i} = \hat{\dot{y}}_i + \hat{x}_i \hat{\theta}_i$ ) and the results, Fig. 10, show their rapid convergence after  $t = 8s$ . The real and estimated lateral velocity and yaw rate are presented in Figs. 9 and 11.

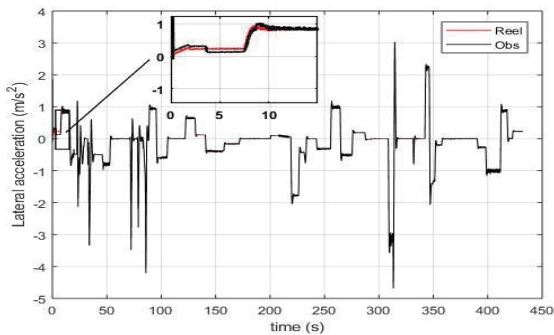


Fig. 10 Lateral acceleration  $a_y$  of the leader, real and observer

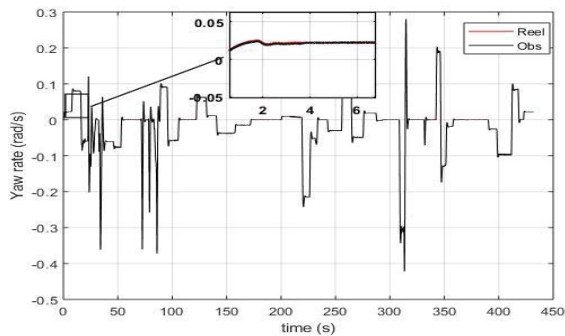


Fig. 11 Yaw rate real and observer

After validation of the observer, we present in the following the results of the control by the sliding mode control applied on the convoy. The leading vehicle was controlled by the control defined in (14) to follow the trajectory presented in Fig. 7, the convoy uses a decentralized global approach to control the longitudinal and lateral movement, follow the desired trajectory in accordance with the constraints defined in Section IV. The law of the control uses the states estimated by the observer.

Fig. 12 shows the movement of the convoy in the reference trajectory. It can be seen that the control law proposed for the fleet of vehicles, allows controlling the convoy in both directions of movement (longitudinal and lateral), to ensure a safe distance between the vehicles to avoid a collision. Each vehicle in the convoy uses information from the leader and the neighbors to calculate its control law, which is part of the

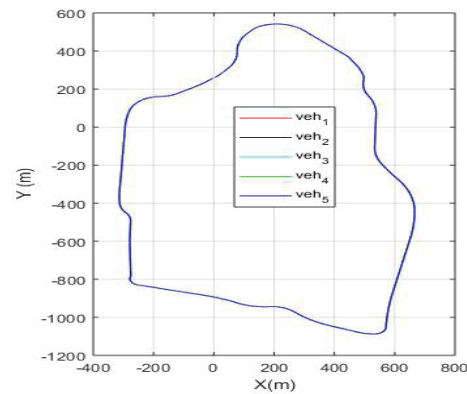


Fig. 12 Trajectory of the convoy

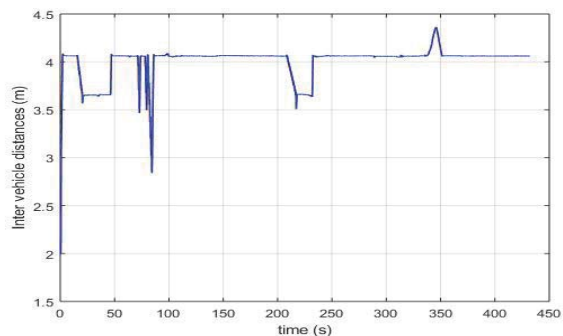


Fig. 13 Inter vehicle distance

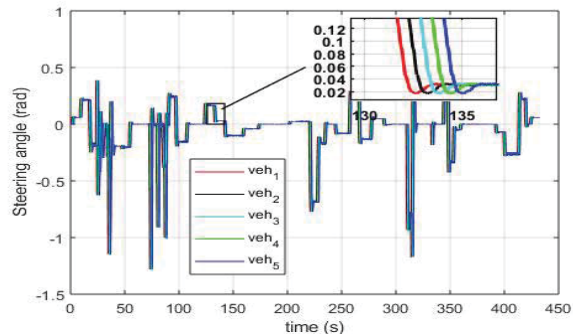


Fig. 14 Steering angle

family of decentralized approaches. It can be seen that the lateral error is practically negligible, i.e., no angular deviation between the convoy vehicles and the reference path, the safety distance is about 4 m between each neighbour, Fig. 13. The safety distance is proportional to the speed. In our case, the fleet moves at a speed ( $v_{x_{max_i}} < 45km/h$ ), so the safety distance is minimal. On the other hand, if the convoy is traveling at a high speed, the distance between the vehicles must increase. In addition, to have a distance proportional to the speed, we have added a delay in the position error, which is related to the longitudinal speed. Figs. 16 and 18 represent the longitudinal and lateral speeds of the fleet. The longitudinal

