Near-Lossless Image Coding based on Orthogonal Polynomials

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Abstract—In this paper, a near lossless image coding scheme based on Orthogonal Polynomials Transform (OPT) has been presented. The polynomial operators and polynomials basis operators are obtained from set of orthogonal polynomials functions for the proposed transform coding. The image is partitioned into a number of distinct square blocks and the proposed transform coding is applied to each of these individually. After applying the proposed transform coding, the transformed coefficients are rearranged into a sub-band structure. The Embedded Zerotree (EZ) coding algorithm is then employed to quantize the coefficients. The proposed transform is implemented for various block sizes and the performance is compared with existing Discrete Cosine Transform (DCT) transform coding scheme.

Keywords—Near-lossless Coding, Orthogonal Polynomials Transform, Embedded Zerotree Coding

I. INTRODUCTION

IMAGE compression has been becoming increasingly important with the development of aviation, communications, internet and space techniques. Especially lossless compression becomes crucial when there is no loss of information is acceptable in applications such as medical image, remote sensing, image archiving, satellite communications and so on. Many embedded DCT [1,2] and wavelet transform [3,4] based image coding algorithms have been reported in the literatures. In [5], a hybrid coding algorithm called Embedded Subband Partitioning Block Arithmetic Coding (ESPBA) which uses quadtree partitioning and clock arithmetic coding to encode the image coefficients based on segmentation is reported. An embedded color image coding is reported in [6], where the set partitioning strategies of hierarchical trees and the blocks are integrated into a single algorithm to improve the coding efficiency. The Set Partitioning Hierarchical Trees (SPIHT) based embedded color image coding in presented in [7]. Various near lossless compression techniques [8-11] were also proposed in the literature. A rate-controllable near-lossless image codec is designed in [12], in which a content adaptive information loss distribution scheme is proposed to solve the conflict problem between image quality and compression efficiency. A progressive Lossless/Near-Lossless image coding is proposed by Avcibas et. al [13], based on the concept of probability mass estimation and successive context refinement. The JPEG-LS is the ISO/ITU standard for lossless and near lossless compression of continuous images, where the Low

Complexity Lossless Compression (LOCO – I) is the core algorithm14]. The JPEG lossless still image compression standard [15] provides different prediction schemes for users to select particular bit rate for image coding. In this work, we modify Shapiro's Embedded Zerotree Wavelet (EZW) algorithm [3] that uses the Discrete Wavelet Transform, with proposed Orthogonal Polynomials transform since wavelet transform could not be well suited to represent 2D singularities along edges or contours.

II. ORTHOGONAL POLYNOMIALS TRANSFORM CODING

In order to a devise near lossless transform coding scheme, a linear 2-D image formation system is considered around a Cartesian coordinate separable, blurring, point spread operator in which the image I results in the superposition of the point source of impulse weighted by the value of the object function f. Expressing the object function f in terms of derivatives of the image function I relative to its Cartesian coordinates is very useful for analyzing the image. The point spread function $M(x, y)$ can be considered to be real valued function defined for $(x, y) \in X$ x Y, where X and Y are ordered subsets of real values. In case of gray-level image of size $(n$ x $n)$ where X (rows) consists of a finite set, which for convenience can be labeled as $\{0, 1, …, n-1\}$, the function $M(x, y)$ reduces to a sequence of functions.

$$M (i, t) = u_i(t), \quad i, t = 0, 1, …, n-1 \qquad (1)$$

The linear two dimensional transformation can be defined by the point spread operator $M(x, y)$ $(M(i, t) = u_i(t))$ as shown in equation (2).

$$\beta'(\zeta,\eta)=\int_{x\in X}\int_{y\in Y}M(\zeta,x)M(\eta,y)I(x,y)dxdy \qquad (2)$$

Considering both X and Y to be a finite set of values $\{0, 1, 2,…, n -1\}$, equation (2) can be written in matrix notation as follows

$$\left|\beta'_{ij}\right| = \left(|M|\otimes|M|\right)^t |I| \qquad (3)$$

where $\otimes$ is the outer product, $|\beta'_{ij}|$ are n^2 matrices arranged in the dictionary sequence, $|I|$ is the image , $|\beta'_{ij}|$ are the coefficients of transformation and the point spread operator $|M|$ is

$$|M| = \begin{vmatrix} u_0(t_1) & u_1(t_1) & \cdots & u_{n-1}(t_1) \\ u_0(t_2) & u_1(t_2) & \cdots & u_{n-1}(t_2) \\ & & \vdots & \\ u_0(t_n) & u_1(t_n) & \cdots & u_{n-1}(t_n) \end{vmatrix} \qquad (4)$$

