

Newtonian Mechanics Descriptions for General Relativity Experimental Tests, Dark Matter and Dark Energy

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Abstract—As the continuation to the previous studies of gravitational frequency shift, gravitational time dilation, gravitational light bending, gravitational waves, dark matter, and dark energy are explained in the context of Newtonian mechanics. The photon is treated as the particle with mass of $h\nu/C^2$ under the gravitational field of much larger mass of M . Hence the quantum mechanics theory could be applied to gravitational field on cosmology scale. The obtained results are the same as those obtained by general relativity considering weak gravitational field approximation; however, the results are different when the gravitational field is substantially strong.

Keywords—Gravitational time dilation, gravitational light bending, gravitational waves, dark matter, dark energy, General Relativity, gravitational frequency shift.

I. INTRODUCTION

THE aspects of theoretical physics have provided a gateway towards the understanding of the physical phenomenon of the Universe. Through years of research, different theories have been developed to explain the experimental observations in the Universe, such as General Relativity Theory (GRT) [1]-[3], String Theory, unification of gravitational theory and most importantly, quantum mechanics [4]-[8]. With the inclusion of different renowned models, GLT and Quantum Field Theory (QFT) differ mainly because of the very understanding of the particles' behavior and incorporation of different masses and degrees as part of the respective studies.

II. BACKGROUND

A number of physicists have attempted to check the compatibility of the GRT and the QFT that can include the adaptation of the String Theory with that of the Quantum Gravity [4]-[7]. The experiments were aimed at integration of the GRT and the QFT functionalities [8]. GRT and QFT have significantly widened the very understanding of the physical world. Since the beginning, the QFT has led to the foundation of particle physics, atomic physics, nuclear physics, condensed matter physics, and quantum optics to name a few. On the other hand, GRT has led to the formulation of GPS technology, relativistic astrophysics, and cosmology.

One of the core features in quantum theory is the superposition principle that has been demonstrated through various systems, including, atoms, neutrons and molecules. As

part of quantum entanglement phenomenon, the particle loses its coherence thereby leading to different models involving gravitational time dilation and gravitational waves to be incorporated in order to completely understand the phenomenon.

III. THE APPROACH

A photon is treated as the particle with mass of $h\nu/C^2$, in which the Newtonian Mechanics is applied to the particle, with the mass as the variable entity. In [9], the author obtained the gravitational frequency shift formula for photons and particles. The results are the same as those obtained by GRT considering weak gravitational field approximation; however, the results are different when the gravitational field is substantially strong.

In this paper, the author is moving further to explain that some of other famous experimental tests for GRT, such as gravitational time dilation, gravitational light bending, and gravitational waves, can be explained with Newtonian Mechanics. Besides, the origins of the dark matter and dark energy have also been discussed along with its relevance with the Newtonian Mechanics.

IV. GRAVITATIONAL FREQUENCY SHIFT

In [9], the author has studied the gravitational frequency shifts for photons and particles using the classical Newtonian mechanics. Fig. 1 depicts the thought experimental approach as used in [9].

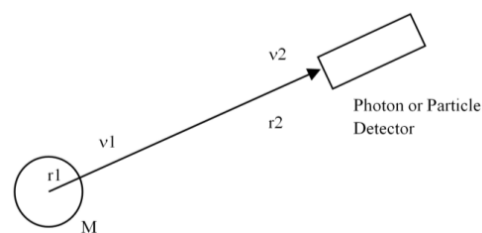


Fig. 1 Photon having frequency v_1 moving away from the surface of object having radius and mass as r_1 and M respectively. Photon detector is placed at r_2 that is capable of capturing the photon redshift [9]

Based on the classical Newtonian approach, the object having radius r_1 and mass M is assumed to be non-rotating and uncharged having spherical symmetry. The photon emission possesses the basic frequency of v_1 from the surface of object

that is traveling away and in the radial direction. Apart from that, we also assume the momentum of the photon ($P=mC$) is subject to vary within the gravitation field of object; however, the photon velocity remains as constant, which is the speed of light C . Subject to the classical Newtonian approach, following equation is obtained:

$$-\frac{dP}{dt} = -\frac{d(mV)}{dt} = -\frac{dm}{dt}V - m\frac{dV}{dt} = \frac{GmM}{r^2} \quad (1)$$

