International Journal of Electrical, Electronic and Communication Sciences
ISSN: 2517-9438
Vol:2, No:1, 2008

Near Perfect Reconstruction Quadrature Mirror Filter

A. Kumar, G. K. Singh, and R. S. Anand

***Abstract*—**In this paper, various algorithms for designing quadrature mirror filter are reviewed and a new algorithm is presented for the design of near perfect reconstruction quadrature mirror filter bank. In the proposed algorithm, objective function is formulated using the perfect reconstruction condition or magnitude response condition of prototype filter at frequency ($\omega = 0.5\pi$) in ideal condition. The cutoff frequency is iteratively changed to adjust the filters coefficients using optimization algorithm. The performances of the proposed algorithm are evaluated in term of computation time, reconstruction error and number of iterations. The design examples illustrate that the proposed algorithm is superior in term of peak reconstruction error, computation time, and number of iterations. The proposed algorithm is simple, easy to implement, and linear in nature.

***Keywords*—**Aliasing cancellations filter bank, Filter banks, quadrature mirror filter (QMF), subband coding.

I. Introduction

DURING last two decades, quadrature mirror filter banks have been extensively used in subband coding of speech signals, image processing and transmultiplexers [1]. Originally, the concept of QMF is introduced for removing aliasing distortion in speech coding [2]. First time, Esteban and Galand [3] used quadrature mirror filter (QMF) in speech coding. Earlier subband coders designed based on set of band pass filters suffered from all three types of distortions: aliasing distortion, phase distortion, and amplitude distortion. Aliasing distortion is removed using suitable design of synthesis filters and phase distortion is eliminated with use of linear phase FIR filters. Amplitude distortion can be minimized using computer aided techniques or equalized by cascading with a filter [1].

There are many techniques to design QMF banks in literature [1-9]. Johnston [4] has introduced the concept of a two band linear phase QMF banks and formulated the objective function using weighted sum of ripple in the system response and out of band rejection. It is minimized by the Hooke and Jeaves optimization algorithm given in [5]. But this approach does not give optimum solution and it requires manual intervention. Also it is not useful for filters with larger taps due to high degree of nonlinearity, slow convergence, and initial start points. A prototype filter for M band pseudo QMF banks based on Parks McClellan algorithm is presented in [6]. In this technique, passband frequency is iteratively changed to minimize the reconstruction error using linear optimization. A simple alias free QMF banks with slight modifications is given in [7]. The author has used same objective function and optimization algorithm as in [6] in which the cutoff frequency is changed iteratively to minimize the objective function instead of passband frequency. Different windows are used for designing the prototype filters with high stopband attenuation. A comparative study of the performance is presented in [8]. Over past few years, many techniques have been proposed for the design linear phase QMF banks using different algorithms [4-9] so that perfect reconstruction can be achieved.

This paper, therefore, presents a new algorithm for the design of near perfect reconstruction QMF bank based on the proposed algorithms [7-9] with some modifications. Finally, a comparative study has been performed.

Authors are with Department of Electrical Engineering, Indian Institute of Technology Roorkee, 247667, India (e-mail: anilkdee@iitr.ernet.ac.in, gksngfee@iitr.ernet.ac.in, anandfee@iitr.ernet.ac.in).

II. Quadrature Mirror Filter (QMF)

Basically, QMF bank is a two-channel filter bank, which consists of analysis filters, down-samplers at transmission end, and up-samplers and synthesis filters at receiving end. The block diagram of QMF bank is shown in Fig. 1.

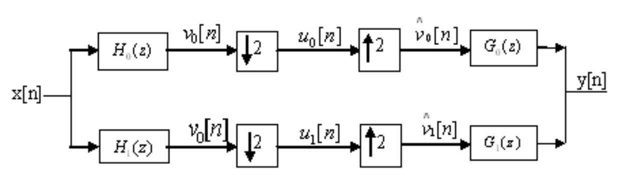

Fig. 1 Quadrature mirror filter

If QMF characteristics [2-3] are used, then system output is reduced to Eq. 1 and design problem becomes prototype filter design problem, as all other filters can be deduced from one single filter.

$$Y(z) = \frac{1}{2}\left\{H^2{}_0(z) - H^2{}_0(-z)\right\}X(z) \tag{1}$$

If $H_0(z)$ is taken as finite impulse responses filter of type 2 and distortion transfer function reduced to Eqn.2.

