

# Natural and Mixed Convection Heat Transfer Cooling of Discrete Heat Sources Placed Near the Bottom on a PCB

Tapano Kumar Hotta, S P Venkateshan

**Abstract**—Steady state experiments have been conducted for natural and mixed convection heat transfer, from five different sized protruding discrete heat sources, placed at the bottom position on a PCB and mounted on a vertical channel. The characteristic length ( $L_h$ ) of heat sources vary from 0.005 to 0.011 m. The study has been done for different range of Reynolds number and modified Grashof number. From the experiment, the surface temperature distribution and the Nusselt number of discrete heat sources have been obtained and the effects of Reynold number and Richardson number on them have been discussed. The objective is to find the rate of heat dissipation from heat sources, by placing them at the bottom position on a PCB and to compare both modes of cooling of heat sources.

**Keywords**—Discrete heat source, mixed convection, natural convection, vertical channel

## I. INTRODUCTION

HEAT transfer from heated surfaces, in relation to cooling of electronic equipments, has been the area of interest since last decade, because of the advances of the integrated circuit technology. Microminiaturization of electronic components for digital computers, has resulted in increased circuit densities with large power dissipation rates per unit area of components. Rapid advances in semiconductor technology have led to miniaturization in circuit design and hence the amount of heat, that must be dissipated per unit volume of a device continue to increase enormously. For reliable operation of electronic components, their operating temperature must not exceed the manufacturer's prescribed temperature limit, that is usually about 85°C. Furthermore, the temperature difference among the electronic components should be maintained as small as possible to keep the thermal stresses to a minimum value. Mixed convection cooling has been used for wide range of engineering domains, like heat exchangers, electronic equipments and similar industrial applications. So the researchers of electronic cooling have had increased interest for the analysis of heat transfer from the surfaces of discrete heat sources, under mixed convection regime. In many of the above situations, vertical channel is a preferred geometry for mounting of heat generating modules.

Convective cooling of electronic components mounted on a substrate has been the subject of large number of papers since last decade. Baskaya et al. [1] have investigated experimentally, the convective heat transfer from an array of discrete heat sources in a rectangular channel and found that, there will be enhancement of heat transfer, with increase in buoyancy induced secondary flow and onset of instability. Steady state experiments have been performed by Bhowmik et al. [2] in a vertical rectangular channel with water as the working fluid. Based on the experimental results, they have proposed an empirical correlations between Nu, Re and Gr.

Choi and Kim [3] have numerically investigated the 3D conjugate mixed convection heat transfer in a rectangular channel and have proposed a modified 5% rule in an effort to define various flow regimes. Culham et al. [4] have made a review for the available models and correlations for natural and mixed convection heat transfer in a vertical, uniformly heated parallel plate channel. Ermolaev and Zhbanov [5] have done numerical solutions of 2D, unsteady Navier-Stokes equations, for mixed convection heat transfer in horizontal channel, with Boussinesq approximation and obtained the solutions by Galerkin finite element method. Ghasemi and Aminossadati [6] have numerically investigated the cooling performance of electronic devices by emphasizing the effects of various arrangements of electronic components and found that, an increase in Rayleigh number, leads to a significant improvement in heat transfer. Kumar and Rao [7] have numerically investigated, the interaction of surface radiation with conjugate mixed convection heat transfer from a discretely heated vertical plate with three non-identical heat sources, placed along a plate, in the descending order of their heights, from bottom to top end of the plate. La Pica et.al [8] have studied experimentally, the free convection of air in a vertical channel with one of the channel walls heated with uniform heat flux. They have proposed an empirical correlation between Nusselt number and Rayleigh number. Lee et al. [9] have carried out the analysis of conjugate mixed convection from a plate with two iso-flux heat sources mounted in-line with the vertical plate. They have studied the effects of flow velocity, strength of heat sources, spacing between heat sources, emissivity and substrate conductivity, on the rate of heat transfer. Rao and Narasimham [10] have studied numerically the conjugate mixed convection heat transfer arising from protruding heat generating ribs attached to the substrate, forming channel walls and found that, the heat transferred to the working fluid through the substrate, accounts to around 41 – 47% of the total heat removal from the ribs. Sawanta and Rao [11] have solved numerically, the problems

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of combined conduction, mixed convection and surface radiation heat transfer from a vertical electronic board provided with three identical, flush mounted, discrete heat sources and studied the effects of various parameters, like modified Richardson number, surface emissivity and thermal conductivity, on temperature distribution along the board. They found that, surface radiation and buoyancy play an important role for the cooling of electronic modules. Turkoglu and Yucel [12] have numerically analyzed the 2D, laminar mixed convection flow in vertical channels, with a discrete heat source and found that, at low Reynolds numbers, cooling is more effective when the channel width is large ( $W/H > 1$ ), and at high Reynolds numbers it is more effective in narrow channels.

