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Multiple soliton solutions of (2+1)-dimensional potential Kadomtsev-Petviashvili equation

Mohammad Najafi, Ali Jamshidi

Abstract—We employ the idea of Hirota's bilinear method, to obtain some new exact soliton solutions for high nonlinear form of (2+1)-dimensional potential Kadomtsev-Petviashvili equation. Multiple singular soliton solutions were obtained by this method. Moreover, multiple singular soliton solutions were also derived.

Keywords—Hirota bilinear method, potential Kadomtsev-Petviashvili equation, Multiple soliton solutions, Multiple singular soliton solutions.

I. Introduction

Many important phenomena and dynamic processes in physics, mechanics, chemistry and biology can be represented by nonlinear partial differential equations. The study of exact solutions of nonlinear evolution equations plays an important role in soliton theory and explicit formulas of nonlinear partial differential equations play an essential role in the nonlinear science. Also, the explicit formulas may provide physical information and help us to understand the mechanism of related physical models.

In recent years, many kinds of powerful methods have been proposed to find solutions of nonlinear partial differential equations, numerically and/or analytically, e.g., the tanh function method [1], the homogeneous balance method [2], the tanhcoth method [3], the Exp-function method [4], the decomposition method [5] and the improved tanh function method [6].

In this paper, by means of the Hirota's bilinear method and wronskian form, we will obtain some exact and new solutions for the (2 + 1)-dimensional potential Kadomtsev-Petviashvili equation. In the following section we have a brief review on the Hirota's bilinear method and in Section 3 and 4, we apply the Hirota's bilinear method to obtain multiple soliton solutions and multiple singular soliton solutions of the (2 + 1)-dimensional potential Kadomtsev-Petviashvili equation . Finally, the paper is concluded in Section 5.

II. The Hirota bilinear method

To formally derive N-soliton solutions for completely integrable equations, we will use the Hirotas direct method combined with the simplified version of [7]–[9]. It was proved by many that soliton solutions are just polynomials of exponentials. This will be also confirmed in the coming discussions. We first substitute

$$u(x,t) = \mathrm{e}^{kx+my-ct}, \tag{1}$$

into the linear terms of any equation under discussion to determine the relation between k and c. We then substitute the Cole-Hopf transformation

$$u(x,y,t) = R\,(\ln f(x,y,t))_{xx}, \tag{2}$$

into the equation under discussion, where the auxiliary function f, for the single soliton solution, is given by

$$f(x,y,t) = 1 + C_1\, f_1(x,y,t) = 1 + C_1\,\mathrm{e}^{\theta_1}. \tag{3}$$

The steps of the Hirotas method as summarized in [10]–[13] are as follows:

(i) For the relation between k_i and c_i, we use

$$u(x,y,t) = \mathrm{e}^{\theta_i} \quad , \quad \theta_i = k_i\,x + m_i\,y - c_i\,t, \tag{4}$$

(ii) For single soliton, we use

$$f = 1 + C_1\,\mathrm{e}^{\theta_1}, \tag{5}$$

to determine R.

(iii) For two-soliton solutions, we use

$$f = 1 + C_1\,\mathrm{e}^{\theta_1} + C_2\,\mathrm{e}^{\theta_2} + C_1\,C_2\,a_{12}\,\mathrm{e}^{\theta_1+\theta_2}, \tag{6}$$

to determine the phase shift coefficient a_{12}, and hence can be generalized for a_{ij},$1 \le i < j \le 3$.

(iv) For three-soliton solutions, we use

$$\begin{aligned} f = {} & 1 + C_1\,\mathrm{e}^{\theta_1} + C_2\,\mathrm{e}^{\theta_2} + C_3\,\mathrm{e}^{\theta_3} + C_1\,C_2\,a_{12}\,\mathrm{e}^{\theta_1+\theta_2} \\ & + C_1\,C_3\,a_{13}\,\mathrm{e}^{\theta_1+\theta_3} + C_2\,C_3\,a_{23}\,\mathrm{e}^{\theta_2+\theta_3} \\ & + C_1\,C_2\,C_3\,b_{123}\,\mathrm{e}^{\theta_1+\theta_2+\theta_3}, \end{aligned} \tag{7}$$

to determine b_{123}. Pekcan proved in [14], $b_{123} = a_{12}\,a_{23}\,a_{13}$, then the equation gives rise to three-soliton solutions.

In the following, we will apply the aforementioned steps to potential Kadomtsev-Petviashvili equation. Multiple soliton solutions are obtained for $C_1 = C_2 = C_3 = 1$. However, multiple singular soliton solutions are obtained if $C_1 = C_2 = C_3 = -1$.