We consider a set of orthogonal polynomials $u_0(t), u_1(t), …, u_{n-1}(t)$ of degrees 0, 1, 2, …, $n-1$ respectively to construct the polynomial operators of different sizes from equation (4) for $n \geq 2$ and $t_i = i$. The generating formula for the polynomials is as follows.

$$u_{i+1}(t)= (t-\mu) u_i(t) - b_i(n) u_{i-1}(t) \text{ for } i \geq 1, \qquad (5)$$
$$u_1(t) = t - \mu, \text{ and } u_0(t) = 1,$$

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where $b_i(n) = \dfrac{\langle u_i, u_i \rangle}{\langle u_{i-1}, u_{i-1} \rangle} = \dfrac{\sum_{t=1}^{n} u_i^2(t)}{\sum_{t=1}^{n} u_{i-1}^2(t)}$

and $\mu = \dfrac{1}{n} \sum_{t=1}^{n} t$

Considering the range of values of t to be $t_i = i$, $i = 1, 2, 3, \ldots, n$, we get

$$b_i(n) = \frac{i^2(n^2 - i^2)}{4(4i^2 - 1)}, \qquad \mu = \frac{1}{n}\sum_{t=1}^{n} t = \frac{n+1}{2}$$

We can construct point-spread operators $|M|$ of different size from equation (4) using the above orthogonal polynomials for $n \geq 2$ and $t_{i=i}$. For the convenience of point-spread operations, the elements of $|M|$ are scaled to make them integers.

III. THE ORTHOGONAL POLYNOMIALS BASIS

For the sake of computational simplicity, the finite Cartesian coordinate set X, Y is labeled as $\{1, 2, 3\}$. The point spread operator in equation (3) that defines the linear orthogonal transformation for image coding can be obtained as $|M| \otimes |M|$, where $|M|$ can be computed and scaled from equation (4) as follows.

$$|M| = \begin{vmatrix} u_0(x_0) & u_1(x_0) & u_2(x_0) \\ u_0(x_1) & u_1(x_1) & u_2(x_1) \\ u_0(x_2) & u_1(x_2) & u_2(x_2) \end{vmatrix} = \begin{vmatrix} 1 & -1 & 1 \\ 1 & 0 & -2 \\ 1 & 1 & 1 \end{vmatrix} \quad (6)$$

The set of polynomial basis operators $O_{ij}^{\,n}$ ($0 \leq i, j \leq n-1$) can be computed as

$$O_{ij}^{\,n} = \hat{u}_i \otimes \hat{u}_j^{\,t}$$

where $\hat{u}_i$ is the $(i + 1)^{st}$ column vector of $|M|$.

The complete set of basis operators of sizes (2x2) and (3x3) are given below.

Polynomial basis operators of size (2x2) are

$$\left[O_{00}^2\right] = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \; \left[O_{01}^2\right] = \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix},$$

$$\left[O_{10}^2\right] = \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix}, \; \left[O_{11}^2\right] = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

Polynomial basis operators of (3x3) are

$$\left[O_{00}^3\right] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \; \left[O_{01}^3\right] = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix},$$

$$\left[O_{02}^3\right] = \begin{bmatrix} 1 & -2 & 1 \\ 1 & -2 & 1 \\ 1 & -2 & 1 \end{bmatrix}, \; \left[O_{10}^3\right] = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix},$$

$$\left[O_{11}^3\right] = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix}, \; \left[O_{12}^3\right] = \begin{bmatrix} -1 & 2 & -1 \\ 0 & 0 & 0 \\ 1 & -2 & 1 \end{bmatrix},$$

$$\left[O_{20}^3\right] = \begin{bmatrix} 1 & 1 & 1 \\ -2 & -2 & -2 \\ 1 & 0 & 1 \end{bmatrix} \; \left[O_{21}^3\right] = \begin{bmatrix} -1 & 0 & 1 \\ 2 & 0 & -2 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\left[O_{22}^3\right] = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 1 \end{bmatrix}$$

having designed the orthogonal polynomials model, the proposed image coding technique is presented in the next section.