The negative (-) sign indicates that the momentum of the photon is reduced because of the gravitation force as the photon moves away from the object source. If the photon of mass $m = h\nu/C^2$ moves away from the center of a large mass M of radius r_1 with speed of C , the shifted frequency for the photon at r_2 can be computed by (1), yielding:

$$v_2 = v_1 \exp \left[\left(\frac{GM}{C^2} \right) \left(\frac{1}{r_2} - \frac{1}{r_1} \right) \right] \quad (2)$$

Similarly based on classical Newtonian mechanics, following is the equation for particles:

$$m = \frac{m_0}{\sqrt{(1 - (V/C)^2)}} \quad (3)$$

Also,

$$V = V(r) = \frac{dr}{dt} \quad (4)$$

It implies that:

$$m = m(V) = m[V(r)] \quad (5)$$

Similarly, the particles have the same frequency shift formula as that of (2). The calculation for the frequency redshift can be initiated by:

$$Z = \frac{\lambda_o - \lambda_e}{\lambda_e} \quad (6)$$

where λ_o and λ_e represent the wavelength observed and emitted pertaining to the light source. Hence, using the classical Newtonian mechanics thereby considering uncharged, spherically symmetric object of mass M having C as the speed of light given by:

$$v = \frac{c}{\lambda} \quad (7)$$

Based on the computations, the frequency redshift can be calculated under weak gravitational field approximation as in [9]:

$$Z \sim \left[\left(\frac{GM}{C^2} \right) \left(\frac{1}{r_1} \right) \right] = \left(\frac{r_s}{2r_1} \right) \quad (8)$$

where $r_s = 2GM/C^2$ is the Schwarzschild radius of M .

As evident from the calculations, the results obtained in (8) are quite identical to the results obtained through GRT in [2]. In this paper, the author is going to try to explain that (1) can

be used to explain the gravitational time dilation, gravitational light bending, and gravitational waves. As evident from the calculated results, the outcomes are the same as those obtained by GRT under weak gravitational field approximation, but different when the gravitational field is strong.

V. GRAVITATIONAL TIME DILATION

In [2], [10], the gravitational time dilation has been discussed that there is an elapsed time difference between two events as measured by observers at different distances from a gravitational mass. With the consideration of particles moving at a slower speed and weak gravitational fields, the results would be obtained through (2).

Assume that an event is occurring during the time period of ΔT in a region with no gravitational field involved. The same ΔT can be counted at r_2 or r_1 from the center of mass M .

$$\Delta T = N_2 \delta t_2 = N_1 \delta t_1 \quad (9)$$

If light photons with frequencies v_1, v_2 are used as the clocks,

$$\delta t_1 = \frac{1}{v_1} \quad (10)$$

$$\delta t_2 = \frac{1}{v_2} \quad (11)$$

From (2), the solution to the time dilation problem in Schwarzschild space for a non-rotating mass M can be given by

$$\delta t_2 = \delta t_1 / \exp \left[\left(\frac{GM}{C^2} \right) \left(\frac{1}{r_2} - \frac{1}{r_1} \right) \right] \quad (12)$$

For $r_2 \gg r_1$, on comparison to GRT solution [2], we get:

$$\delta t_2 = \frac{\delta t_1}{\sqrt{1 - \frac{2GM}{r_1 c^2}}} \quad (13)$$

Under the weak field approximation, it is implied that:

$$\frac{2GM}{r_1 c^2} \leq 1$$

Both of the theories get the same results

$$\delta t_2 = \delta t_1 \left(1 + \frac{GM}{r_1 c^2} \right) \quad (14)$$

From (9) and (14),

$$N_2 = \frac{N_1}{\left(1 + \frac{GM}{r_1 c^2} \right)} \quad (15)$$

Therefore, the observer at r_2 will feel time moves faster than that at r_1 ($N_2 < N_1$).

VI. GRAVITATIONAL LIGHT BENDING

In GRT, light follows the curvature of space-time. The light will be bent when it passes around a massive object M [2]. The deflection angle:

$$\theta = \frac{4GM}{rc^2} = 2\frac{r_s}{r} \quad (16)$$

In [11], [12], the authors have shown that the Mahajan and Yarman's Approaches can be used for the prediction of the gravitational lens with half of the deflection angle compared to that of the GRT; however, they are based on quite a different philosophy. Originally, the Yarman's approach has employed Fermat Principle for the derivation of the deflection angles in contrast to the Newtonian Deflection [12]. The authors obtained the light bending formula by adaptation of Yarman's approach that relies on the momentum vector's deflection of the photon because of the gravitational pull. The approach can greatly enhance the very understanding of the physics and mathematics for the light bending phenomenon.