$$T(e^{j\omega}) = \frac{e^{-j\omega N}}{2}\left\{\left|H_0(e^{j\omega})\right|^2 - (-1)^N \left|H_0(e^{j(\omega-\pi)})\right|^2\right\} \tag{2}$$

There are two cases for selection of the length of filter. If length of filter is odd, then at $\omega = 0.5$, transfer function of system reduced to zero which is not required for perfect reconstruction of the signal. If length of filter is even, then transfer function of the system is reduced to Eqn.3. The perfect reconstruction (PR) condition is given by Eqn. 4:

$$T(e^{j\omega}) = \frac{e^{-j\omega N}}{2}\left\{\left|H_0(e^{j\omega})\right|^2 + \left|H_0(e^{j(\omega-\pi)}\right|^2\right\} \quad (3)$$

$$\left|H_0(e^{j\omega})\right|^2 + \left|H_0(e^{j(\omega-\pi)}\right|^2 = 1 \quad (4)$$

If this condition is not satisfied, then it is amplitude distortion and termed as reconstruction error. Peak reconstruction error (PRE) is given by Eqn. (5).

$$\text{PRE} = \max\left\{20\log\left(\left|H_0(e^{j\omega})\right|^2 + \left|H_0(e^{j(\omega-\pi)}\right|^2\right)\right\} \quad (5)$$

III. Proposed Algorithm

Over past few years, many methods [4-9] have been proposed to minimize the reconstruction error (amplitude distortion) so that perfect reconstruction could be achieved. For perfect reconstruction, prototype filter in QMF bank must satisfy these conditions [6]:

$$\left|H_0(e^{j\omega})\right|^2 + \left|H_0(e^{j(\omega-\frac{\pi}{2})}\right|^2 = 1 \text{ For } 0 < \omega < \frac{\pi}{2} \quad (6)$$

$$\left|H_0(e^{j\omega})\right| = 0 \quad \text{For } \omega > \frac{\pi}{2} \quad (7)$$

Johnston [4] used both conditions in single objective function and it is minimized using search algorithm [5]. Authors in [6] have also used first condition and formulated the objective function given by equation (8) and second condition is satisfied by taking high stopband attenuation.

$$\phi = \max\left\{\left|H_0(e^{j\omega})\right|^2 + \left|H_0(e^{j(\omega-\frac{\pi}{2})})\right|^2 - 1\right\}$$

$$\text{For } 0 < \omega < \frac{\pi}{2} \quad (8)$$

The perfect reconstruction is also possible if Eqn. (5) is satisfied. If magnitude response of distortion transfer function is evaluated at ω = 0.5π using condition (5), which is equal to 0.707 in ideal condition as given in Eqn. (9). Therefore, filter coefficients are adjusted so that coefficients values are near to 0.707 at ω = 0.5π.

$$T(e^{j\frac{\pi}{2}}) = \left|H_0(e^{j\frac{\pi}{2}})\right|^2 + \left|H_0(e^{j(-\frac{\pi}{2})}\right|^2 = 1 \text{ At } \omega = \frac{\pi}{2} \quad (9)$$

In the proposed algorithm, objective function is formulated using condition given in Eqn. 9 and cutoff frequency is varied to satisfy the objective function with use of optimization algorithm [7-9] with some modification. Flowchart of the proposed algorithm is shown in Fig. 2.