IV. ZERO TREE CODING AND SUBBAND STRUCTURE

After the proposed Orthogonal Polynomials based transform is performed, the transformed coefficients are rearranged from individual localized blocks to a global hierarchical sub-band structure for quantization and coding. This is achieved by grouping together proposed transformed coefficients of similar spatial orientation and frequency to create the larger blocks necessary in the hierarchical sub-band structure. In an (nxn) block of OPT transformed coefficients, the C_{00} coefficients make up the lowest frequency sub-band, with the C_{01}, C_{10} and C_{11} coefficients respectively assembled into vertical, horizontal and diagonal sub-bands at the same scale. Larger, higher frequency sub-bands are created by interleaving the higher frequency transformed coefficients, as illustrated in Fig. 1. In a practical implementation this rearrangement may be more efficiently accomplished by redefining the spatial relationship between parent and children. The number of scales in the sub-band image, M, is related to the (NxN) block size by

$$M = \log_2 N \qquad (7)$$

Once the blocks are rearranged into sub-band structure, the EZW algorithm is applied on the proposed transformed co-efficients. This algorithm works well with the proposed coding scheme because the zerotree structure is effective in describing the significance map of the transform coefficients, as it exploits (i) the inherent self similarity of the sub-band image over the range of scales, and (ii) the positioning of majority of (near) zero valued coefficients in the higher frequency sub-bands. The EZW algorithm applies Successive Approximation Quantization (SAQ) in order to provide multi-precision representation of the transformed coefficients and to facilitate the embedded coding. The algorithm codes the transformed coefficients in decreasing order in several scans. Each scan of the algorithm consists of two passes: significant map encoding and refinement pass. The dominant pass scans the sub-band structure in zig-zag, right-to-left and then top-to-bottom within each scale, before proceeding to the next higher scale of sub-band structure as presented in fig.2. For every pass, a threshold (T) is chosen against which all the coefficients are measured and encoded as one of the following four symbols.

(i) Significant positive – If the coefficient value is greater than threshold T.
(ii) Significant negative – If the magnitude of the coefficient value is greater than threshold T

(iii) Zerotree root – A coefficient is encoded as zerotree root if the coefficient and all its descendents are insignificant with respect to threshold T.

(iv) Isolated zero – If the coefficient is insignificant but some of its descendents are significant.

The initial threshold is obtained as $T_0 = 2^{\lfloor \log_2 C_{max} \rfloor}$, where C_{max} is the maximum coefficient in the sub-band structure. The successive approximation quantization uses a monotonically decreasing set of thresholds and encodes the transformed coefficients as one of the above four labels with respect to any given threshold. For successive significant pass encoding, the threshold is lowered as $T_K = \dfrac{T_K - 1}{2}$ and only those coefficients not yet found to be significant in the previous pass are scanned for encoding, and the process is repeated until the threshold reached zero, results in complete encoded bitstreams. Once all the coefficients are encoded, the initial threshold T_0 and the encoded bitstreams are sent to the receiver. The decompression is achieved by EZW decoding followed by sub-band rearrangement and inverse quantization with the proposed polynomials basis operators as described in section III. The EZW decoding algorithm decodes the bitstreams with the set of threshold values in successive refinement pass until the threshold reaches zero. In order to achieve better results for near-lossless compression, the proposed transform is tested for various blocks such as (8x8), (16x16), (32x32) and (64x64).

A_{00}	A_{01}	A_{02}	A_{03}	B_{00}	B_{01}	B_{02}	B_{03}
A_{10}	A_{11}	A_{12}	A_{13}	B_{10}	B_{11}	B_{12}	B_{13}
A_{20}	A_{21}	A_{22}	A_{23}	B_{20}	B_{21}	B_{22}	B_{23}
A_{30}	A_{31}	A_{32}	A_{33}	B_{30}	B_{31}	B_{32}	B_{33}

(a) Two 4 x 4 transformed blocks

A_{00}	B_{00}	A_{01}	B_{01}	A_{02}	A_{03}	B_{02}	B_{03}
A_{10}	B_{10}	A_{11}	B_{11}	A_{12}	A_{13}	B_{12}	B_{13}
A_{20}	A_{21}	B_{20}	B_{21}	A_{22}	A_{23}	B_{22}	B_{23}
A_{30}	A_{31}	B_{30}	B_{31}	A_{32}	A_{33}	B_{32}	B_{33}