Although number of papers have been published in the general area of cooling of similar size of discrete heat sources mounted on a PCB and placed in horizontal channels but not much work has been done in this area by considering different size of heat sources, which is closely related to a practical problem, used in electronic industries and by taking into account the effect of surface radiation heat transfer on them. Again the vertical channel geometry is rarely studied for the cooling of electronic modules. The earlier work done by sudhakar et al. [13] has been based on heat transfer and optimization studies, by considering similar size of heat sources. So the present work emphasizes the importance of surface radiation along with natural and mixed convection heat transfer, from the surfaces of different size of discrete heat sources mounted on a vertical channel.

## II. EXPERIMENTAL SETUP

The experimental set up used for the present study is shown in Fig. 1. The test section is made by a parallelepiped (1) and consists of four identical wooden boxes (2), whose dimensions are given in Table I. The wooden boxes are filled with silica glass wool to reduce the heat loss through the heat sources (4). A substrate (PCB) (3) of low thermal conductivity material (Bakelite), is fixed on one of the faces of each box. These faces are exposed to ambient. The position of the boxes are adjusted by nut and bolt arrangement (5). So the four boxes form a channel, with heat sources mounted on the substrate of one of the wooden box. The top and bottom face of the test section are open to atmosphere. Rubber gaskets have been provided on the sides of the channel, to prevent the air leak from the corners of the channel. A hole is made in the parallelepiped to insert the anemometer probe, for the measurement of velocity.

The heat sources are made up of aluminum, which are cut into rectangular blocks of sizes as given in Table I, with a cavity made at their center for placing of heater wires and thermocouples. A 80/20 coil type Nichrome wire is used as the heating element and is inserted into the cavity with Teflon tape wound around it, to avoid metal to metal contact. The temperature of heat sources are measured by placing chromel-alumel K-type thermocouples at 4 different positions on the heat source.

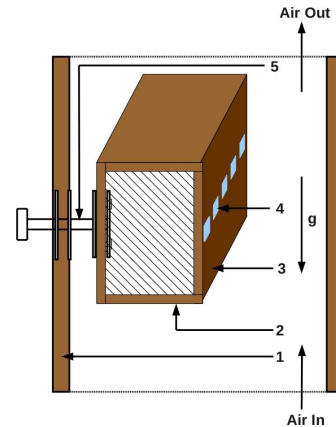


Fig. 1 (a)

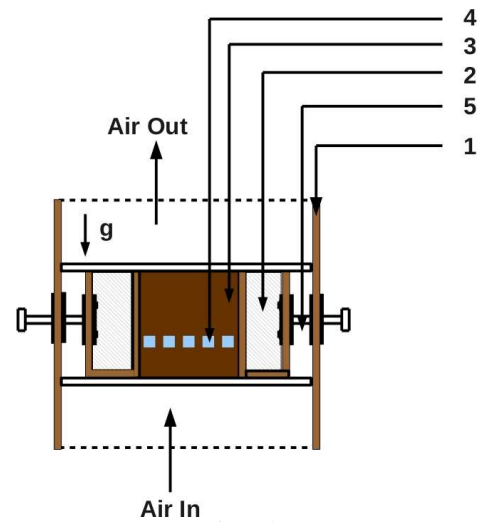


Fig. 1 (b)

Fig. 1 Schematic of the test section (a) Natural convection (b) Mixed convection 1. Parallelepiped 2. Wooden box packed with insulation 3. Substrate 4. Heat source 5. Nut and bolt arrangement

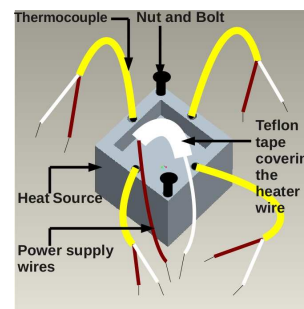


Fig. 2 Schematic of the heat source with heater wires and thermocouples

The beads are prepared by using a capacitance discharge type bead making apparatus. All the thermocouples have been calibrated with a standard thermometer, at ice point, boiling point of water and at room temperature and found that, they are having a maximum measurement error of  $\pm 0.2^\circ \text{C}$ . The

heat sources are embedded onto the substrate and are fitted to it, with four screws of 2 mm diameter. The schematic of the heat source with heater wires and thermocouples is shown in Fig. 2.