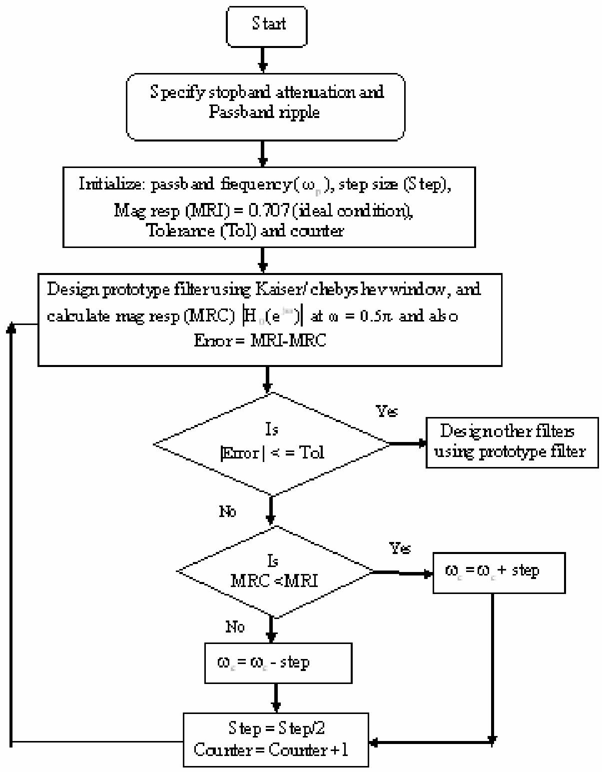

Fig. 2 A flowchart for the proposed algorithm

Filter order and stopband attenuation are fixed, and cutoff frequency is iteratively adjusted so that filter coefficients are near to 0.707 at ω = 0.5π. Specify tolerance at ω = 0.5π and magnitude response in ideal condition and design the prototype using initial cutoff frequency and filter order using different windows and Chebyshev). If tolerance is not satisfied, initial cutoff frequency is varied using step size. Prototype filter is redesigned using new cutoff frequency, same order and stopband attenuation. In every iteration, step size is halved. There is no problem of initial start point and convergence. Therefore, the proposed approach can be effectively used for prototype filter of larger taps that results in low reconstruction error so that perfect reconstruction could be achieved.

IV. Result And Comparative Performance

A two channel near perfect reconstruction QMF bank is designed using proposed algorithm and the algorithm given in the [7-8] for same design specifications. The proposed algorithm is found to be superior in terms of number of iterations (NOI), peak reconstruction Error (PRE), and computation time (CPU Time).

Table I and II illustrate the comparative performance study of the proposed algorithm and the algorithm given in [7-9]. Kaiser and chebyshev window [10] is used for this purpose.

TABLE I
COMPARISON OF PERFORMANCE USING KAISER WINDOW

Type	N	A_s	A_p	PRE (db)	NOI	ω_c	CPU Time (s)
Proposed	78	60	0.00607	0.0376	16	0.5109	0.156
Jain et al	78	60	0.00607	0.0647	139	0.5107	0.530
Proposed	82	60	0.00434	0.0354	18	0.5104	0.171
Jain et al	82	60	0.00434	0.0605	141	0.5102	0.546
Proposed	92	60	0.00173	0.0368	16	0.5092	0.156
Jain et al	92	60	0.00173	0.0634	147	0.5091	0.546
Proposed	52	80	0.00078	0.0091	16	0.5193	0.156
Jain et al	52	80	0.00078	0.0170	153	0.5192	0.577
Proposed	56	80	0.00043	0.0094	17	0.5179	0.156
Jain et al	56	80	0.00043	0.0175	150	0.5178	0.577
Proposed	66	80	0.00008	0.0090	16	0.5151	0.156
Jain et al	66	80	0.00008	0.0168	152	0.5151	0.561

TABLE II
COMPARISON OF PERFORMANCE USING CHEBYSHEV WINDOW

Type	N	A_s	A_p	PRE (db)	NOI	ω_c	CPU Time (s)
Proposed	60	60	0.00017	0.0113	17	0.5153	0.1404
Jain et al	60	60	0.00017	0.0207	150	0.5152	0.4992
Proposed	78	60	0.00607	0.0108	15	0.5117	0.1404
Jain et al	78	60	0.00607	0.0199	156	0.5116	0.5304
Proposed	82	60	0.00434	0.0063	17	0.5111	0.1404
Jain et al	82	60	0.00434	0.0115	160	0.5111	0.5304
Proposed	52	80	0.00078	0.0130	17	0.5204	0.1404
Jain et al	52	80	0.00078	0.0177	150	0.5204	0.4992
Proposed	56	80	0.00043	0.0130	17	0.5189	0.1404
Jain et al	56	80	0.00043	0.0178	151	0.5190	0.4992
Proposed	66	80	0.00008	0.0130	17	0.5160	0.1404
Jain et al	66	80	0.00008	0.0179	156	0.5161	0.5304

In all the cases, initial cutoff frequency used is 0.45 and transition width used is 0.1 for stopband attenuation of 60 and 0.2 for the stopband attenuation of 80. From the tables, it is evident that the reconstruction error is less, and conversed very fast as it required and very low computation. Results depicted in Table I and II clearly shows the superiority the proposed algorithm.

In example-1, length of the prototype filter (N) = 66, stopband attenuation (A_s) = 80 db, passband ripple (A_p) = 0.000086 db, initial cutoff frequency (ω_c) = 0.45 and transition width is 0.2. Frequency responses of analysis and synthesis filters are shown in Fig. 3 and 4 respectively and the reconstruction error is shown in Fig. 5. Finally, impulse response of QMF bank is shown in Fig. 6.

