International Journal of Engineering, Mathematical and Physical Sciences
ISSN: 2517-9934
Vol:6, No:4, 2012

Natural Convection Boundary Layer Flow of a Viscoelastic Fluid on Solid Sphere with Newtonian Heating

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Abstract**—**The present paper considers the steady free convection boundary layer flow of a viscoelastic fluid on solid sphere with Newtonian heating. The boundary layer equations are an order higher than those for the Newtonian (viscous) fluid and the adherence boundary conditions are insufficient to determine the solution of these equations completely. Thus, the augmentation an extra boundary condition is needed to perform the numerical computational. The governing boundary layer equations are first transformed into non-dimensional form by using special dimensionless group and then solved by using an implicit finite difference scheme. The results are displayed graphically to illustrate the influence of viscoelastic K and Prandtl Number Pr parameters on skin friction, heat transfer, velocity profiles and temperature profiles. Present results are compared with the published papers and are found to concur very well.

Keywords**—**boundary layer flow, Newtonian heating, sphere, viscoelastic fluid.

I. Introduction

IN recent years, the flow and heat transfer phenomena over sphere have received a considerable attention due to its practical needs in numerous engineering applications including solving the cooling problems in turbine blades, electronic systems and manufacturing processes [1]. Extensive studies on the topic natural convection specifically on sphere have been conducted by several researchers. For example, Chiang *et al*. [2] have studied an exact analysis of the laminar free convection from a sphere by considering prescribed surface temperature and surface heat flux and their work have been continued by Huang and Chen [3] who considered the effects of blowing and suction. Nazar *et al*. [4], [5] considered the problem of free convection boundary layer on an isothermal sphere in a micropolar fluid for the case of constant wall temperature and also constant heat flux. Molla and Hossain [6] have investigated the effects of chemical reaction, heat and mass diffusion in natural convection flow from an isothermal sphere with temperature dependent viscosity while Cheng [7] has considered the problem of natural convection heat and mass transfer from a sphere in micropolar fluids with constant wall temperature and concentration.

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Generally, the studies on the problems of convective boundary layer and heat transfer focus to the problem that related to the prescribed wall temperature and heat flux. However, in 1994, Merkin [8] has considered the problem of free convection boundary layer over vertical surfaces for the case of Newtonian heating in his study. The investigation reveaedl the situation with Newtonian heating arises in what are usually termed conjugate convective flows, where the heat is supplied to the convective fluid through a bounding surface with a finite heat capacity.

Recently, the problem of Newtonian heating has been studied extensively due to the large application and demand in engineering field. Salleh *et al.* [9], [10] have considered the case of Newtonian heating in their studied about convection in a sphere. Very recently, Merkin *et al.* [11] have investigated the problem of forced convection heat transfer near a forward stagnation point with Newtonian heating. Polymers are complex rheological materials that exhibit both viscous and elastic (viscoelastic) properties under varying conditions of stress, strain and temperature. The viscoelastic materials behave more like solids at low temperatures and having fast deformation speeds. They exhibit liquid properties at high temperatures and slow deformation speeds. A literature survey indicated there has been an extensive research available regarding the viscoelastic fluid. Starting from Thomas *et al*. [12], they have presented the unsteady motion of a sphere in a viscoelastic liquid where they considered the unsteady motion of a sphere moving under a constant force. Verma [13] has derived the boundary layer equations near a body of revolution in a uniform stream and a case of the boundary layer over the surface of sphere and found that the increase in the elasticity of the liquid causes a shift in the point of separation towards the forward stagnation point. Carew *et al.* [14] have considered the problem of a sphere falling along the axis of vertical cylindrical tube containing a viscoelastic fluid. In chemical engineering systems, viscoelastic flows arise in numerous processes. Such flows possess both viscous and elastic properties and can exhibit normal stresses and relaxation effects.

Recently, the numerical studies of transient free convective mass transfer in a Walters-B viscoelastic flow with wall suction and free convection boundary layer flow of a viscoelastic fluid in the presence of heat generation have been investigated by Chang *et al.* [15] and Kasim *et al.* [16] respectively. Velocity is found to increase with a rise in viscoelasticity parameter with both time and distance close to the plate surface.

