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New scheme in determining nth order diagrams for cross multiplication method via combinatorial approach

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Abstract—In this paper, a new recursive strategy is proposed for determining $\frac{(n-1)!}{2}$ of nth order diagrams. The generalization of nth diagram for cross multiplication method were proposed by Pavlovic and Bankier but the specific rule of determining $\frac{(n-1)!}{2}$ of the nth order diagrams for square matrix is yet to be discovered. Thus using combinatorial approach, $\frac{(n-1)!}{2}$ of the nth order diagrams will be presented as $\frac{(n-1)!}{2}$ starter sets. These starter sets will be generated based on exchanging one element. The advantages of this new strategy are the discarding process was eliminated and the sign of starter set is alternated to each others.

Keywords—starter sets, permutation, exchanging one element, determinant.

I. Introduction

The most commonly used techniques for finding determinant are cross multiplication, cofactor expansion, and Gauss elimination. All these techniques were discussed in great details in many textbooks such as [1],[2], [4], [5], [8], [10]-[12]. Among these three methods, cross multiplication method works only for the order of matrix $n \leq 3$. Then Bankier[3] and Pavlovic[7] attempted to generalise cross multiplication method. However they failed to generate a specific rule for determining $\frac{(n-1)!}{2}$ of the nth order diagram.
Thus in this paper, we tackled the determining $\frac{(n-1)!}{2}$ for the nth order diagram problem by applying a new technique which based on our new permutation method[9].
This paper is organized as follows. Section 2 provides an problem formulation for determining $\frac{(n-1)!}{2}$ of the nth order diagrams. In section 3, some preliminaries definition are provided. We propose a new strategy for starter sets generation and the general algorithm is given in section 4. The construction of nth order diagram is provided in section 5. Finally, the conclusion is in section 6.

II. Problem formulation

The nth order diagram for any $n \times n$ matrix is developed by appending the first $(n-1)$ column to the right of that matrix. Thus based on [7], there are $\frac{(n-1)!}{2}$ of the nth order diagram for matrix order n.

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Now we explore the cross multiplication method for case $n = 3$ with single 3rd order diagram.

$$det(A) = \left|\begin{array}{ccc|cc} a_{11} & a_{12} & a_{13} & a_{11} & a_{12} \\ a_{21} & a_{22} & a_{23} & a_{21} & a_{22} \\ a_{31} & a_{32} & a_{33} & a_{31} & a_{32} \end{array}\right|$$

$= [a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32}] - [a_{13}a_{22}a_{31} + a_{11}a_{23}a_{32} + a_{12}a_{21}a_{31}]$.
where the number of the terms (diagonals) are $3! = 6 = 2 \times 3$. In other hand, any nth diagram, the number of terms is $2n$. Then when all $\frac{(n-1)!}{2}$ of the nth order diagram were constructed, the number of terms are $\frac{(n-1)!}{2} \times 2n = n!$ which is follow to Leibniz formula [6]:

$$det(A) = \sum_{(\sigma(1),\sigma(2),\ldots,\sigma(n)) \in S_n} sign(\sigma) \cdot a_{1\sigma(1)} \cdot a_{2\sigma(2)} \cdots a_{n\sigma(n)}.$$

The summation is the set of all $n!$ permutations σ of n elements.
Consider case $n = 4$, there are three distinct of 4th order diagrams.
Example 2.1:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$$

The three of 4th diagrams where the first $(n-1)$ columns are append to the right of matrices :

$$\left|\begin{array}{cccc|ccc} a_{11} & a_{12} & a_{13} & a_{14} & a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{41} & a_{42} & a_{43} \end{array}\right|$$

$$\left|\begin{array}{cccc|ccc} a_{11} & a_{13} & a_{14} & a_{12} & a_{11} & a_{13} & a_{14} \\ a_{21} & a_{23} & a_{24} & a_{22} & a_{21} & a_{23} & a_{24} \\ a_{31} & a_{33} & a_{34} & a_{32} & a_{31} & a_{33} & a_{34} \\ a_{41} & a_{43} & a_{44} & a_{42} & a_{41} & a_{43} & a_{44} \end{array}\right|$$

$$\left|\begin{array}{cccc|ccc} a_{11} & a_{14} & a_{12} & a_{13} & a_{11} & a_{14} & a_{12} \\ a_{21} & a_{24} & a_{22} & a_{23} & a_{21} & a_{24} & a_{22} \\ a_{31} & a_{34} & a_{32} & a_{33} & a_{31} & a_{34} & a_{32} \\ a_{41} & a_{44} & a_{42} & a_{43} & a_{41} & a_{44} & a_{42} \end{array}\right|$$

Every 4th diagram consists 8 terms (diagonals). Then the total of terms are $8 \times 3 = 24 = 4!$. The sign of each terms was discussed well in [3] and [7]. The main problem was determining $\frac{(n-1)!}{2}$ of the nth order diagrams.