(b) Sub-band structure of fig 1(a)

Fig. 1 Example of the rearrangement of transformed blocks to form a hierarchical sub-band structure
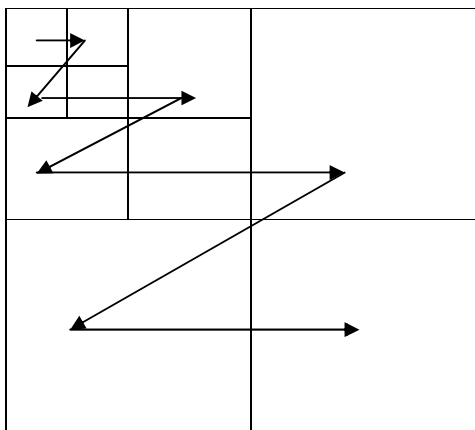

Fig. 2 EZW sub-band structure scanning order

V. EXPERIMENTS AND RESULTS

For near lossless compression, the proposed Orthogonal Polynomials transform coding has been experimented with more than 2000 monochrome images of different types. Two sample images viz. lena and boat images, which are of size (256x256) with pixel values in the range (0–255) are shown in Fig. 3 (a) and (b), respectively. The input images are partitioned into non-overlapping (nxn) regions and are applied with the orthogonal polynomials based transformation in each sub-image region, as described in Section II. The resulting transformation coefficients are then rearranged into sub-band structure as given in Fig.1 (b) and EZW algorithm is applied as described in section IV result to compressed bit stream. The experiments have been conducted for various block sizes such as (8x8), (16x16), (32x32), (64x64) and the results are measured in bits per pixel (bpp). When the block size is (8x8), the proposed OPT transform obtains 3.86bpp and 3.98bpp respectively for lena and boat images where as DCT transform gives 4.04bpp and 4.26bpp for the same images. Similarly for the block size of (16x16), the proposed transform achieves 3.87bpp and 4.01bpp for lena and boat images, the DCT transform achieves 4.05bpp and 4.28bpp respectively for the same images. The experiments have been conducted for various test image and the results are presented in table 2 including test images. From the experimental results, it is observed that the (8x8) block size gives minimum bit rate as compared with other block sizes. The decompression is done by first Embedded Zerotree decoding followed by sub-band structure rearrangement for block size of (nxn). Then, the inverse transform is applied on the rearranged coefficients using Orthogonal Polynomials basis as described in section III and the reconstructed image is obtained are shown in fig 4(a) and (b) for the same test images when the block size is (8x8).

(a)

(b)

Fig. 3 Original test images

(a)

(b)

Fig. 4 Results of the proposed transform for near lossless coding
when block size is (8x8)

TABLE I
PERFORMANCE OF NEAR-LOSSLESS COMPRESSION WITH PROPOSED AND DCT
TRANSFORM CODING SCHEME

Imgs	Proposed transform(bpp)				DCT (bpp)			
	(8x8)	(16x16)	(32x32)	(64x64)	(8x8)	(16x16)	(32x32)	(64x64)
Lena	3.86	3.87	3.89	4.04	4.04	4.05	4.09	4.19
Boat	3.98	4.01	4.03	4.14	4.26	4.28	4.30	4.48
Frs	4.06	4.07	4.09	4.25	4.39	4.40	4.43	4.55
Pepr	3.91	3.93	3.95	4.10	4.21	4.22	4.25	4.43
Bab.	4.02	4.04	4.06	4.38	4.32	4.34	4.36	4.50

VI. CONCLUSION

In this paper, a near lossless Orthogonal Polynomials based image coding has been implemented. From the set of polynomials functions, the polynomials operators and polynomials basis operators are obtained for the proposed transform coding. The transformed coefficients are then rearranged into sub-band structure and the embedded zerotree coding is been applied on the transformed coefficients. Since no training of any kind is required for embedded zerotree coding, it performs remarkably well for most of the images with the proposed Orthogonal Polynomials transform. The experiments are conducted with proposed transform obtains better results as compared with DCT transform and gives minimum bits per pixel when the block size is (8x8).

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