TABLE I  
SPECIFICATION OF THE TEST SECTION WITH DISCRETE HEAT SOURCES

Objects	Specification in mm
Heat source 1	15 × 10 × 1
Heat source 2	15 × 7 × 4
Heat source 3	15 × 14 × 6
Heat source 4	30 × 10 × 4
Heat source 5	40 × 15 × 4
Substrate	200 × 200 × 5
Wooden box	200 × 130 × 200 × 130
Parallelepiped	550 × 450 × 450 × 20

The heat input to each of the five heat sources are independently controlled by DC power supplies, which are having a voltage range of 0 - 12 V and a current range of 0 - 2 A. The temperature data of heat sources are recorded by a PC based data acquisition system. The data logger model is 34970A, manufactured by Agilent technologies limited, which can display and also store the data recorded by thermocouples. A digital multimeter is used to cross check the data with DC power supplies. It is connected in series with heater for measuring the current and parallel with heater for voltage measurement. The velocity of air at the inlet of test section is measured by an anemometer (*Airflow™* T A5).

The experiments have been conducted in a low speed vertical wind tunnel, as shown in Fig. 3.

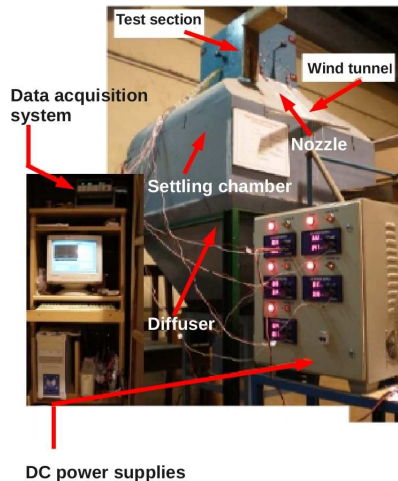


Fig. 3 Vertical wind tunnel used for the experiment, with data acquisition system and DC power supplies

The experimental set up consists of a diffuser, settling chamber, nozzle and test section. All these parts are assembled into a single unit and are supported on a stand. An axial flow

fan is mounted on a separate stand just below the diffuser of the wind tunnel, to avoid vibration, when the fan is ON. The fan is connected to a variable frequency drive to facilitate the variable flow rates in the test section, as required by the experiment. The variable frequency drive runs the fan from a speed range of 60 rpm

to 2500 rpm, giving a flow velocity range of 0.06 m/s to 8 m/s. The settling chamber consists of honeycomb structure, fabricated from Aluminum E channels of dimensions 25 mm × 12.5 mm and two fine wire meshes, which act as anti-turbulent screens, to reduce the velocity fluctuations in the test section and to straighten the flow.

#### A. Experimental Procedure

The procedure of conducting the experiment is as follows. For Natural convection heat transfer experiment, the DC power supply is switched ON and the voltage is adjusted in order to obtain the desired power input for each heat source. The data logger is switched ON and the scan is initiated. When the variation in thermocouple temperature readings are within a range of  $\pm 0.1^\circ\text{C}$  in ten minutes, it is assumed that steady state has been reached and the scanned data are stored in the computer. The voltage, current from the DC power supplies and the temperature from the computer are recorded. The experiments have been repeated for two different power inputs of 0.5 W and 1 W and for both polished and black painted surfaces. For mixed convection heat transfer experiment, the above procedures are repeated by switching ON the axial fan and the fan speed is adjusted to get the required rpm. After steady state is reached, the above parameters are measured along with the air velocity in the channel, which is measured by using an anemometer. Here the experiments have been repeated for power ranges of 1.5 to 3 W in an interval of 0.5 W and for fan speeds of 60 to 300 rpm in an interval of 40 rpm.