Fixing the first column of the matrix, then permuting the remaining columns in all possible ways and discarding any permutation which is generated the similar nth diagram was suggested by [3]. The permutations were discussed represent as the column indices of the matrix. Pavlovic [7] also proposed the similar way. This procedure is easy for hand-computing. However discarding the permutation which is a permutation has already been written in reverse order becomes tedious when the number of n incremented. Other from that, they never develop the algorithm and compared to the existing method (cofactor expansion)
Thus, before $\frac{(n-1)!}{2}$ of nth order diagrams are constructed, we need to list all $\frac{(n-1)!}{2}$ matrix which can be generated based on $\frac{(n-1)!}{2}$ distinct starter sets. Then in order to generate that starter sets, we will employ the exchanging one element technique for generating the starter sets inductively from $(n-2)th$ element until 2nd element selected.

III. Preliminary definitions

In this section, we started by providing some definitions that will be needed in the method derivation.

Definition 3.1: A starter set is a set that is used as a basis to enumerate other permutations.

Definition 3.2: An equivalence starter set is a set that can produce the same permutation from other starter set.
We name the permutation which is a permutation had already written in reverse order except the first column, as equivalence starter sets.

Definition 3.3: A starter set matrix is generated matrix based on starter set.

IV. Starter sets generation

Let S be the set of n elements such that $S = \{1, 2, 3, ..., n\}$. Cases $n = 2$ and 3 are trivial. In this section, we illustrate how the exchanging procedure works for cases $n = 4$ and $n = 5$. The procedure is constructed recursively.

A. Exchanging one element strategy

Case $n = 4$

Step 1: **Fix the first element**.
Permutation $\{1, 2, 3, 4\}$ is produced.

Step 2: **Determine starter sets**.
Identify the element in the $(n-2)$th position i.e element '2'. Exchange this element until it reaches the nth (last) position (see Table I). We produce three distinct starter sets.

TABLE I
List of starter sets for $n = 4$

1	2	3	4
1	3	2	4
1	3	4	2

Case $n = 5$

Step 1: **Fix the first element**.
Permutation $\{1, 2, 3, 4, 5\}$ is produced.

Step 2 : Identify the element in the $(n-2)$th position i.e element '3'. Exchange this element until it reaches the nth (last)position (see Table II). We produce three distinct starter sets.

TABLE II
Starter sets from the exchange of the $(n-2)$th element

1	2	3	4	5
1	2	4	3	5
1	2	4	5	3

Step 3 : Identify the element in the $(n-3)$th position i.e '2' in each starter sets from Step 2. Exchange this element until it reaches the nth (last)position (see Table III). We produce other 12 distinct starter sets.

TABLE III
All starter set for $n = 5$

1	2	3	4	5
1	3	2	4	5
1	3	4	2	5
1	3	4	5	2
1	2	4	3	5
1	4	2	3	5
1	4	3	2	5
1	4	3	5	2
1	2	4	5	3
1	4	2	5	3
1	4	5	2	3
1	4	5	3	2

B. General recursive algorithm

General algorithm for generate $\frac{(n-1)!}{2}$ starter sets as followed

Step 1 : Set $1, 2, 3, 4, \ldots, k, k+1, \ldots, n-2, n-1, n$ as initial permutation and without loss of generality, the first element is fixed.

Step 2 : Identify the element in the $(n-2)$th position of the initial permutation in step 1. Exchange this element until it reaches the nth (last) position. Hereby three distinct starter sets are obtained.

Step 3 : Identify the element in the $(n-3)$th position of the initial permutation in step 2. Exchange this element until it reaches the n th (last) position. Hereby 12 distinct starter sets are obtained.

$\vdots$

Step n-2 : Identify the element of in the 2nd position of each starter sets in step (n-3). Exchange this element until it reaches the nth (last) position At this step, the $\frac{(n-1)!}{2}$ distinct starter sets are obtained.

The main idea is select the element in the $(n-2)$th until 2nd position. Since the each step is quite similar, this algorithm is developed in recursive procedure.

V. Construction of nth order diagrams based on starter sets

In this section, we generate the $\frac{(n-1)!}{2}$ of nth order diagram from $\frac{(n-1)!}{2}$ distinct starter sets matrices. Each $\frac{(n-1)!}{2}$ of nth order diagram is developed by rewriting the first $(n-1)$ columns to the right of $\frac{(n-1)!}{2}$ starter set matrices. We illustrate with the Example 5.1.