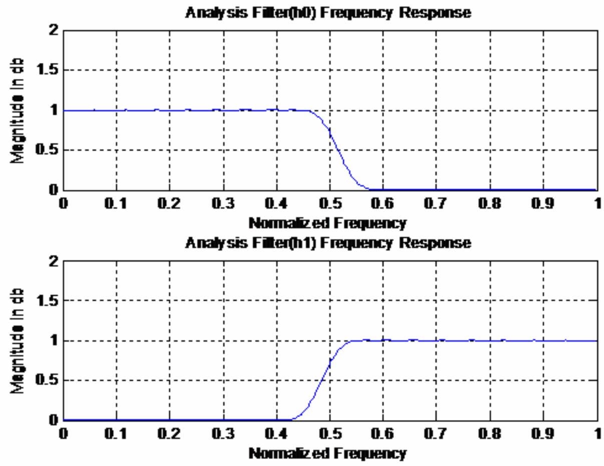

Fig. 3 Frequency response of analysis filters (h_0, h_1)

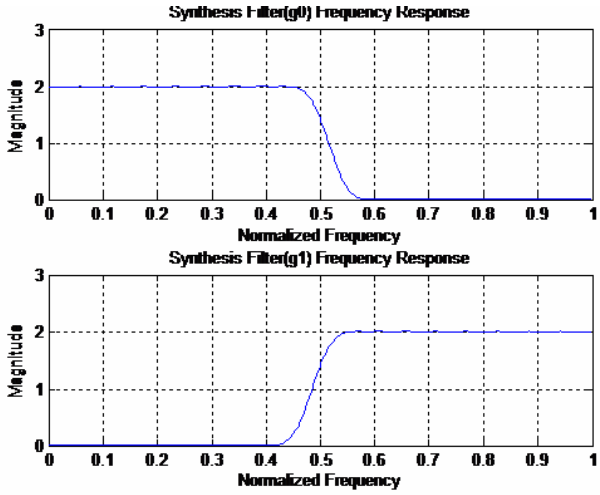

Fig. 4 Frequency response of synthesis filter (g_0, g_1)

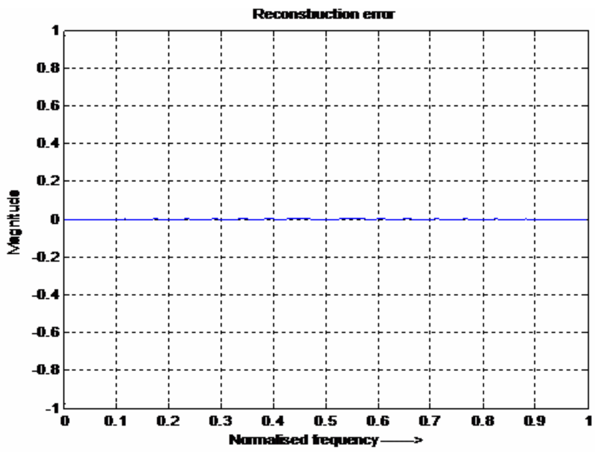

Fig. 5 Reconstruction error in the proposed algorithm

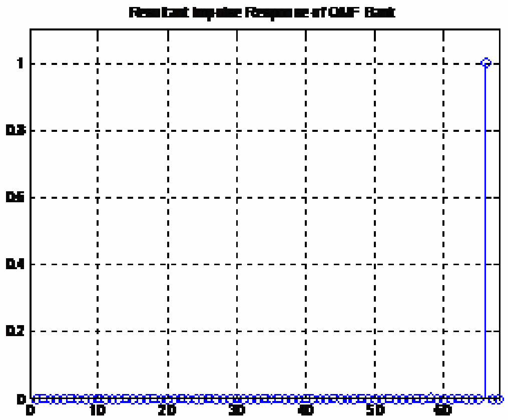

Fig. 6 Impulse response of QMF bank

It is observed that the all filter coefficients obtained by the proposed algorithm are non zero as compared to algorithm given in [9] in which only first and last coefficients are zero. Because of low peak reconstruction error, small number of iterations and less computation time, the proposed algorithm can be used for filters with larger taps too.