In each experiment, the temperature of all the five heat sources and the ambient temperature are recorded and the temperature excess is calculated. The heat transfer coefficient due to convection is calculated and a comparison has been made for all the five heat sources. Then the non-dimensional numbers for natural and mixed convection heat transfer have been calculated and an empirical correlation has been proposed between Nusselt number (Nu), Richardson number (Ri) and characteristic length of the heat sources ( $L_h$ ). The calculations of the above parameters have been done in the following way.

$$\begin{aligned}
 Q_{\text{supplied}} &= VI & (1) \\
 Q_{\text{radiation}} &= F\epsilon\sigma A (T_{\text{heat source}}^4 - T_{\text{ambient}}^4) & (2) \\
 Q_{\text{conduction}} &= k_{\text{substrate}} A (T_{\text{heat source}} - T_{\text{substrate}})/t & (3) \\
 Q_{\text{insulation}} &= k_{\text{insulation}} A (T_{\text{substrate}} - T_{\text{insulation}})/t_i & (4) \\
 Q_{\text{convection}} &= Q_{\text{supplied}} - Q_{\text{radiation}} - Q_{\text{conduction}} - Q_{\text{insulation}} & (5) \\
 h_{\text{convection}} &= Q_{\text{convection}}/A (T_{\text{heat source}} - T_{\text{ambient}}) & (6) \\
 h_{\text{radiation}} &= Q_{\text{radiation}}/A (T_{\text{heat source}} - T_{\text{ambient}}) & (7)
 \end{aligned}$$

### III. RESULTS AND DISCUSSION

#### A. Natural Convection

Five heat sources are placed at the bottom of the substrate in ascending order of their areas as shown in Fig. 4. The

spacing between the heat sources follow the golden mean ratio of 1.618, as reported by Choi and Kim [3], by which they found a significant temperature drop of heat sources. Experiments have been conducted, by supplying a uniform power of 0.5 W to all the heat sources in the first case and in second case, by giving a higher power of 1 W to one of the heat sources and by maintaining others at a uniform power of 0.5 W. So six set of experiments have been conducted for both polished heat sources and by painting them with black paint.

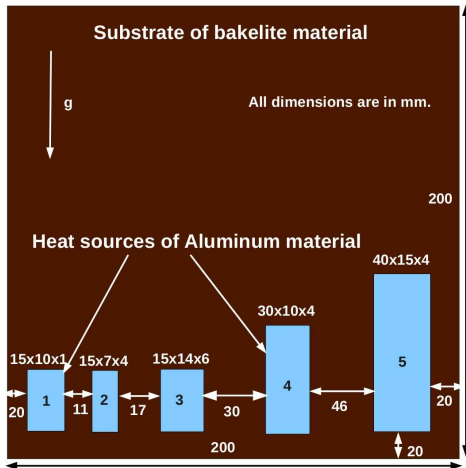


Fig. 4 Placing of five different size of heat sources horizontally at the bottom of the substrate

1. Variation of maximum temperature excess and maximum convective heat transfer coefficient of five heat sources

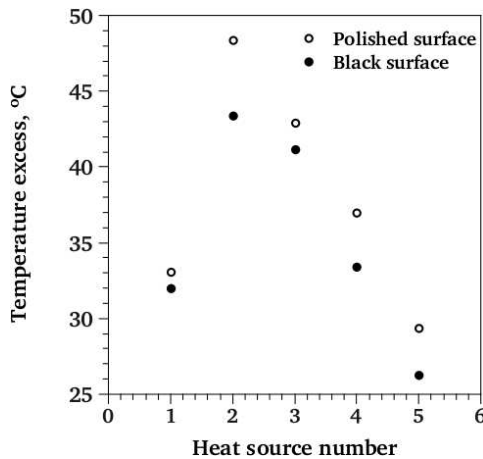


Fig. 5 Variation of maximum temperature excess of five heat sources

It is seen from Fig. 5 that, the minimum temperature among the heat sources have been obtained for heat source 5, in both cases of polished and black painted surfaces. This is because of the fact that, heat source 5 is having highest area among other heat sources, so that the rate of heat transfer from the surface of heat source 5 to surrounding is more, which led to decrease the temperature of this heat source. Also it is surrounded by large cool ambiance and doesn't interact with

the thermal environment of other heat sources. It has been seen that, the heat source 2 is having the highest temperature among other heat sources, because of the fact that it is having least area, and is under the thermal interaction of other heat sources. The temperature drops down from heat source 2 to 5 (in the ascending order of their characteristic length). So it has been found that, size of the heat sources play an important role for their cooling. There is a temperature drop of 3 – 7% from polished to black painted surface, because the emissivity of black painted surface is quite high that is 0.85, because of which the rate of heat transfer from surface of these heat sources to surrounding is more and greater cooling is achieved.