Motivated by the work above, this paper investigates the problem of natural convection boundary layer flow of viscoelastic fluid on solid sphere with Newtonian Heating. The full governing boundary layer equations are first transformed into a system of non-dimensional equations via the non-dimensional variables, and then into non-similar equations before they are solved numerically by the Keller-box method as described in the books by Na [16] and Cebeci and Bradshaw [17]. Results of this study such as the local skin friction coefficient, the local wall temperature as well as the velocity and temperature profiles are presented in this paper. To the best of our knowledge, this problem has not been considered before; hence the reported results are newly found.

II. Problem Formulation

The problem of steady natural convection boundary layer flow for an isothermal horizontal circular cylinder placed in a viscoelastic fluid is considered in this paper. Fig. 1 illustrates the geometry of the problem and the corresponding coordinate system. The problem is considered as a heated sphere of radius a, which is immersed in a viscous and incompressible fluid of ambient temperature T_∞. The surface of the sphere is subjected to Newtonian heating (NH).

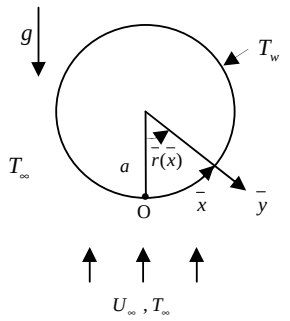

Fig. 1 Physical model and coordinate system

Under the usual Boussinesq and boundary layer approximations, the equations for mass continuity,(continuity / mass conservation) momentum and energy can be written in the following form:

$$\frac{\partial}{\partial \bar{x}}\left(\bar{r}\ \bar{u}\right)+\frac{\partial}{\partial \bar{y}}\left(\bar{r}\ \bar{v}\right)=0, \tag{1}$$

$$\begin{aligned}&\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}\\&=\upsilon\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}+g\beta(T-T_\infty)\sin(\bar{x}/a)\\&+\frac{k_0}{\rho}\left[\frac{\partial}{\partial \bar{x}}\left(\bar{u}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right)+\bar{v}\frac{\partial^3 \bar{u}}{\partial \bar{y}^3}+\frac{\partial \bar{u}}{\partial \bar{y}}\frac{\partial^2 \bar{v}}{\partial \bar{y}^2}\right],\end{aligned} \tag{2}$$

$$\bar{u}\frac{\partial T}{\partial \bar{x}}+\bar{v}\frac{\partial T}{\partial \bar{y}}=\alpha\frac{\partial^2 T}{\partial \bar{y}^2}, \tag{3}$$

subject to the boundary conditions;

$$\begin{aligned}&\bar{u}=\bar{v}=0, \qquad \frac{\partial T}{\partial \bar{y}}=-h_s T\ , \ \text{ on } \ \bar{y}=0,\\&\bar{u}=0,\ \frac{\partial \bar{u}}{\partial \bar{y}}=0,\ \ T=T_\infty,\ \text{ as } \ \bar{y}\to\infty.\end{aligned} \tag{4}$$

where ρ, g, β, μ, k_0, α, T, and h_s are the density, gravitational acceleration, coefficient of thermal expansion, dynamic viscosity, vortex viscosity, thermal diffusivity of the fluid, local temperature and heat transfer parameter for Newtonian heating (NH), respectively. In this problem, it is considered that $\bar{r}(x)=a\sin\left(\frac{\bar{x}}{a}\right)$, $\bar{u}$ and $\bar{v}$ are the velocity components along x- and y-direction respectively.

The non-dimensional variables are introduced in this study as follow:

$$\begin{aligned}&x=\frac{\bar{x}}{a},\quad y=Gr^{1/4}\left(\frac{\bar{y}}{a}\right),\quad r(x)=\frac{\bar{r}(\bar{x})}{a},\quad u=\frac{a}{\upsilon}Gr^{-1/2}\bar{u},\\&v=\frac{a}{\upsilon}Gr^{-1/4}\bar{v},\qquad \theta=\frac{(T-T_\infty)}{T_\infty},(NH)\end{aligned} \tag{5}$$

where $Gr=\frac{g\beta(T_\infty)a^3}{\nu^2}$ is the Grashof number for the case of Newtonian heating.