III. Multiple soliton solutions of the potential Kadomtsev-Petviashvili equation:

In this paper, we investigate explicit formula of soliton solutions of the following high nonlinear form of (2 + 1)-dimensional potential Kadomtsev-Petviashvili equation given in [1],

$$u_{xt} + \frac{1}{4}\,u_{xxxx} + \frac{3}{2}\,u_x\,u_{xx} + \frac{3}{4}\,u_{yy} = 0 \tag{8}$$

M. Najafi is with the Department of Mathematics, Khorasgan Branch, Islamic Azad University, Isfahan, Iran e-mail: (mnajafi82@gmail.com).

A. Jamshidi is with the Department of Mathematics, Khorasgan Branch, Islamic Azad University, Isfahan, Iran.

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where $u = u(x, y, z, t) : \mathbb{R}_x \times \mathbb{R}_y \times \mathbb{R}_t \to \mathbb{R}$.
To determine multiple-soliton solutions for Eq. (8), we follow the steps presented above. We first consider $C_1 = C_2 = C_3 = 1$. Substituting

$$u\,(x, y, t) = \mathrm{e}^{\theta_i}, \quad \theta_i = k_i x + m_i y - w_i t \tag{9}$$

into the linear terms of Eq.(8) to find the relation

$$w_i = \frac{{k_i}^4 + 3\,{m_i}^2}{4\,k_i}, \qquad i = 1, 2, \ldots, N \tag{10}$$

and consequently, θ_i becomes

$$\theta_i = k_i x + m_i y - \frac{{k_i}^4 + 3\,{m_i}^2}{4\,k_i}\,t. \tag{11}$$

To determine R, we substitute

$$u(x, y, t) = R\,(\ln f(x, y, t))_x \tag{12}$$

where

$$f(x, y, t) = 1 + f_1(x, y, t) = 1 + \mathrm{e}^{k_1 x + m_1 y - \frac{{k_1}^4 + 3\,{m_1}^2}{4\,k_1}}$$

into Eq.(8) and solve to find that $R = 2$.
This means that the single singular soliton solution is given by

$$u\,(x, y, t) = 2\,\frac{k_1 \mathrm{e}^{k_1 x + m_1 y - \frac{{k_1}^4 + 3\,{m_1}^2}{4\,k_1}}}{1 + \mathrm{e}^{k_1 x + m_1 y - \frac{{k_1}^4 + 3\,{m_1}^2}{4\,k_1}}}. \tag{13}$$

For the two-soliton solutions, we substitute

$$u(x, y, t) = 2\,(\ln f(x, y, t))_x, \tag{14}$$

where

$$f(x, y, t) = 1 + \mathrm{e}^{\theta_1} + \mathrm{e}^{\theta_2} + a_{12}\,\mathrm{e}^{\theta_1 + \theta_2}, \tag{15}$$

into Eq.(8), where θ_1 and θ_2 are given in Eq.(11) to obtain

$$a_{12} = \frac{-4\,(k_1 - k_2)\,(w_1 - w_2) + (k_1 - k_2)^4 + 3\,(m_1 - m_2)^2}{4\,(k_1 + k_2)\,(w_1 + w_2) - (k_1 + k_2)^4 - 3\,(m_1 + m_2)^2}, \tag{16}$$

and

$$w_s = \frac{{k_s}^4 + 3\,{m_s}^2}{4\,k_s}, \qquad s = 1, 2,$$

for $|k_i| \neq |k_2|$ and $|m_1| \neq |m_2|$, hence

$$a_{ij} = \frac{-4\,(k_i - k_j)\,(w_i - w_j) + (k_i - k_j)^4 + 3\,(m_i - m_j)^2}{4\,(k_i + k_j)\,(w_i + w_j) - (k_i + k_j)^4 - 3\,(m_i + m_j)^2}, \tag{17}$$

and

$$w_s = \frac{{k_s}^4 + 3\,{m_s}^2}{4\,k_s}, \qquad s = 1, 2, 3,$$

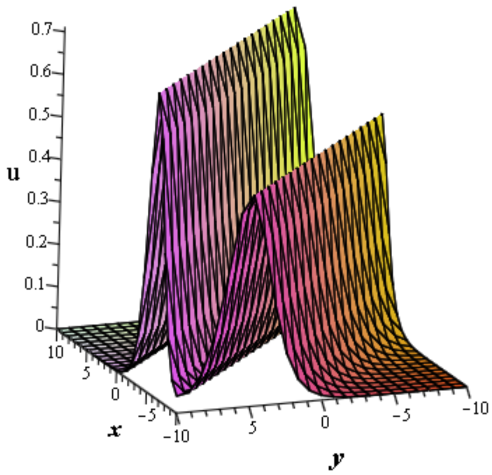

Fig. 1. The two soliton solution with $k_1 = 1$ and $k_2 = -1.2$.

