Noise-Improved Signal Detection in Nonlinear Threshold Systems

Youguo Wang, and Lenan Wu

Abstract—We discuss the signal detection through nonlinear threshold systems. The detection performance is assessed by the probability of error *Per* . We establish that: (1) when the signal is complete suprathreshold, noise always degrades the signal detection both in the single threshold system and in the parallel array of threshold devices. (2) When the signal is a little subthreshold, noise degrades signal detection in the single threshold system. But in the parallel array, noise can improve signal detection, i.e., stochastic resonance (SR) exists in the array. (3) When the signal is predominant subthreshold, noise always can improve signal detection and SR always exists not only in the single threshold system but also in the parallel array. (4) Array can improve signal detection by raising the number of threshold devices. These results extend further the applicability of SR in signal detection.

Keywords—Probability of error, signal detection, stochastic resonance, threshold system.

I. INTRODUCTION

NOISE can bring beneficial help to signal processing or signal transmission in some nonlinear systems. This phenomenon is called stochastic resonance (SR) [1]. Most studies of SR involve a subthreshold (weak) signal through a single nonlinear system. The nonlinear system can produce a stronger beneficial response by the addition of noise [1]-[4]. Very recently, another new form of SR has been introduced under the name of suprathreshold stochastic resonance (SSR) through a parallel array [5]-[12], where the input signal is suprathreshold with respect to the common threshold. In a single threshold system, noise always degrades signal transmission for suprathreshold signal. However, in a parallel array, when different and independent noises are added on every device in the array, every device will in general produce a different response. When all these responses are summed, the output can be improved and SR may occur. SR and SSR are two distinct forms of improvement by noise [8]. The study of SR in signal detection has received some attentions [13]-[21]. Here, we study further SR in signal detection through the array of threshold devices. We use the probability of error *Per* to assess the performance. We establish that: (1) when the signal is

complete suprathreshold, noise always degrades the signal detection both in the single threshold system and in the parallel array. (2) When the signal is a little subthreshold, noise degrades signal detection in the single threshold system. But noise can improve signal detection in the parallel array, i.e., SR exists in the parallel array. (3) When the signal is predominant subthreshold, SR always exists not only in the single threshold system but also in the parallel array. (4) Array can improve signal detection by raising the number of threshold devices. These results extend further the applicability of SR in signal detection.

II. OPTIMAL NONLINEAR DETECTION IN A PARALLEL ARRAY

A continuous signal x is with probability density function (PDF) $f_0(x)$ (hypothesis H_0) or with PDF $f_1(x)$ (hypothesis H_1) with prior probability P_0 and P_1 ($P_0 + P_1 = 1$). A noise η_i , independent of x , can be added to x before quantization by threshold device i , which is with a threshold level u_i , delivers the output $y_i = H[x + \eta_i - u_i], i = 1,2,...,N$, where $H(u) = 1$ if $u \succ 0$ and is zero otherwise. We consider here that N noises η_i are mutually independent, and identically distributed with PDF $f_\eta(x)$. The response of the parallel array is obtained by summing the outputs of all the threshold devices as $y = \sum_{i=1}^{N} y_i$.

When N is one, the array is the single threshold system. Such conditions, with arrays of threshold devices, are relevant for existing and future multi-sensor networks having to cope with limited time and resources for data processing, storage, communication, and for energy supply [8]. The detection of signal x is based on the observation y , which assumes integer values between 0 and N ($y \in R = \{0,1,\cdots,N\}$). As discussed in [21], we can obtain the optimal detector, also known as the maximum a posterior probability (MAP) detector, which implements the test $L(y = n) \underset{H_0}{\overset{H_1}{\underset{\prec}{\succ}}} \dfrac{P_0}{P_1}$, by use of the likelihood ratio $L(y = n) = \dfrac{\Pr(y = n \mid H_1)}{\Pr(y = n \mid H_0)}$. The probability of error *Per* reached by the MAP detector is expressed as:

$$Per = \frac{1}{2} - \frac{1}{2}\sum_{n \in R}\left| P_0 \Pr(y = n \mid H_0) - P_1 \Pr(y = n \mid H_1) \right| \quad (1)$$