Substitution of (5) into (1) - (3) led to the following non-dimensional equations:

$$\frac{\partial}{\partial x}(ru)+\frac{\partial}{\partial y}(rv)=0, \tag{6}$$

$$\begin{aligned}u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}-K\left[\frac{\partial}{\partial x}\left(u\frac{\partial^2 u}{\partial y^2}\right)+v\frac{\partial^3 u}{\partial y^3}-\frac{\partial u}{\partial x}\frac{\partial^2 v}{\partial y^2}\right]\\+\theta\sin(x),\end{aligned} \tag{7}$$

$$u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\frac{1}{\mathrm{Pr}}\frac{\partial^2 \theta}{\partial y^2}, \tag{8}$$

and the boundary conditions (4) become

$$\begin{aligned}&u=v=0, \qquad \frac{\partial T}{\partial \bar{y}}=-\gamma(1+\theta)(NH,)\ \ \text{on}\ \ y=0,\\&u=0,\ \ \frac{\partial \bar{u}}{\partial \bar{y}}=0,\ \ \theta=0,\ \ \text{as}\ \ y\to\infty.\end{aligned} \tag{9}$$

where $K = \frac{k_0 \mathrm{Gr}^{5/2}}{a^2}$ represents the viscoelastic parameter and $\gamma = a h_s Gr^{-1/4}$ denotes conjugate parameter for Newtonian heating. From Salleh *et al*. [10], as $\gamma = 0$ gives $\theta = 0$, that corresponding to having $h_s = 0$ and hence no heating existed from the sphere.

III. SOLUTION PROCEDURES

In order to solve (6) – (8), according to the boundary condition (9), the following variables are assumed:

$$\psi = xr(x) f(x,y), \qquad \theta = \theta(x,y). \tag{10}$$

where ψ is the stream function defined as

$$u = \frac{1}{r}\frac{\partial \psi}{\partial y} \text{ and } v = -\frac{1}{r}\frac{\partial \psi}{\partial x}. \tag{11}$$

which satisfies (6), thus (7) and (8) become

$$\begin{aligned} &\frac{\partial^3 f}{\partial y^3} - \left(\frac{\partial f}{\partial y}\right)^2 + \left(1 + x\frac{\cos x}{\sin x}\right) f \frac{\partial^2 f}{\partial y^2} \\ &+ K\left[2\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3} - \left(1 + x\frac{\cos x}{\sin x}\right)\left(f\frac{\partial^4 f}{\partial y^4} + \left(\frac{\partial^2 f}{\partial y^2}\right)^2\right)\right] + \theta\frac{\sin x}{x} \\ &= Kx\left[\frac{\partial f}{\partial x}\frac{\partial^4 f}{\partial y^4} - \frac{\partial^2 f}{\partial x \partial y}\frac{\partial^3 f}{\partial y^3} - \frac{\partial f}{\partial y}\frac{\partial^4 f}{\partial x \partial y^3} + \frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial x \partial y^2}\right] \\ &+ x\left(\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x \partial y} - \frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}\right), \end{aligned} \tag{12}$$

$$\frac{1}{\mathrm{Pr}}\frac{\partial^2 \theta}{\partial y^2} + \left(1 + x\frac{\cos x}{\sin x}\right) f \frac{\partial \theta}{\partial y} = x\left(\frac{\partial f}{\partial y}\frac{\partial \theta}{\partial x} - \frac{\partial f}{\partial x}\frac{\partial \theta}{\partial y}\right), \tag{13}$$

with respect to the following boundary conditions

$$\begin{aligned} &f = 0, \quad \frac{\partial f}{\partial y} = 0, \quad \frac{\partial \theta}{\partial y} = -\gamma(1+\theta)(NH), \text{ on } \quad y = 0, \\ &\frac{\partial f}{\partial y} = 0, \quad \frac{\partial^2 f}{\partial y^2} = 0, \quad \theta = 0, \text{ as } \quad y \to \infty. \end{aligned} \tag{14}$$

At the lower stagnation point of the cylinder $(x \approx 0)$, (12)-(13) are reduced to the following ordinary differential equations:

$$f''' + 2ff'' - f'^2 + \theta + K(ff'''' - 2f'f''' + f''^2) = 0, \tag{15}$$

$$\frac{1}{\mathrm{Pr}}\theta'' + f\theta' = 0, \tag{16}$$

with the boundary conditions

$$\begin{aligned} &f(0) = f'(0) = 0, \quad \theta'(0) = -\gamma(1+\theta(0))(NH), \\ &f'(\infty) = 0, \quad f''(\infty) = 0, \quad \theta(\infty) = 0. \end{aligned} \tag{17}$$

where primes denote the differentiation with respect to y.