In this paper, the author is trying to prove that the full deflection angle by a photon with mass of $h\nu/C^2$ passing over large mass M can be derived from the Newtonian Mechanics equation (1), and the result is the same as that of (16) obtained from GRT [2]:

From [9], it can be shown that:

$$m = m_0 \exp\left[\frac{GM}{c^2}\left(\frac{1}{R} - \frac{1}{R_0}\right)\right] \approx m_0 \left[1 + \frac{GM}{c^2}\left(\frac{1}{R} - \frac{1}{R_0}\right)\right] \quad (17)$$

$$-\frac{dm}{dt}V - m\frac{dV}{dt} = \frac{GmM}{R^2} \approx \frac{GM}{R^2}m_0 \left[1 + \frac{GM}{c^2}\left(\frac{1}{R} - \frac{1}{R_0}\right)\right] \quad (18)$$

$$m\frac{dV_{vr}}{dt} \approx -\frac{GMm_0r}{R^3} \quad (19)$$

or

$$\frac{dV_{vr}}{dt} \approx -\frac{GMr}{R^3} \quad (20)$$

where

$$R = \sqrt{r^2 + x^2} \quad \text{and} \quad \cos(\Phi) = \frac{r}{R} \quad (21)$$

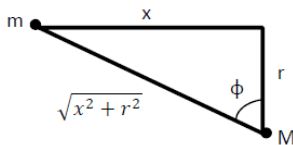


Fig. 2 Photon m is passing mass M along x direction, and will be bent along r direction

Hence, from [11], the downwards fraction of the acceleration to the center of M is

$$a_r(x) = \frac{Gmr}{(r^2 + x^2)^{3/2}} \quad (22)$$

The downward velocity is

$$dV_{vr} = a_r(x) \times \text{time to cross } dx = a_r(x)dx/c \quad (23)$$

Taking the contributions from the whole path, the downward velocity can be given by:

$$V_{vr} = \int_{-\infty}^{\infty} \frac{Gmr}{c(r^2 + x^2)^{3/2}} dx \quad (24)$$

Having $m \approx m_0$ approximation, following result can be obtained:

$$\alpha_v \approx \frac{V_{vr}}{c} = \frac{GM}{c^2r} \int_{-\pi/2}^{\pi/2} \cos\Phi d\Phi = \frac{2GM}{c^2r} \quad (25)$$

On the other hand, from (18), it can be deduced that:

$$\frac{dm}{dt}V_{mr} = -\frac{GM}{R^3}r \left[\frac{GM}{c^2}\left(\frac{1}{R} - \frac{1}{R_0}\right)\right] \quad (26)$$

By taking the derivatives on both sides, and ignore the higher order derivatives, it implies that:

$$\frac{d^2m}{dt^2}V_{mr} \sim 0 \quad \text{and} \quad \frac{GM^2m_0}{c^2R^4} \sim 0$$

We get:

$$\frac{dm}{dt}\frac{dV_{mr}}{dt} = -\frac{GMr}{R^3}m_0 \left[\frac{GM}{c^2}\left(-\frac{1}{R^2}\right)\right]\frac{dR}{dt} \quad (27)$$

On further simplification, we get:

$$\frac{dV_{mr}}{dt} = \frac{-\frac{GMr}{R^3}m_0 \left[\frac{GM}{c^2}\left(-\frac{1}{R^2}\right)\right]\frac{dR}{dt}}{m_0 \frac{GM}{c^2}\left(-\frac{1}{R^2}\right)\frac{dR}{dt}} = \frac{GMr}{R^3} \quad (28)$$

Equation (28) is similar to (20). Hence, it can be estimated that:

$$\alpha_m \approx \frac{V_{mr}}{c} = \frac{2GM}{c^2r} \quad (29)$$

The total deflection angle can be given by:

$$\alpha = \alpha_m + \alpha_v = \frac{4GM}{c^2r} \quad (30)$$

In (30), the attained results are quite identical to the one obtained from (16) as part of GRT analysis [2].

VII. DARK MATTER AND DARK ENERGY

Dark matter, as a hypothesized matter, is thought to account for approximately 85% of the matter in the Universe. Some cosmology observations cannot be explained unless more matter is present than can be seen, which provides more gravitational effects. Dark matter is invisible and is very difficult to be detected [13].