Where $\Pr(y = n \mid H_0)$ and $\Pr(y = n \mid H_1)$ are the conditional probabilities. Given a fixed value x_0 of input signal x, we have the conditional probability $\Pr(y_i = 0 \mid x_0) = \Pr(x_0 + \eta_i \le u_i) = q_i(x_0)$, and $\Pr(y_i = 1 \mid x_0) = 1 - q_i(x_0)$. Because the effect of threshold level distribution to the performance of the array is little, especially at larger noise intensity [12], we also can assume that all the thresholds share the same value u as in [8-10]. Then we have $q_i(x_0) = q(x_0)$ and the $\Pr(y = n \mid x_0)$ follows, according to the binomial distribution, as $\Pr(y = n \mid x_0) = \binom{N}{n}[1 - q(x_0)]^n q(x_0)^{N-n}$, where $\binom{N}{n}$ is the binomial coefficient. We therefore obtain

$$\Pr(y = n \mid H_0) = \int_{-\infty}^{+\infty} \Pr(y = n \mid x_0) f_0(x_0) dx_0 . \quad (2)$$

$$\Pr(y = n \mid H_1) = \int_{-\infty}^{+\infty} \Pr(y = n \mid x_0) f_1(x_0) dx_0 . \quad (3)$$

The probability of error Per in (1) follows directly from (2) and (3), possibly through numerical integration, in broad conditions concerning the noises η_i and the input signal x.

III. NOISE CAN IMPROVE SIGNAL DETECTION IN THE ARRAY

For illustration of the possibility of SR and array improve signal detection, we choose signal x is with Gaussian PDF $f_0(x) = \frac{1}{\sqrt{2\pi}\sigma_x}\exp(-\frac{(x-\mu_0)^2}{2\sigma_x^2})$ or with Gaussian PDF $f_1(x) = \frac{1}{\sqrt{2\pi}\sigma_x}\exp(-\frac{(x-\mu_1)^2}{2\sigma_x^2})$, and noises η_i are with Gaussian PDF $f_\eta(x) = \frac{1}{\sqrt{2\pi}\sigma}\exp(-\frac{x^2}{2\sigma^2})$. We adjust the threshold level from zero to a bigger value in steps of 0.10 for different threshold number in the parallel array. Figs. 1-4 show evolutions of the probability of error Per of (1), as a function of the noise root-mean-squared (rms) amplitude σ for some typical parameters. When threshold level $\mu = P_0\mu_0 + P_1\mu_1 = 0$ and the input signal is complete suprathreshold, i.e., the signal spends a half of its time above (or below) the threshold, Fig.1 shows that noise always degrades the signal detection both in the single threshold system and in the parallel array, and SSR does not occur. When the threshold level $u \, (> P_0\mu_0 + P_1\mu_1)$ is changed from 0.10 to 2 and there are a little subthreshold ingredients in the input signal, i.e., the signal spend more time below the threshold, Figs. 2-3 show that noise also degrades signal detection in the single threshold system. However, in the parallel array ($N > 1$), noise can improve signal detection and SR occurs. This condition shares the same mechanism with previous SSR [5]-[11]. When the threshold level $u \, (\gg P_0\mu_0 + P_1\mu_1)$ is bigger than 2 and the input signal is predominant subthreshold, i.e., the signal spend many time

below the threshold, Fig. 4 shows that noise always can improve signal detection and SR always exists not only in the single threshold system but also in the parallel array. In addition, Figs. 1-4 also show that when the number of threshold devices increases, the Per always decreases and the array can improve the signal detection for any fixed threshold level and noise intensity.
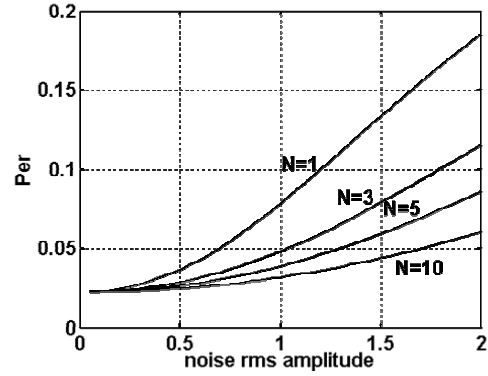

Fig. 1 Per is a function for different N. x is with PDF $f_0(x) = \frac{1}{\sqrt{2\pi}}\exp[-\frac{(x-2)^2}{2}]$ or $f_1(x) = \frac{1}{\sqrt{2\pi}}\exp[-\frac{(x+2)^2}{2}]$ and noises η_i are with PDF $f_\eta(x) = \frac{1}{\sqrt{2\pi}\sigma}\exp(-\frac{x^2}{2\sigma^2})$. The other parameters are $u_i = u = 0$, $P_0 = P_1 = 0.5$.