The physical quantities of principal interest are shearing stress, and the rate of heat transfer in terms of skin friction coefficient C_f and the local wall temperature $\theta_w(x)$ respectively, which can be written as:

$$C_f = x\frac{\partial^2 f}{\partial y^2}(x,0), \qquad \theta_w(x) = -1 - \frac{\partial \theta}{\partial y}(x,0)(NH) \tag{18}$$

where

$$\begin{aligned} &C_f = \tau_w \Big/ \rho U_\infty^2, \\ &\tau_w = \mu\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)_{\bar{y}=0} + k_0\left(\bar{u}\frac{\partial^2 \bar{u}}{\partial \bar{x}\partial \bar{y}} + \bar{v}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + 2\frac{\partial \bar{u}}{\partial \bar{x}}\frac{\partial \bar{u}}{\partial \bar{y}}\right)_{\bar{y}=0}. \end{aligned} \tag{19}$$

IV. RESULT AND DISCUSSION

The systems of equations (12),(13) and (15),(16) are solved numerically for some values of the viscoelastic parameter *K* and Prandtl number Pr using the implicit finite-difference method known as the Keller-box method. In this paper, the case in question is when the Prandtl number, Pr is 7 only in order to save the space. The present results for the skin friction coefficient C_f and the wall temperature $\theta_w(0)$ were compared with Salleh *et al.* [10] in order to validate the numerical results obtained. The comparison shows that the numerical solutions (see Table 1) obtained by the present authors give a good aggreement with existing results obtained by previous authors. Therefore the present authors are confident that the results obtained are very accurate.

TABLE I
VALUES OF SKIN FRICTION $f''(0)$ AND THE WALL TEMPERATURE $\theta_w(0)$ AT THE LOWER STAGNATION POINT OF THE WALL $(x \approx 0)$ WHEN PRANDTL NUMBER PR=0.7 AND 7, VISCOELASTIC PARAMETER *K*=0 (NEWTONIAN FLUID) AND PARAMETER FOR NEWTONIAN HEATING $\gamma = 1$

Prandtl number	**Skin Friction** $f''(0)$		**Wall Temperature** $\theta_w(0)$	
	Salleh *et al.*[10]	present	Salleh *et al.* [10]	present
0.7	8.9606	8.961327	26.4584	26.454302
7.0	1.2489	1.248393	3.3651	3.360566

In this paper, we include the numerical result of skin friction coefficient C_f and the wall temperature θ_w for different position x. The numerical solution starts at the lower stagnation point of the sphere ($x \approx 0$) and proceeds around the sphere up to the point $x = 170°$ with value of viscoelastic parameter is 1 (See Table 2). We can conclude from the table, as x increase, i.e. from the lower stagnation point of the sphere ($x \approx 0$) and proceed around the sphere up to the point 170°, both values of skin friction and wall temperature are increasing.

TABLE II
VALUES OF SKIN FRICTION $f''(x)$ AND THE WALL TEMPERATURE $\theta(x)$ FOR VARIOUS VALUES OF POSITION x WHEN PRANDTL NUMBER PR=7, VISCOELASTIC PARAMETER K=1 AND PARAMETER FOR NEWTONIAN HEATING $\gamma = 1$

Position x	Skin Friction $f''(x)$	Wall Temperature $\theta_w(x)$
0°	0.000000	4.410999
10°	0.192349	4.421824
20°	0.383475	4.454580
30°	0.572146	4.510096
40°	0.757113	4.589812
50°	0.937095	4.695874
60°	1.116628	4.836514
70°	1.282309	5.006562
80°	1.438746	5.215692
90°	1.584305	5.471331
100°	1.717142	5.783590
110°	1.835103	6.166503
120°	1.935571	6.640135
130°	2.015199	7.234383
140°	2.069450	7.996580
150°	2.091716	9.008849
160°	2.071366	10.436873
170°	2.983848	12.837161

Fig. 2 and Fig. 3 illustrate the behavior of skin friction and wall temperature for various values of viscoelastic parameter K at Prandtl number 7. It shows that when the viscoelastics parameter K increased, it reduced the values of skin friction. Conversely, the opposite effect is observed on values of wall temperature.