Example 5.1: Let $n = 5$ and 12 starter sets matrices constructed from Table III. The list of 5th order diagrams are given:

$$\left|\begin{array}{ccccc|cccc} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{41} & a_{42} & a_{43} & a_{44} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{51} & a_{52} & a_{53} & a_{54} \end{array}\right|$$

$$\left|\begin{array}{ccccc|cccc} a_{11} & a_{13} & a_{12} & a_{14} & a_{15} & a_{11} & a_{13} & a_{12} & a_{14} \\ a_{21} & a_{23} & a_{22} & a_{24} & a_{25} & a_{21} & a_{23} & a_{22} & a_{24} \\ a_{31} & a_{33} & a_{32} & a_{34} & a_{35} & a_{31} & a_{33} & a_{32} & a_{34} \\ a_{41} & a_{43} & a_{42} & a_{44} & a_{45} & a_{41} & a_{43} & a_{42} & a_{44} \\ a_{51} & a_{53} & a_{52} & a_{54} & a_{55} & a_{51} & a_{53} & a_{52} & a_{54} \end{array}\right|$$

$$\left|\begin{array}{ccccc|cccc} a_{11} & a_{13} & a_{14} & a_{12} & a_{15} & a_{11} & a_{13} & a_{14} & a_{12} \\ a_{21} & a_{23} & a_{24} & a_{22} & a_{25} & a_{21} & a_{23} & a_{24} & a_{22} \\ a_{31} & a_{33} & a_{34} & a_{32} & a_{35} & a_{31} & a_{33} & a_{34} & a_{32} \\ a_{41} & a_{43} & a_{44} & a_{42} & a_{45} & a_{41} & a_{43} & a_{44} & a_{42} \\ a_{51} & a_{53} & a_{54} & a_{52} & a_{55} & a_{51} & a_{53} & a_{54} & a_{52} \end{array}\right|$$

$$\left|\begin{array}{ccccc|cccc} a_{11} & a_{13} & a_{14} & a_{15} & a_{12} & a_{11} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{23} & a_{24} & a_{25} & a_{22} & a_{21} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{33} & a_{34} & a_{35} & a_{32} & a_{31} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{43} & a_{44} & a_{45} & a_{42} & a_{41} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{53} & a_{54} & a_{55} & a_{52} & a_{51} & a_{53} & a_{54} & a_{55} \end{array}\right|$$

The first fours of 5th order diagram are generated based on result in first four rows in Table III. Then the next fours are generated from second four rows in Table III.

$$\left|\begin{array}{ccccc|cccc} a_{11} & a_{12} & a_{14} & a_{13} & a_{15} & a_{11} & a_{12} & a_{14} & a_{13} \\ a_{21} & a_{22} & a_{24} & a_{23} & a_{25} & a_{21} & a_{22} & a_{24} & a_{23} \\ a_{31} & a_{32} & a_{34} & a_{33} & a_{35} & a_{31} & a_{32} & a_{34} & a_{33} \\ a_{41} & a_{42} & a_{44} & a_{43} & a_{45} & a_{41} & a_{42} & a_{44} & a_{43} \\ a_{51} & a_{52} & a_{54} & a_{53} & a_{55} & a_{51} & a_{52} & a_{54} & a_{53} \end{array}\right|$$

$$\left|\begin{array}{ccccc|cccc} a_{11} & a_{14} & a_{12} & a_{13} & a_{15} & a_{11} & a_{14} & a_{12} & a_{13} \\ a_{21} & a_{24} & a_{22} & a_{23} & a_{25} & a_{21} & a_{24} & a_{22} & a_{23} \\ a_{31} & a_{34} & a_{32} & a_{33} & a_{35} & a_{31} & a_{34} & a_{32} & a_{33} \\ a_{41} & a_{44} & a_{42} & a_{43} & a_{45} & a_{41} & a_{44} & a_{42} & a_{43} \\ a_{51} & a_{54} & a_{52} & a_{53} & a_{55} & a_{51} & a_{54} & a_{52} & a_{53} \end{array}\right|$$

$$\left|\begin{array}{ccccc|cccc} a_{11} & a_{14} & a_{13} & a_{12} & a_{15} & a_{11} & a_{14} & a_{13} & a_{12} \\ a_{21} & a_{24} & a_{23} & a_{22} & a_{25} & a_{21} & a_{24} & a_{23} & a_{22} \\ a_{31} & a_{34} & a_{33} & a_{32} & a_{35} & a_{31} & a_{34} & a_{33} & a_{32} \\ a_{41} & a_{44} & a_{43} & a_{42} & a_{45} & a_{41} & a_{44} & a_{43} & a_{42} \\ a_{51} & a_{54} & a_{53} & a_{52} & a_{55} & a_{51} & a_{54} & a_{53} & a_{52} \end{array}\right|$$

$$\left|\begin{array}{ccccc|cccc} a_{11} & a_{14} & a_{13} & a_{15} & a_{12} & a_{11} & a_{14} & a_{13} & a_{15} \\ a_{21} & a_{24} & a_{23} & a_{25} & a_{22} & a_{21} & a_{24} & a_{23} & a_{25} \\ a_{31} & a_{34} & a_{33} & a_{35} & a_{32} & a_{31} & a_{34} & a_{33} & a_{35} \\ a_{41} & a_{44} & a_{43} & a_{45} & a_{42} & a_{41} & a_{44} & a_{43} & a_{45} \\ a_{51} & a_{54} & a_{53} & a_{55} & a_{52} & a_{51} & a_{54} & a_{53} & a_{55} \end{array}\right|$$