Fig. 2 Per is a function for different N. x is with PDF $f_0(x) = \frac{1}{\sqrt{2\pi}}\exp[-\frac{(x-2)^2}{2}]$ or PDF $f_1(x) = \frac{1}{\sqrt{2\pi}}\exp[-\frac{(x+2)^2}{2}]$, and noises η_i are with PDF $f_\eta(x) = \frac{1}{\sqrt{2\pi}\sigma}\exp(-\frac{x^2}{2\sigma^2})$. The other parameters are $u_i = u = 0.1$, $P_0 = P_1 = 0.5$.

Fig. 3 *Per* is a function for different N . x is with

$$\text{PDF } f_0(x) = \frac{1}{\sqrt{2\pi}} \exp[-\frac{(x-2)^2}{2}] \text{ or}$$

$$f_1(x) = \frac{1}{\sqrt{2\pi}} \exp[-\frac{(x+2)^2}{2}] \text{ and noises } \eta_i \text{ are with}$$

$$\text{PDF } f_\eta(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp(-\frac{x^2}{2\sigma^2}) . \text{ The other parameters}$$
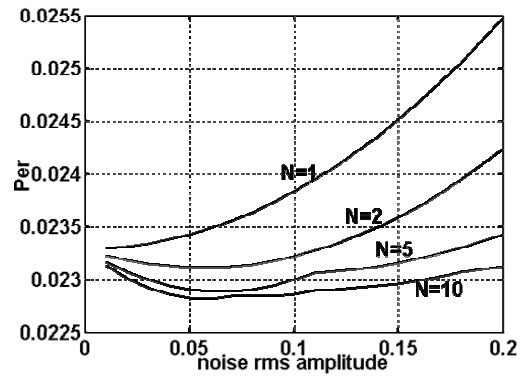
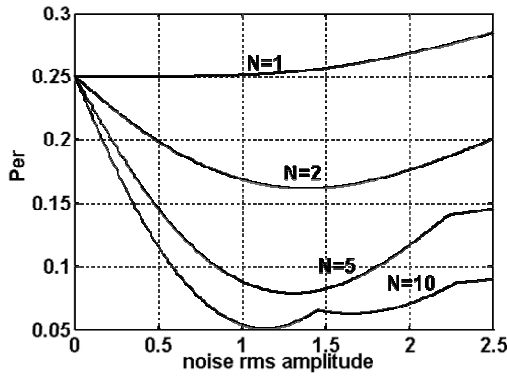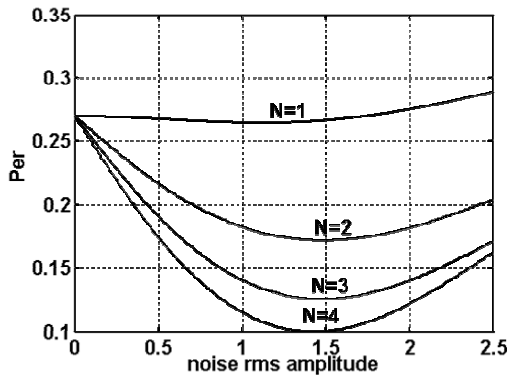$$\text{are } u_i = u = 2 , \ P_0 = P_1 = 0.5.$$

Fig. 4 *Per* is a function for different N . x is with

$$\text{PDF } f_0(x) = \frac{1}{\sqrt{2\pi}} \exp[-\frac{(x-2)^2}{2}] \text{ or}$$

$$f_1(x) = \frac{1}{\sqrt{2\pi}} \exp[-\frac{(x+2)^2}{2}] \text{ and noises } \eta_i \text{ are with}$$

$$\text{PDF } f_\eta(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp(-\frac{x^2}{2\sigma^2}) . \text{ The other parameters}$$

$$\text{are } u_i = u = 2.1 , \ P_0 = P_1 = 0.5.$$

IV. CONCLUSION

In this paper, we discuss further the signal detection through a parallel array of threshold devices. When the signal is complete suprathreshold, noise always degenerate signal detection and SSR does not occur. However, when the signal is a little subthreshold, although noise still degenerate signal detection in the single threshold system, noise can improve signal detection in the parallel array and SR occurs in the array,

where the SR mechanism is similar with previous SSR. When the signal is predominant subthreshold, SR always occurs not only in the single threshold system but also in the parallel array. Other PDFs of signal and noise (for instance, uniform PDF or other PDF) may also have the similar effects on signal detection through nonlinear threshold system.

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