The velocity and temperature distributions at the lower stagnation point are given at Prandtl number Pr = 7 with various values of viscoelastics parameter K. These profiles are illustrated in Fig. 4 and Fig. 5. Based on Fig. 4 it is noticed that the velocity distributions are decreased when the values of viscoelastic parameter K are increased until one point (boundary layer thickness $\eta = 1.6$). Later, we can see the profile of velocity distribution increases with the increase of values of viscoelastic parameter.

The values of these profiles are lower for viscoelastic fluid to be compared to Newtonian fluid (viscoelastic parameter K = 0) for the range values of boundary layer thickness $0 < \eta < 1.6$. Therefore, the thickness of the velocity boundary layer for a viscoelastic fluid is higher than the velocity boundary layer thickness for a Newtonian fluid. Fig. 5 shows that increasing the value of viscoelastic parameter leads to higher temperature distribution.

This behavior reflects the coupling of the energy equation to the momentum equation through the temperature dependent body forces. The effect of Pr on the velocity and temperature profiles is illustrated by Fig. 6 and Fig. 7 respectively, near the lower stagnation point of the sphere when Prandtl number Pr = 3, 5 and 7 (comparison purposes), viscoelastic parameter K = 1 and Newtonian heating parameter γ = 1. From Fig. 6, it is found that as Prandtl number Pr increases, the temperature profiles decrease and also the thermal boundary layer thickness. This is because the fluid is highly conductive for small values of the Prandtl number. Physically, if Prandtl number Pr increases, the thermal diffusivity decreases and this phenomenon lead to the decreasing manner of the energy transfer ability that reduces the thermal boundary layer. As expected from Fig. 7**,** it is shown that for viscoelastic parameter K = 1 and Newtonian heating parameter γ = 1, as Prandtl number Pr increases, the velocity profiles are also decrease. It may also be observed that the absolute maxima of the local skin-friction shifts toward the middle of the surface.

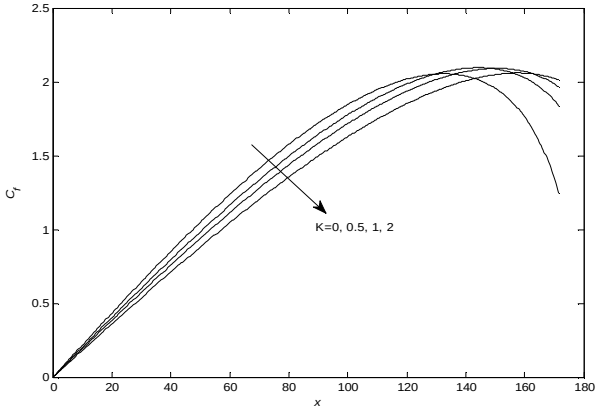

Fig. 2 Variation of skin friction $f''(x)$ for various values of viscoelastic parameter K when Prandtl number Pr=7 and Newtonian heating parameter $\gamma = 1$

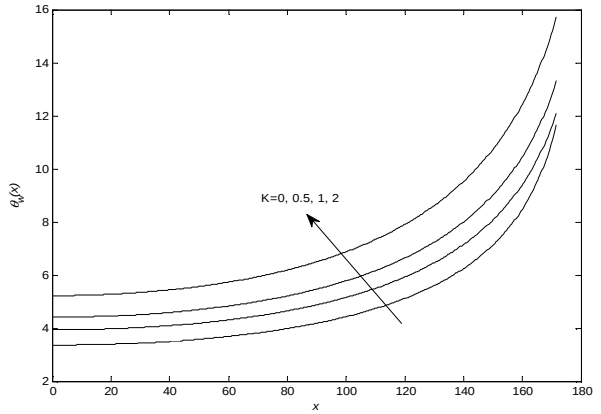

Fig. 3 Variation of wall temperature $\theta(x)$ for various values of viscoelastic parameter K when Prandtl number Pr=7 and Newtonian heating parameter $\gamma = 1$