for $|k_i| \neq |k_j|$ and $|m_i| \neq |m_j|$.
It is interesting to point out that for $m_s = k_s, s = 1, 2, 3$, the phase shift reduces to

$$a_{12} = \frac{(k_1 - k_2)^2}{(k_1 + k_2)^2}, \tag{18}$$

for $|k_1| \neq |k_2|$, hence

$$a_{ij} = \frac{(k_i - k_j)^2}{(k_i + k_j)^2}, \tag{19}$$

for $|k_i| \neq |k_j|$. This in turn gives

$$f\,(x, y, t) = 1 + \mathrm{e}^{\theta_1} + \mathrm{e}^{\theta_2} + \frac{(k_1 - k_2)^2}{(k_1 + k_2)^2}\,\mathrm{e}^{\theta_1 + \theta_2}, \tag{20}$$

where

$$\theta_i = k_i x + k_i y - \frac{1}{4} k_1 \left({k_1}^2 + 3\right) t, i = 1, 2, \tag{21}$$

which is a two soliton solution(Fig. 1).
Similarly, to determine the three soliton solutions, we set

$$f(x, y, t) = 1 + \mathrm{e}^{\theta_1} + \mathrm{e}^{\theta_2} + \mathrm{e}^{\theta_3} + a_{12}\,\mathrm{e}^{\theta_1 + \theta_2} + a_{13}\,\mathrm{e}^{\theta_1 + \theta_3} + a_{23}\,\mathrm{e}^{\theta_2 + \theta_3} + a_{12}\,a_{23}\,a_{13}\,\mathrm{e}^{\theta_1 + \theta_2 + \theta_3}. \tag{22}$$

To determine the three soliton solutions explicitly, we substitute the last result for $f(x, y, t)$ into Eqs. (28), (See Fig. 2).

The higher level soliton solutions, for $n \geq 4$ can be obtained in a parallel manner. The obtained results confirm that the (2+1)-dimensional potential Kadomtsev-Petviashvili equation is completely integrable and possesses multiple soliton solutions of any order.

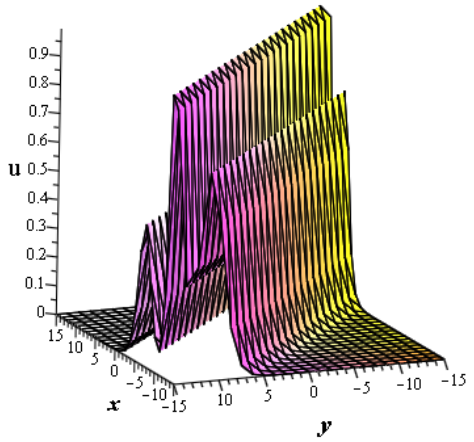

Fig. 2. The three soliton solution with $k_1 = -1, k_2 = 1.2$ and $k_3 = 1.6$.

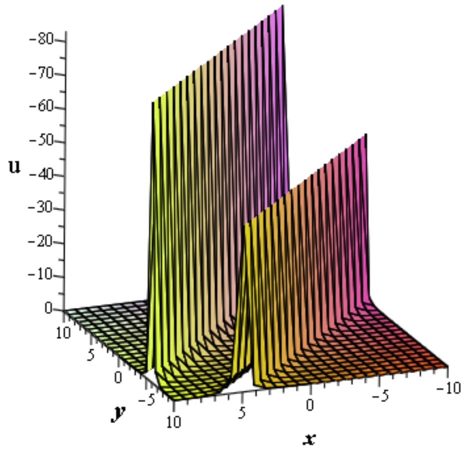

Fig. 3. The two soliton solution with $k_1 = 1$ and $k_2 = -1.2$.

IV. Multiple singular soliton solutions of the potential Kadomtsev-Petviashvili equation :

We first consider $C_1 = C_2 = C_3 = -1$. Substituting

$$u\,(x,y,t) = \mathrm{e}^{\theta_i}, \quad \theta_i = k_i x + m_i y - w_i t \tag{23}$$

into the linear terms of Eq.(8) to find the relation

$$w_i = \frac{{k_i}^4 + 3\,{m_i}^2}{4\,k_i}, \qquad i = 1, 2, \ldots, N \tag{24}$$

and consequently, θ_i becomes

$$\theta_i = k_i x + m_i y - \frac{{k_i}^4 + 3\,{m_i}^2}{4\,k_i}\,t. \tag{25}$$

To determine R, we substitute

$$u(x,y,t) = R\,(\ln f(x,y,t))_x, \tag{26}$$

where $f(x,y,t) = 1 - f_1(x,y,t) = 1 + \mathrm{e}^{k_1 x + m_1 y + k_1^3\,t}$ into Eq.(8) and solve to find that $R = 2$.
This means that the single singular soliton solution is given by

$$u\,(x,y,t) = -2\,\frac{k_1 \mathrm{e}^{k_1 x + m_1 y - \frac{{k_1}^4 + 3\,{m_1}^2}{4\,k_1}}}{1 + \mathrm{e}^{k_1 x + m_1 y - \frac{{k_1}^4 + 3\,{m_1}^2}{4\,k_1}}}. \tag{27}$$