Dark energy is the hypothesized unknown form of energy, which exists in the Universe, and tends to accelerate the expansion of the Universe [14], [15].

In this paper, the author is going to explain that the dark matter and dark energy can be related with the photon or particle gravitational frequency shifts.

Let's assume that a photon of mass m is emitted away from the surface of the object with mass M and radius of r_0 . At

distance r , the photon will have gravitational redshift that implies that the photon lose energy as highlighted by [9]:

$$m_r C^2 - m_0 C^2 = m_0 \left\{ \exp \left[\frac{GM}{c^2} \left(\frac{1}{r} - \frac{1}{r_0} \right) \right] - 1 \right\} C^2 \quad (31)$$

Under the weak gravitational field approximation

$$m_r C^2 - m_0 C^2 \approx m_0 C^2 \left\{ 1 + \left[\frac{GM}{c^2} \left(\frac{1}{r} - \frac{1}{r_0} \right) \right] - 1 \right\} = m_0 C^2 \left[\frac{GM}{c^2} \left(\frac{1}{r} - \frac{1}{r_0} \right) \right] \quad (32)$$

By the law of conservation of energy, M with its gravitational field will gain the same amount of energy from the photon m . However, the gained energy is spread over the course of $(r - r_0)$:

$$\Delta E_{dm} = \int_{r_0}^r d \left\{ -m_0 C^2 \left[\frac{GM}{c^2} \left(\frac{1}{r} - \frac{1}{r_0} \right) \right] \right\} \quad (33)$$

The individual quantum frequency is too low to be observed, however, the mass summation of all quantum gained will contribute to the gravitational field for object M . Moreover, it will also be considered as the dark matter (invisible, with extra gravitational field). Similarly, if the photon of mass m is moving toward the object M from distance of r_1 to r_2 , there will be gravitational blueshift, having the photon gaining energy that is given by:

$$m_1 \left\{ \exp \left[\frac{GM}{c^2} \left(\frac{1}{r_2} - \frac{1}{r_1} \right) \right] - 1 \right\} C^2 \sim m_1 C^2 \left[\frac{GM}{c^2} \left(\frac{1}{r_2} - \frac{1}{r_1} \right) \right] \quad (34)$$

By conservation of energy, M with its gravitational field will lose the same amount of energy to the photon m . Since the lost energy is spread over the course of $(r_2 - r_1)$:

$$\Delta E_{de} = \int_{r_1}^{r_2} d \left\{ -m_1 C^2 \left[\frac{GM}{c^2} \left(\frac{1}{r_2} - \frac{1}{r_1} \right) \right] \right\} \quad (35)$$

The individual quantum frequency is too low to be observed, but the mass summation of all quanta will be emitted away from the gravitational field of object M . They can be considered as the dark energy propagating away from the gravitational field of M .

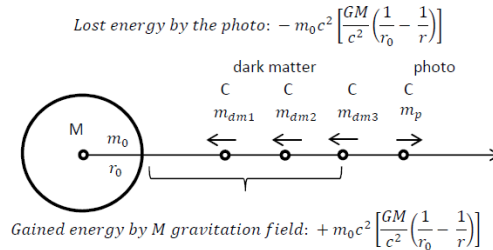


Fig. 3 Dark matter generated by photon with gravitational red shift

In [9], it is proved that the particles will depict the similar frequency shift equations, thereby making the discussion pertaining to dark matter, dark energy quite similar in nature.

Based on the result obtained from (34), (35), the collision of two large objects (binary star or black hole system) would result in the release of large amount of dark energy that travels with the speed C as in the case of photons and/or particles. The radiation can be generated from the dipole, or quadrupole or higher order perturbation system, since the radiation of the dark energy is from the photon or particle gravitational blueshift and their orbiting is reducing, eventually collide together.

Dark energy can also be associated with the gravitational waves. It will be discussed in Section VIII.