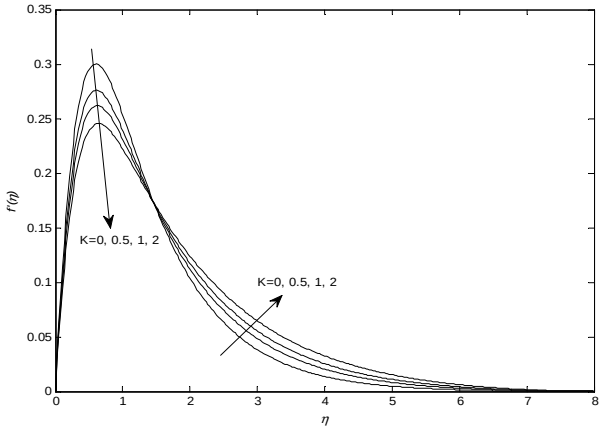

Fig. 4 Velocity profile near the lower stagnation point of the sphere, $(x \approx 0)$ at Prandtl number Pr=7 and Newtonian heating parameter $\gamma = 1$ with various values of viscoelastic parameter K

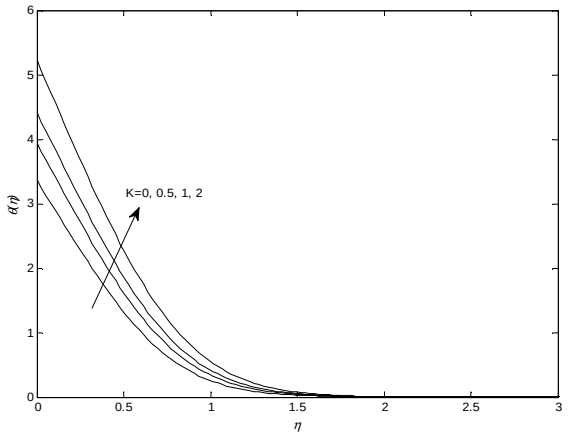

Fig. 5 Temperature profile near the lower stagnation point of the sphere $(x \approx 0)$ at Prandtl number Pr=7 and Newtonian heating parameter $\gamma = 1$ with various values of viscoelastic parameter K

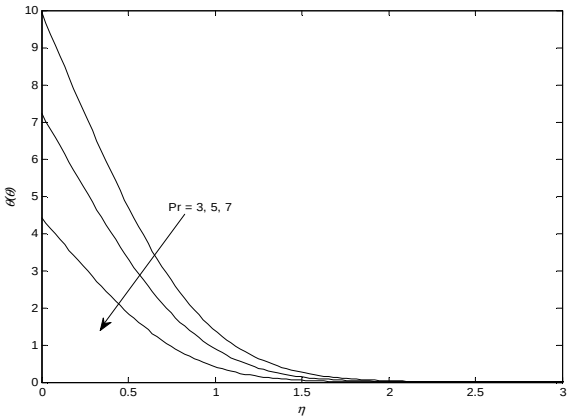

Fig. 6 Temperature profile near the lower stagnation point of the sphere $(x \approx 0)$ at viscoelastic parameter K=1 and Newtonian heating parameter $\gamma = 1$ with various Prandtl number Pr

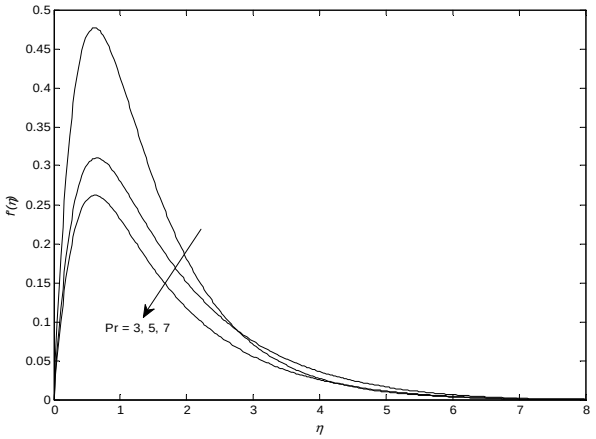

Fig. 7 Velocity profile near the lower stagnation point of the sphere $(x \approx 0)$ at viscoelastic parameter K=1 and Newtonian heating parameter $\gamma = 1$ with various values of Prandtl number Pr

V. Conclusion

The steady natural convection boundary layer flow of a viscoelastic fluid on solid sphere with Newtonian heating has been investigated numerically in this paper. The governing boundary layer equations are transformed into a non-dimensional form and the resulting nonlinear system of partial differential equations is solved numerically using the Keller-box method. This paper has revealed how the viscoelastic parameter K and the Prandtl number Pr affect the flow and heat transfer characteristics.