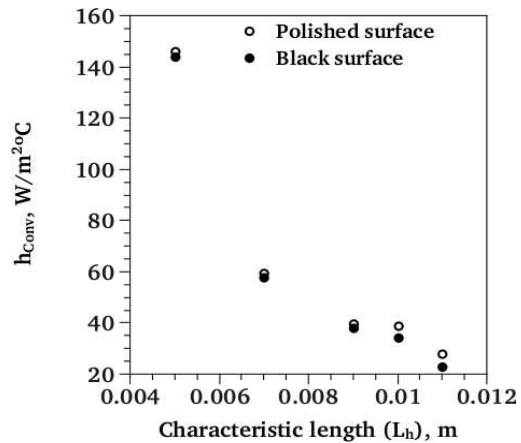


Fig. 6 Variation of maximum convective heat transfer coefficient with the characteristic length of five heat sources

It is seen from Fig. 6 that, the convective heat transfer coefficient for the smallest sized polished heat source is very high and decreases as the characteristic length of the heat sources increase. This is due to the fact that, for smaller width of the heat source, the boundary layer formed on it is very thin, which led to increase the value of heat transfer coefficient. It is seen from the experimental calculation that, the percentage contribution of radiation heat transfer for polished heat sources is only 0.5 – 2%, as their emissivity is very low, that is 0.08 but the contribution is 3 – 7% for black painted surfaces, having an emissivity of 0.85. By increasing the characteristic length of the heat sources, the contribution of radiation heat transfer increases.

#### B. Mixed Convection

Mixed Convection heat transfer experiments have been carried out in a vertical channel for the same configuration as shown in Fig. 4. To delineate the three different regimes of heat transfer, experiments have been conducted for different power ranges of 1.5 - 3 W in an interval of 0.5 W and for different speed ranges of 60 - 300 rpm in an interval of 40 rpm. The experiments have been conducted for a modified Grashof number range of  $2.7 \times 10^{10} \leq Gr^* \leq 1.4 \times 10^{11}$ , Reynold number range of  $5.4 \times 10^3 \leq Re \leq 2.5 \times 10^4$  and Richardson number range of  $0.04 \leq Ri \leq 4.5$ . The results have been discussed below.

1. Variation of temperature excess of five heat sources for different speeds at constant heat input

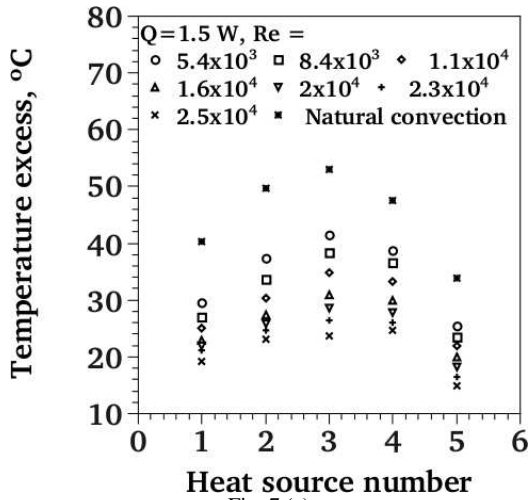


Fig. 7 (a)

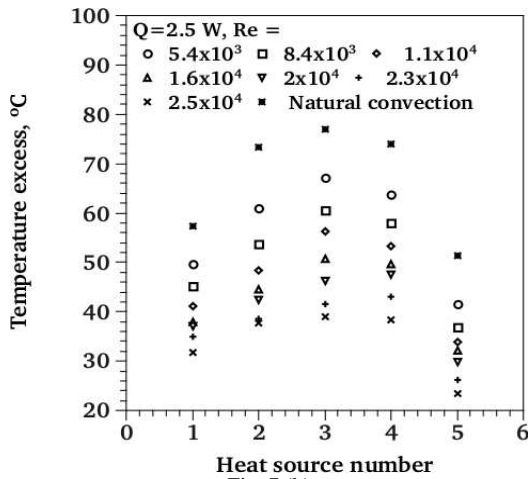


Fig. 7 (b)

Fig. 7 Variation of temperature excess of five heat sources for different speeds, at constant heat input. (a)  $Q=1.5$  W (b)  $Q=3$  W (Only two heat inputs have been shown)