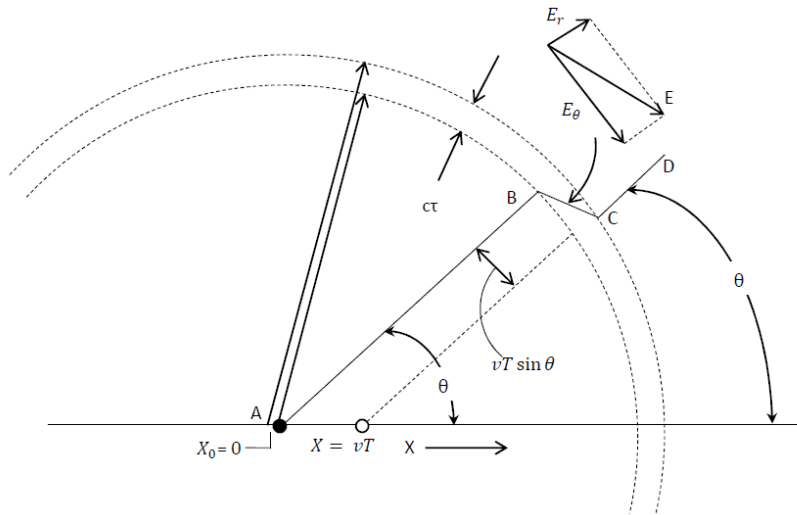


Fig. 4 Object M travels along X direction with acceleration a , from 0 to $\frac{1}{2}vT$, and then to vT . The gravitational field line will draw from A to B to C to D

VIII. GRAVITATIONAL WAVES

In GRT, the curvature of spacetime will cause the gravitational forces between two massive bodies.

Gravitational waves, traveling with the speed of light, are propagating disturbances in the curvature of spacetime, caused by some violent and energetic physical processes in the Universe.

In this paper, the author explains that the gravitational waves can be generated by accelerated massed particles, similarly to the generation of electromagnetic waves by accelerated charged particles [16], with the assumption that the gravitational field perturbation will travel with the speed of light C.

As evident from Fig. 4, suppose that an object with mass M is accelerated over a short time τ , along X direction from $X_0 = 0$ to $X_\tau = \frac{1}{2}v\tau$, where $v \ll C$, $v \sim a\tau$. Later at time T, reaches to $X = Vt$, $T = r/C$. The perturbation of the gravitational field will travel with the speed of light C, and the gravitational field line will draw from point A to B to C to D, therefore the following equation can be obtained:

$$\frac{E_\theta}{E_r} = \frac{vT \sin \theta}{C\tau} = \frac{ar \sin \theta}{C^2} \quad (36)$$

where:

$$E_r = \frac{GM}{r^2} \quad (37)$$

It implies that:

$$E_\theta = \frac{GM}{r^2} * \frac{ar}{C^2} \sin \theta = \frac{GMa \sin \theta}{rC^2} \quad (38)$$

For a binary or quadrupole system [17], [18], where two stars or black holes are orbiting around their center of mass, assuming that each with mass of M, angular frequency of ω , radius to the center of mass R, at distance r, the accelerations along x and y directions can be given by:

$$a_x = -R\omega^2 \cos[\omega(t - \frac{r}{c})] \quad (39)$$

Also,

$$a_y = -R\omega^2 \sin[\omega(t - \frac{r}{c})] \quad (40)$$

As witnessed in Section VII, the energy release or absorption, depend on compression in one direction, along with simultaneously stretching the separation in the orthogonal direction. The equivalent mass from the released or absorbed energy can be

$$mC^2 = m_x C^2 = -m_y C^2 \quad (41)$$

As they irradiate gravitational waves (and thus energy) throughout their orbit, the radius of orbit decreases until the stars/black holes collide. Hence, at distance r, the radiation energy for mass $m_x = m$ along x direction and $m_y = -m$ along

y directions can be given by:

$$\begin{aligned} Energy_{\theta x} &= 2 \int_0^x m E_{\theta x} dx = -2 \int_0^{\omega(t-\frac{r}{c})} m \frac{GM \sin \theta}{rC^2} R \omega^2 \cos[\omega(t - \frac{r}{c})] d[\omega(t - \frac{r}{c})] \\ &= m \frac{GM \sin \theta}{rC^2} R^2 \omega^2 \{1 - \cos^2[\omega(t - \frac{r}{c})]\} = \\ &= m \frac{GM \sin \theta}{2rC^2} R^2 \omega^2 [1 - \cos[2\omega(t - \frac{r}{c})]] \quad (42) \end{aligned}$$

Similarly,

$$\begin{aligned} Energy_{\theta y} &= 2 \int_0^y (-m) E_{\theta y} dy = \\ &= -(-m) \frac{GM \sin \theta}{rC^2} R^2 \omega^2 \sin^2[\omega(t - \frac{r}{c})] = m \frac{GM \sin \theta}{2rC^2} R^2 \omega^2 \{1 - \cos[2\omega(t - \frac{r}{c})]\} \quad (43) \end{aligned}$$