From the present investigation, the following conclusions can be drawn:

- An increase in the value of Prandtl number leads to decrease both value of velocity and temperature distribution
- It may be observed that the absolute maxima of the local skin-friction shifts toward the middle of the surface.
- As viscoelastic parameter K increase, the value of local skin-friction coefficient C_f decreases and also the value of wall temperatute increase $\theta_w(x)$ against the curvature parameter from the lower stagnation point of the circular cylinder ($x \approx 0$).

Acknowledgment

This work is supported by a research grant (Vot FRGS No. 4F019 & 4F109 and RUG:03J62) from Universiti Teknologi Malaysia, UTM and MOHE.

References

[1] L. Tham, R. Nazar, and I. Pop, "Mixed convection boundary-layer flow about an isothermal solid sphere in a nanofluid," Phys. Scr., vol. 84, 2011.

[2] T. Chiang, A. Ossin, and C. L. Tien, "Laminar free convection from a sphere," J. Heat Trans., vol. 86C, pp. 537–542, 1964.

[3] M. J. Huang, and C. L. Chen, "Laminar free convection from a sphere with blowing and suction," J. Heat Transf., vol. 109, pp. 529–532 ,1987.

[4] R. Nazar, N. Amin, T. Grosan, and I. Pop, "Free convection boundary layer on an isothermal sphere in a micropolar fluids," Int. Commun. Heat Mass Transf., vol. 29, no. 3, pp.377–386, 2002.
[5] R. Nazar, N. Amin, T. Grosan, and I. Pop, "Free convection boundary layer on an isothermal sphere with constant surface heat flux in a micropolar fluids," Int. Commun. Heat Mass Transf., vol. 29, no. 8, pp. 1129–1138, 2002.
[6] M. M. Molla, and M. A. Hossain, "Effects of chemical reaction, heat and mass diffusion in natural convection flow from and isothermal sphere with temperature dependent viscosity," Int. J. Comput.-Aided Eng. Softw., vol. 23, pp. 840–857, 2006.
[7] C. Y. Cheng, "Natural convection heat and mass transfer from a sphere in micropolar fluids with constant wall temperature and concentration," Int. Commun. Heat Mass Transf., vol. 35, pp. 750–755, 2008.
[8] J. H. Merkin, "Natural Convection Boundary-layer Flow on a Vertical Surface with Newtonian Heating," Int. J. Heat Fluid Flow, vol. 15, pp. 392–398, 1994.
[9] M. Z. Salleh, R. Nazar, and I. Pop, "Mixed convection boundary layer flow about a solid sphere with Newtonian heating," Arch. Mech., vol. 62, pp. 283–303, 2010.
[10] M. Z. Salleh, R. Nazar, and I. Pop, "Modeling of free convection boundary layer flow on a sphere with Newtonian heating," Acta Appl. Math., vol. 112, pp. 263–274, 2010.
[11] J. H. Merkin, R. Nazar, and I. Pop, "The development of forced convection heat transfer near a forward stagnation point with Newtonian heating," J. Eng.Math., 2011.
[12] R. H. Thomas, and K. Walters, "The unsteady motion of a sphere in a elastico-viscous liquid," Adv. Tech., 1965.
[13] R. L. Verma, "Elastico-viscous boundary-layer flow on the surface of sphere," Dr. Dietrich Steinkopff Verlag Darmstadt., vol. 16, pp. 510-515, 1977.
[14] E. O. A. Carew and P. Townsend, "Non-Newtonian flow past a sphere in a long cylindrical tube," Rheol. Acta, vol. 27, pp. 125-129, 1988.
[15] T. B. Chang, A. Mehmood, O. Anwar Bég, M. Narahari, M. N. Islam, and F. Ameen, " Numerical study of transient free convective mass transfer in a Walters-B viscoelastic flow with wall suction," Commun. Nonlinear Sci. Numer. Simulat., vol. 16, pp. 216–225, 2011.
[16] A. R. M. Kasim, M. A. Admon, and S.Sharidan, "Free convection boundary layer flow of a viscoelastic fluid in the presence of heat generation," World Academy of Sci. Eng. Tech., vol. 75, pp. 492–499, 2011.
[17] T. Y. Na, Computational Methods in Engineering Boundary Value Problem. New York: Academic Press, 1979.
[18] T. Cebeci, and P. Bradshaw, Physical and Computational Aspects of Convective Heat Transfer. New York: Springer, 1988.