It is seen from Fig. 7 that, the maximum temperature of around  $85^{\circ}\text{C}$  is obtained for heat source 3 (having a characteristic length of  $0.009$  m) at a power input of  $3$  W, at a speed of  $60$  rpm and its temperature drops down as the speed increases ( $Re$  increases) and the power input lowers down. This is due to the fact that, heat source 3 is under the interaction of other two heat sources which are not thermally interacting with ambient. In all cases the heat source 5 (having highest characteristic length of  $0.011$  m) attains the lowest temperature, as it is surrounded by large cool ambiance. It is seen that, when the fan is switched off, that is under the case of natural convection, the heat source temperature goes up to  $95^{\circ}\text{C}$  at the same power input of  $3$  W, which is going beyond the safe temperature limit of heat source. So mixed convection heat transfer is a better method, than that of natural convection heat transfer, as far as cooling of discrete heat sources are considered.

2. Variation of convective heat transfer coefficient with different speeds for five heat sources at constant heat input

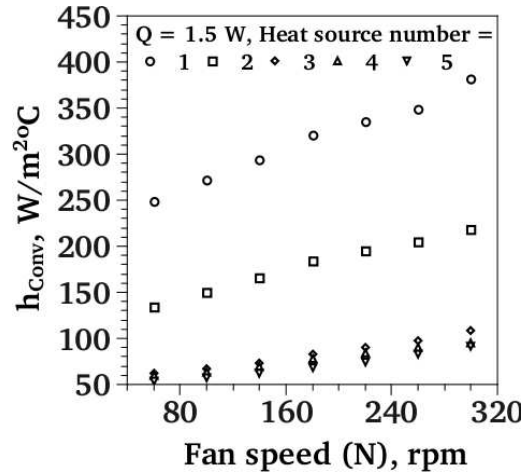


Fig. 8 (a)

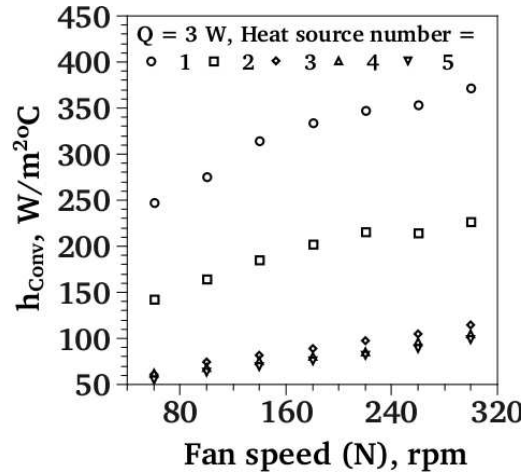


Fig. 8 (b)

Fig. 8 Variation of convective heat transfer coefficient with different speeds for five heat sources, at constant heat input (a)  $Q=1.5$  W (b)  $Q=3$  W (Only two heat inputs have been shown)

It is seen from Fig. 8 that, by increasing the fan speed, the heat transfer coefficient of heat sources increase, leading to better cooling. This is because of the fact that, by increasing the speed, the temperature of heat sources fall down, which led to increase the value of heat transfer. It is seen that, for different heat inputs, there is no significant change in the heat transfer coefficient of heat sources, so the heat input to the heat sources has no effect on the heat transfer coefficient. Among the heat sources, the heat transfer coefficient drops down from heat source 1 to 5 (in the ascending order of their characteristic length). So the size of the heat sources plays an important role for their cooling.

3. Variation of Nusselt Number (Nu) with Richardson Number (Ri) and Reynold number at different heat inputs for five heat sources

When the Nusselt number has been plotted against Richardson number for different heat sources, it is seen from

Fig. 9 that, the value of Nusselt number decreases with increase of Richardson number. From this graph, the three regimes of heat transfer have been delineated, as  $Ri \leq 1$  leads to forced convection,  $Ri \approx 1$  leads to mixed convection and  $Ri \geq 1$  leads to natural convection heat transfer. Similar trends have been found for all the heat sources. The value of Nusselt number decreases from heat source 1 to 5 (in the ascending order of their characteristic length), which is due to the fact that by increasing the size of the heat sources, their convective heat transfer coefficient reduce, leading to decrease in the value of Nusselt number. So it is seen that, size of the heat sources play an important role for their cooling.