The approach and solution are similar to those for accelerated charged particle radiation as evidenced from [16]. The magnitude of strains at distance r and time t on a released photon or particle energy of mC^2 can be given by:

$$\begin{aligned} h &= \frac{Energy_\theta}{2mC^2} = \frac{Energy_{\theta x} + Energy_{\theta y}}{2mC^2} = \\ \frac{\Delta v}{v} &= \frac{GM R^2 \omega^2 \{1 - \cos[2\omega(t - \frac{r}{c})]\} \sin \theta}{rC^4} = \frac{\Delta L}{L} \quad (44) \end{aligned}$$

On 11 February 2016, LIGO and Virgo have announced the first observation of gravitational waves. For the signal named as GW150914 from the merge of a binary black hole system observed on earth [19], [20], the gravitational wave amplitude of strains h_{cal} can be estimated from (44)

$$h_{cal} \sim 0.98 * 10^{-21} \quad (45)$$

Here, the gravitational constant G is $\sim 6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$, for the binary black hole system of GW150914, the average

black hole mass M is $\sim 35 M_\odot$ (solar mass) $\sim 70 \times 10^{30} \text{ kg}$, the orbital separation R is $\sim 350 \text{ km}$, the angular orbiting frequency ω is $\sim 2\pi \times 75 \text{ Hz}$, the distance to earth r is ~ 1.9 billion light year $\sim 1.8 \times 10^{25} \text{ meter}$, and the speed of light C is $\sim 3 \times 10^8 \text{ m/sec}$.

The calculated h_{cal} value is close to that published from LIGO and Virgo observations [19] with $h \sim 1 * 10^{-21}$. For 4km arm of LIGO apparatus [17], [20], the estimated distortion is $\Delta L \sim 4 * 10^{-17} \text{ meters}$. The distortion is quite small considering that the radius of proton is only $\sim 10^{-15} \text{ meters}$.

IX. SUMMARY AND FURTHER STUDIES

In this paper, the author is assuming that the photon can be treated as the particle with variable mass of $h\nu/C^2$, similar to particles. Thus, some of the famous experimental tests for GRT can be explained with Newtonian Mechanics:

1. The photon or particle will have gravitational frequency shifts under a large mass of M gravitational field, described as the quantum energy loss for redshift or quantum energy gain for blueshift;
2. Gravitational time dilation can be explained if we assume that the counting clock frequency has different gravitational frequency shifts at different locations respect

to the center of M;

3. When a photon is passing through an object with mass M, the photon will be bent by M gravitational field, also will have gravitational frequency shift or equivalent mass changes;
4. Similar to radiation waves can be generated by accelerated charge particles, gravitational waves can be generated by accelerated particles with mass of M. The obtained solution has the similar $1/r$ dependency. The amplitude of strains for the energy propagating waves (gravitational waves) can be estimated to be closed to those from GRT, LIGO, and Virgo observations.

Newtonian Mechanics can also be used to explain the generation of dark matter (when photons or particles have gravitational frequency redshift), and the generation of dark energy (when photons or particles have gravitational frequency blueshift).

GRT is based on the similar forms of the Principle of Least Action (in the form of "geodesics" [2]), with the assumption that the gravitational mass is equal to the acceleration mass, and the 4 dimensional spacetime domain warps with mass. Newtonian mechanics results can be obtained through the directives of equations based on the Lagrangian mechanics, which starts with the Principle of Least Action, with variable mass for photons and particles. It could be a sound evidence pertaining to the similar results, however, it cannot be exactly the same for both theories.

In the future, the author is planning to apply similar Newtonian Mechanics approaches to other gravitational effects, Cosmology observations, and the expansion of the Universe, and more detailed analyses for dark matter and dark energy. The author is also interested in the integration of gravitational field theories with those of Quantum Mechanics with Newtonian Mechanics approaches.

X.CONCLUSION

In this paper, the author assumes that the photon can be treated as the particle with variable mass of $h\nu/C^2$, similar to particles, thus Newtonian Mechanics can be used to explain some of the famous experimental tests for GRT, such as gravitational frequency shifts, gravitational time dilation, gravitational light bending, and gravitational waves. The results are the same as those obtained with GRT under the weak gravitational field assumption, but different when the gravitational field is significantly strong. The potential application of this approach will bring the possibility of integrating GRT and Quantum Mechanics together. Also, the origins of the dark matter and dark energy are explained with this approach.

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