V. Conclusion

A new algorithm for design of a two channel near perfect reconstruction QMF bank is proposed in this paper. In the proposed algorithm, objective function is formulated using perfect reconstruction condition and prototype filter has been designed using different window techniques. The cutoff frequency (ω_c) is varied to get optimum value so that filter coefficients value is near to 0.707 at $\omega = 0.5\pi$. A comparative study of the proposed algorithm and algorithm given in the [7-9] has been also carried out. It is found that the peak reconstruction error is less in the proposed algorithm for same design specifications. It is also observed that the proposed algorithm converged very fast in low number of iterations, and can be effectively used for filters with larger taps.

References

[1] P. P. Vaidyanathan, "*Multirate and filter bank,*" Englewood Cliffs, NJ: Prentice-Hall, 1993.

[2] A. Croisier, D. Esteban, C. Galand, "Perfect channel splitting by use of interpolation /decimation/tree decomposition techniques," *International conference on Information Sciences and Systems*, Patras, 1976.

[3] D. Esteban and C. Galand, "Application of quadrature mirror filters to split band voice coding schemes," in *Proceedings IEEE International Conference on Acoustics, Speech, and Signal Processing*, May 1977, pp. 191-195.

[4] J. D. Johnston, "A filter family designed for use in quadrature mirror filter banks," in *Proceedings IEEE International Conference on Acoustics, Speech, and Signal Processing*, pp. 291-294, April 1980.

[5] R. Hooke and T. Jeaves, "Direct search solution of numerical and statistical problems," J. *Ass. Comp. Mach.*, Vol. 8, pp. 212-229, April 1961.

[6] C. D. Creusere and S.K. Mitra, "A simple method for designing high quality prototype filters for M- band pseudo QMF bank," *IEEE Transactions on Signal Processing,* 1995, vol. 43, pp. 1005-1007.

[7] A. Jain, R. Saxena, and S. C. Saxena, "A simple alias free QMF system with near perfect reconstruction," Journal of Indian Institute Science, 2005, 85, 1-10.

[8] A. Ramakrishna and M. J. Nigam, "A Simple Method to Design FIR QMF Bank," Fourth International Conference on intelligent sensing and information processing, 2006 ICISIP, pp. 236-239.

[9] http://www.engineer.tamuk.edu/Spark/chap5.pdf.

[10] S. K. Mitra, "*Digital signal processing: A computer based approach*," Tata McGraw Hill Edition, 2006.

A. Kumar received the B.E. (Electronics & Telecommunication) degree from Army Institute of Technology, Pune, India, in 2002, the M. Tech. (Solid State Electronic Material) degree from Indian Institute of Technology, Roorkee, India in 2006. He is currently pursuing the Ph.D. degree at Indian Institute of Technology, Roorkee, India, in Electrical Engineering.

G. K. Singh received the B.Tech. degree from G. B. Pant University of Agriculture and Technology, Pantnagar, India, in 1981, and the Ph.D. degree from Banaras Hindu University, Varanasi, India, in 1991, both in electrical engineering. He worked in industry for nearly five and a half years. In 1991, he joined M. N. R. Engineering College, Allahabad, India, as a Lecturer. In 1996, he moved to the University of Roorkee, Roorkee, India. Currently, he is a Professor in the Electrical Engineering Department, Indian Institute of Technology, Roorkee, India. He has been involved in design and analysis of electrical machines in general and high-phase-order ac machines in particular and power system harmonics and power quality. He has coordinated a number of research projects sponsored by the CSIR and UGC, Government of India. He has served as a Visiting Associate Professor in the Department of Electrical Engineering, Pohang University of Science and Technology (POSTECH), Pohang, Korea, and as Visiting Professor in the Department of Electrical and Electronics Engineering, Middle East Technical University, Ankara, Turkey. Prof. Singh received the Pt. Madan Mohan Malaviya Memorial Medal and the Certificate of Merit Award 2001–2002 of The Institution of Engineers (India).

R. S. Anand received B.E., M.E. and Ph.D. Degrees in Electrical Engineering from Indian Institute of Technology Roorkee, India in 1985, 1987 and 1992, respectively. He joined in Department of Electrical Engineering, IIT Roorkee as Assistant Professor in June 1998. Currently, he is an Associate Professor in the Electrical Engineering Department, Indian Institute of Technology, Roorkee, India. Previous to IIT Roorkee he has been the lecturer at IIT Kharagpur, India for about four years. His research areas of interest are intelligent systems, microprocessor & computer based instrumentation, biomedical signals and image processing, and ultrasonic applications in NDE and medical diagnosis.