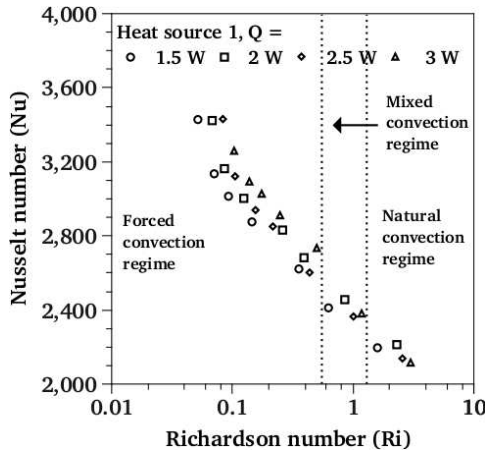


Fig. 9 (a)

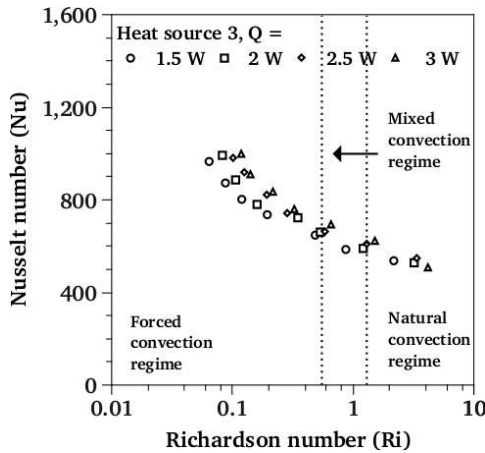


Fig. 9 (b)

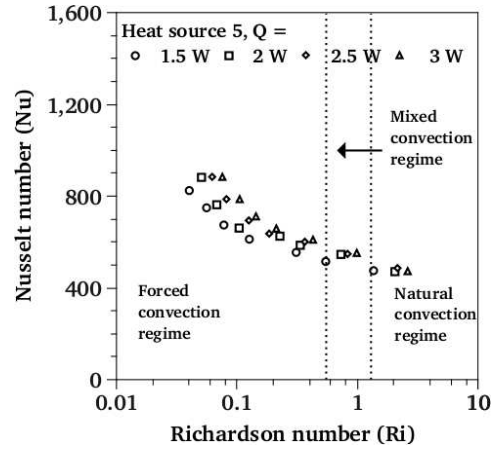


Fig. 9 (c)

Fig. 9 Variation of Nusselt number (Nu) with Richardson number (Ri) at different heat inputs for five heat sources (a) Heat source 1 (b) Heat source 3 (c) Heat source 5 (Only three heat sources have been shown)

When the Nusselt number has been plotted against Reynold number for different heat sources, it is found from Fig. 10 that, the value of Nusselt number increases with the increase in Reynold number and all the values lie on a line. This is due to the fact that, by increasing the speed, the value Reynold number increases and also there is a rise in the value of heat transfer coefficient. The Nusselt number value falls down from heat source 1 to 5 (in the ascending order of their characteristic length) and are due to the same reason as explained earlier.

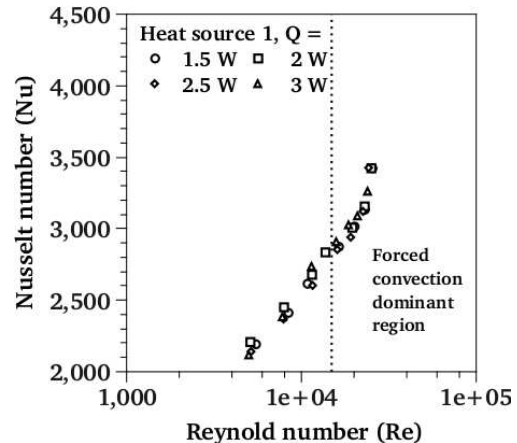


Fig. 10 (a)



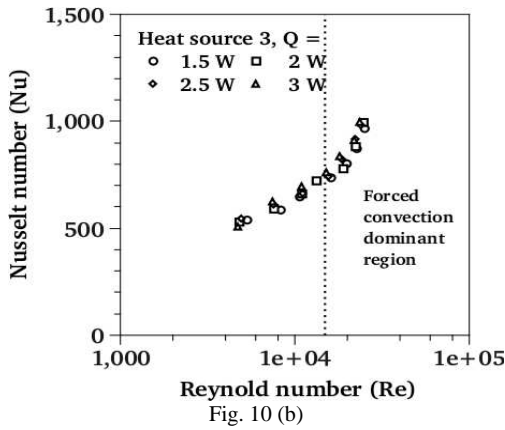


Fig. 10 (b)