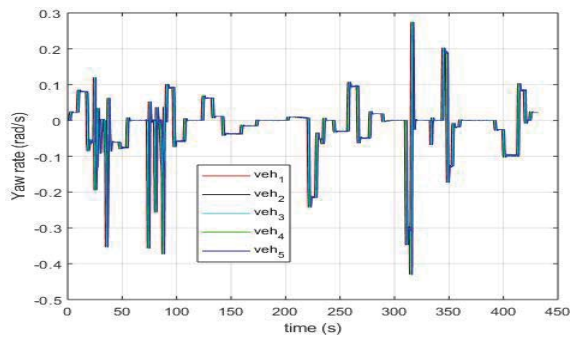
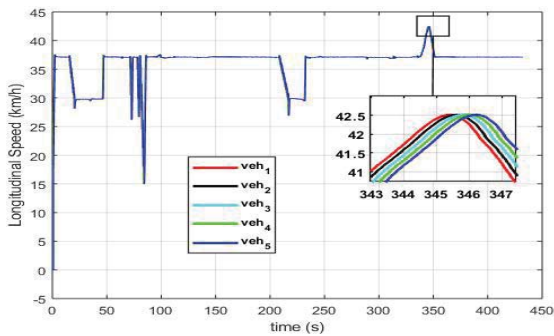
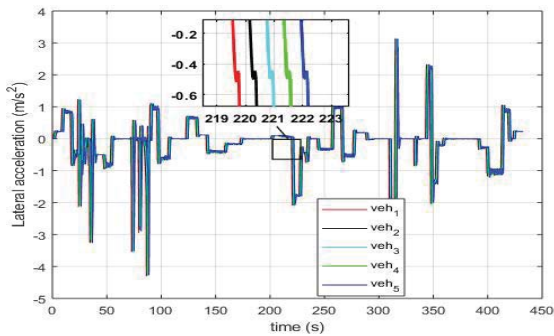
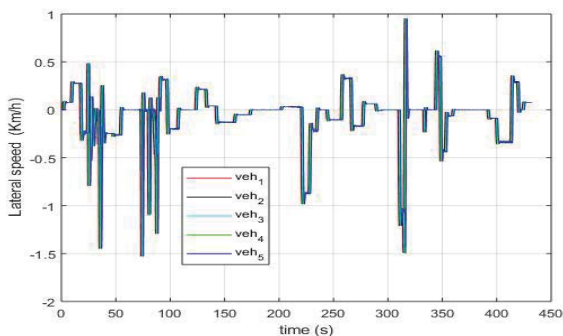


Fig. 15 Yaw rate of the convoy

Fig. 16 Velocity  $v_x$  of the convoyFig. 17 Lateral acceleration  $a_y$  of the convoyFig. 18 Velocity  $v_y$  of the convoy

movement is controlled by the torque of the drive/braking wheels, and the steering angle controls the lateral movement, the latter being calculated as:

$$\delta_i = u_{2i} / (2C_{\alpha fi} - 2 \frac{I_{wi}}{R_{ti}^2} \ddot{x}_i)$$

The steering angle is shown in Fig. 14. It represents the lateral movement for each vehicle in the convoy. This angle is proportional to the lateral acceleration, Fig. 17 and the yaw rate, Fig. 15.

The control approach we proposed, controls both movements of the convoy, as shown in Fig. 12, the accumulation errors are almost negligible, which proves a better performance for the path tracking. With the decentralized global approach, we have that the tracking errors are not accumulated to the other vehicles.

## VI. CONCLUSION

In this article, we have controlled a convoy of autonomous vehicles using the states reconstructed by a first-order sliding mode observer, to reduce the number of sensors. The model used, represents a non-linear system with bilateral couplings for a fleet of road vehicles, that can be used for a large number of vehicles. The observer developed is to estimate the position, speed, and acceleration of each vehicle based on the position sensors in order to calculate the law of control of the convoy. The practical results show the rapid convergence of this observer with the real state. A sliding mode control based on this observer, which solves the tracking problem for a set of unconnected coordinated cars, has been proposed. Two controls are calculated for the Leader and Convoy vehicles. In our case, the leader is driven autonomously. The second control for the convoy is based on the decentralized approach, as each vehicle in convoy uses the information from the leader and the preceding vehicle. The results of the simulation show robustness of following using this control approach, and the errors of accumulations towards other vehicles are almost neglected, and the vehicles stay on the leader's trajectory, respecting a safe distance between vehicles to ensure safety.

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