Then for the last fours are

$$\left|\begin{array}{ccccc|cccc} a_{11} & a_{12} & a_{14} & a_{15} & a_{13} & a_{11} & a_{12} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{24} & a_{25} & a_{23} & a_{21} & a_{22} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{34} & a_{35} & a_{33} & a_{31} & a_{32} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{44} & a_{45} & a_{43} & a_{41} & a_{42} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{54} & a_{55} & a_{53} & a_{51} & a_{52} & a_{54} & a_{55} \end{array}\right|$$

$$\left|\begin{array}{ccccc|cccc} a_{11} & a_{14} & a_{12} & a_{15} & a_{13} & a_{11} & a_{14} & a_{12} & a_{15} \\ a_{21} & a_{24} & a_{22} & a_{25} & a_{23} & a_{21} & a_{24} & a_{22} & a_{25} \\ a_{31} & a_{34} & a_{32} & a_{35} & a_{33} & a_{31} & a_{34} & a_{32} & a_{35} \\ a_{41} & a_{44} & a_{42} & a_{45} & a_{43} & a_{41} & a_{44} & a_{42} & a_{45} \\ a_{51} & a_{54} & a_{52} & a_{55} & a_{53} & a_{51} & a_{54} & a_{52} & a_{55} \end{array}\right|$$

$$\left|\begin{array}{ccccc|cccc} a_{11} & a_{14} & a_{15} & a_{12} & a_{13} & a_{11} & a_{14} & a_{15} & a_{12} \\ a_{21} & a_{24} & a_{25} & a_{22} & a_{23} & a_{21} & a_{24} & a_{25} & a_{22} \\ a_{31} & a_{34} & a_{35} & a_{32} & a_{33} & a_{31} & a_{34} & a_{35} & a_{32} \\ a_{41} & a_{44} & a_{45} & a_{42} & a_{43} & a_{41} & a_{44} & a_{45} & a_{42} \\ a_{51} & a_{54} & a_{55} & a_{52} & a_{53} & a_{51} & a_{54} & a_{55} & a_{52} \end{array}\right|$$

$$\left|\begin{array}{ccccc|cccc} a_{11} & a_{14} & a_{15} & a_{13} & a_{12} & a_{11} & a_{14} & a_{15} & a_{13} \\ a_{21} & a_{24} & a_{25} & a_{23} & a_{22} & a_{21} & a_{24} & a_{25} & a_{23} \\ a_{31} & a_{34} & a_{35} & a_{33} & a_{32} & a_{31} & a_{34} & a_{35} & a_{33} \\ a_{41} & a_{44} & a_{45} & a_{43} & a_{42} & a_{41} & a_{44} & a_{45} & a_{43} \\ a_{51} & a_{54} & a_{55} & a_{53} & a_{52} & a_{51} & a_{54} & a_{55} & a_{53} \end{array}\right|$$

The process of determining sign for each diagonals, one can follows the rule which was formulated in [3] and [7]. We only highlight the determining sign of each starter sets. Since the strategy is based on exchanging one element, the sign of starter set is alternated to previous starter set. The detail in Table IV.

TABLE IV
THE SIGN OF ALL STARTER SET FOR $n = 5$

					sign
1	**2**	**3**	**4**	**5**	+
1	3	2	4	5	−
1	3	4	2	5	+
1	3	4	5	2	−
1	**2**	**4**	**3**	**5**	−
1	4	2	3	5	+
1	4	3	2	5	−
1	4	3	5	2	+
1	**2**	**4**	**5**	**3**	+
1	4	2	5	3	−
1	4	5	2	3	+
1	4	5	3	2	−

Remark 5.2: The **bold** mark of the starter sets in Table IV shows that their signs are also alternated to each other.
Since the specific scheme for determining $\frac{(n-1)!}{2}$ of nth order diagram was successfully developed, determining the determinant via cross multiplication for any $n \times n$ matrix become possible.

VI. Conclusion

The advantage of this new recursive strategy is the equivalence starter sets are not generated. Only the starter sets are obtained. On the other hand, in process constructing the $\frac{(n-1)!}{2}$ of nth order diagram, the discarding equivalence starter sets are not occur. Then it will be contributed to time computation reduction. Thus we can claim that our work has improved the [3] and [7] methods. For the future research, a new algorithm will be designed for parallel computation.

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