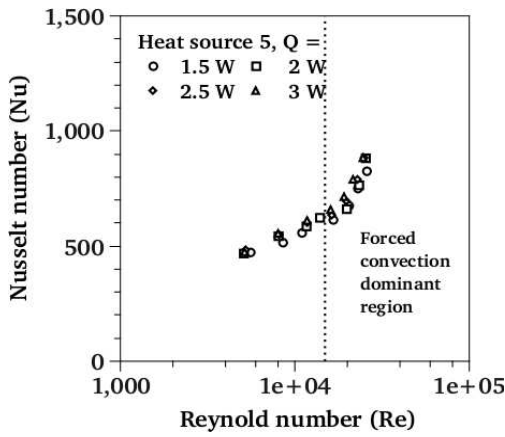


Fig. 10 (c)

Fig. 10 Variation of Nusselt number (Nu) with Reynold number (Re) at different heat inputs for five heat sources (a) Heat source 1 (b) Heat source 3 (c) Heat source 5 (Only three heat sources have been shown)

C. Proposed Correlations

Based on the above results an empirical correlation between Nusselt number, Richardson number and the characteristic length of heat sources has been proposed. This correlation is based on 140 data points and have an index of correlation of 0.97. The percentage RMS error is found to be 15%. The relation is as follows

$$Nu = 6.5 \left( \frac{Ri}{1 + Ri} \right)^{-0.18} \left( 1 + \frac{H}{L_h} \right)^{3.22}$$

This equation is valid for following range of parametes.

$$472 \leq Nu \leq 3430$$

$$0.04 \leq Ri \leq 4.5$$

$$2.8 \leq H/L_h \leq 5$$

Based on the above correlation a parity plot has been plotted between Nusselt number data and Nusselt number fit, as shown in Fig. 11 and it has been found that, the scatter in the data falls within a range of ± 20%.

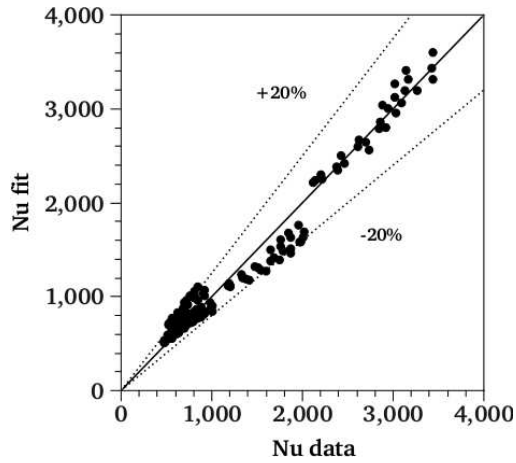


Fig. 11: Parity plot showing the agreement of Nusselt number data (Nu data) and Nusselt number fit (Nu fit)

IV. EXPERIMENTAL UNCERTAINTY

The uncertainty in the primary quantities have been obtained by calibration of the instruments and for derived quantities it is calculated, by using the uncertainty value of primary quantities and the formula given in [14]. The uncertainty value of different quantities are shown in Table II. The formula for calculation of uncertainty is given below.

$$\Delta y = \sqrt{\sum \left( \frac{\partial y}{\partial x} \right)^2 (\Delta x)^2}$$

TABLE II  
UNCERTAINTY IN PHYSICAL QUANTITIES

Sl. No	Measured quantities	% Uncertainties
1	Current	± 3.23
2	Voltage	± 1.33
3	Temperature	± 6.25
4	Power input	± 7

where y is the derived quantity, x is the measured quantity and Δx is the error involved in the measured quantity.

V. CONCLUSION

The following observations have been made from the experimental study. For both natural and mixed convection heat transfer regime, when the heat sources are placed horizontally near the bottom on the PCB, the largest size heat generating module should be placed at the position 5 or 1, where the minimum of the maximum temperature excess is comparable with other positions. Radiation heat transfer plays a significant role, as the temperature of heat sources reduce by 3 – 7%, by painting them with black paint, leading to better cooling. The size of the heat sources has a great impact on the heat transfer coefficient. From mixed convection heat transfer experiment it is seen that, the heat input to the heat sources has

no effect on their heat transfer coefficient, and the value increases by increasing the speed and characteristic length of heat sources. The three regimes of heat transfer have been delineated on the basis of Richardson number (Ri) and is seen that the value of Nusselt number decreases with increase in Richardson number and increases with increase in Reynold number. Mixed convection is a better method than the natural convection heat transfer, as far as cooling of discrete heat sources concerned.

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