For the two-soliton solutions, we substitute

$$u(x,y,t) = 2\,(\ln f(x,y,t))_x \tag{28}$$

where

$$f(x,y,t) = 1 - \mathrm{e}^{\theta_1} - \mathrm{e}^{\theta_2} + a_{12}\,\mathrm{e}^{\theta_1+\theta_2}, \tag{29}$$

into Eq.(8), where θ_1 and θ_2 are given in Eq.(25) to obtain

$$a_{12} = \frac{-4\,(k_1-k_2)\,(w_1-w_2) + (k_1-k_2)^4 + 3\,(m_1-m_2)^2}{4\,(k_1+k_2)\,(w_1+w_2) - (k_1+k_2)^4 - 3\,(m_1+m_2)^2}, \tag{30}$$

and

$$w_s = \frac{{k_s}^4 + 3\,{m_s}^2}{4\,k_s}, \qquad s = 1, 2,$$

for $|k_i| \neq |k_2|$ and $|m_1| \neq |m_2|$, hence

$$a_{ij} = \frac{-4\,(k_i-k_j)\,(w_i-w_j) + (k_i-k_j)^4 + 3\,(m_i-m_j)^2}{4\,(k_i+k_j)\,(w_i+w_j) - (k_i+k_j)^4 - 3\,(m_i+m_j)^2}, \tag{31}$$

and

$$w_s = \frac{{k_s}^4 + 3\,{m_s}^2}{4\,k_s}, \qquad s = 1, 2, 3,$$

for $|k_i| \neq |k_j|$ and $|m_i| \neq |m_j|$.

It is interesting to point out that for $m_s = k_s, s = 1, 2, 3,$ the phase shift reduces to

$$a_{12} = \frac{(k_1-k_2)^2}{(k_1+k_2)^2}, \tag{32}$$

for $|k_1| \neq |k_2|$, hence

$$a_{ij} = \frac{(k_i-k_j)^2}{(k_i+k_j)^2}, \tag{33}$$

for $|k_i| \neq |k_j|$. This in turn gives

$$f\,(x,y,t) = 1 - \mathrm{e}^{\theta_1} - \mathrm{e}^{\theta_2} + \frac{(k_1-k_2)^2}{(k_1+k_2)^2}\,\mathrm{e}^{\theta_1+\theta_2}, \tag{34}$$

where

$$\theta_i = k_i x + k_i y - \frac{1}{4} k_1\left({k_1}^2 + 3\right)t, i = 1, 2, \tag{35}$$

which is a two soliton solution(Fig. 3).

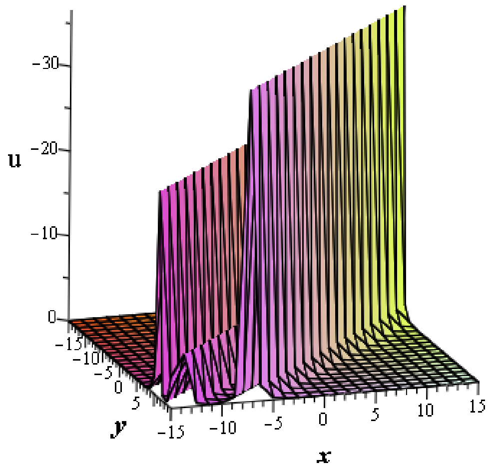

Fig. 4. The three soliton solution with $k_1 = -1, k_2 = 1.2$ and $k_3 = -1.6$.

Similarly, to determine the three soliton solutions, we set

$$f(x,y,t) = 1 - \mathrm{e}^{\theta_1} - \mathrm{e}^{\theta_2} - \mathrm{e}^{\theta_3} + a_{12}\,\mathrm{e}^{\theta_1+\theta_2} + a_{13}\,\mathrm{e}^{\theta_1+\theta_3}$$

$$+a_{23}\,\mathrm{e}^{\theta_2+\theta_3} - a_{12}\,a_{23}\,a_{13}\,\mathrm{e}^{\theta_1+\theta_2+\theta_3}. \tag{36}$$

To determine the three soliton solutions explicitly, we substitute the last result for $f(x,y,t)$ into Eqs. (28),(See Fig. 4).

The higher level singular soliton solutions, for $n \geq 4$ can be obtained in a parallel manner.

V. Conclusion

In this paper, by using the Hirota bilinear method, we obtained some explicit formulas of solutions for the (2+1)-dimensional potential Kadomtsev-Petviashvili equation. Multiple soliton solutions were formally derived. Moreover, multiple singular soliton solutions of any order was derived as well. The results of other works are special cases of our results.

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