Multiobjective Optimal Power Flow Using Hybrid Evolutionary Algorithm

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Abstract—This paper solves the environmental/ economic dispatch power system problem using the Non-dominated Sorting Genetic Algorithm-II (NSGA-II) and its hybrid with a Convergence Accelerator Operator (CAO), called the NSGA-II/CAO. These multiobjective evolutionary algorithms were applied to the standard IEEE 30-bus six-generator test system. Several optimization runs were carried out on different cases of problem complexity. Different quality measure which compare the performance of the two solution techniques were considered. The results demonstrated that the inclusion of the CAO in the original NSGA-II improves its convergence while preserving the diversity properties of the solution set.

Keywords—optimal power flow, multiobjective power dispatch, evolutionary algorithm

I. INTRODUCTION

THE basic goal of any solution to the optimal power flow (OPF) problem is to determine the dispatch of generators so as to meet the load demand while minimizing the total fuel cost, subject to the satisfaction of all constraints on the system. More objectives have recently been incorporated into the OPF problem. These include optimization of active/reactive losses, power plant emissions, voltage profile and stability. This has extended the definition of the OPF problem from a single objective case to a multiobjective one. Many strategies have been adopted to obtain an optimal solution for this multiobjective OPF problem. One approach is to form a weighted sum of the multiple objectives in order to create a composite objective function [1]. Another approach chooses one of the objective functions and treats the rest of the objectives as constraints by limiting each of them within certain pre-defined limits [2]. The problem with these methods is that they require specification of weighting coefficients or pre-defined limits which affect the quality of solutions obtained. Also, as common with analytical methods, several runs have to be performed in order to obtain different and acceptable non-dominated solutions for the decision maker to choose from. Recently, multiobjective evolutionary algorithms (MOEAs) have been applied to solve the multiobjective OPF. Their population-based nature makes it possible for them to yield multiple Pareto-optimal solutions in a single run.

The Environmental/Economic Dispatch (EED) multiobjective problem seeks to simultaneously minimize both fuel cost and the emissions produced by power plants. Environmental concerns on the effect of SO_2 and NO_X emissions produced by the fossil-fueled power plants led to the inclusion of

minimization of emissions as an objective in the OPF formulation. The Niche Pareto Genetic Algorithm (NPGA), the Non-dominated Sorting Genetic Algorithm (NSGA) and the Strength Pareto Evolutionary Algorithm (SPEA) were used in [3],[4],[5] to solve the EED problem and in [6], a comparative study of the performance of the mentioned approaches was done. The EED problem has also been solved using the NSGA-II [7] while in [8], a stochastic approach developed around the NSGA-II was used.

The set of solutions achieved by a multiobjective optimization algorithm is required to satisfy both convergence and diversity criteria. The convergence criterion requires that the obtained approximation set should be as close as possible to the true Pareto front while the diversity criterion requires that it should be well-spread and covering wide areas of the Pareto front. Moreover, when dealing with real-world applications, it is desired that the approximation set be achieved within an acceptable amount of time and a limited budget of objective function evaluations. Although elitist MOEAs such as the NSGA-II do well in progressing close to the Pareto-optimal set with a good distribution of solutions because of the inclusion of explicit diversity-preservation operator in them, they do not guarantee true convergence to the Pareto-optimal set [9]. This is because the diversity-preservation operator always emphasizes the less crowded solutions in the non-dominated set causing some solutions that are already Pareto-optimal to be replaced by non-Pareto ones. It can be concluded from the foregoing that elitist multiobjective optimization evolutionary algorithms cannot simultaneously meet the requirements of convergence to the Pareto front and maintenance of a diverse solution set; hence the need to incorporate a mechanism that improves convergence to the Pareto front while maintaining a good spread of solutions [10].

A portable Convergence Accelerator Operator (CAO) for incorporation into multiobjective optimization evolutionary algorithms was proposed in [10],[11] and was incorporated into both NSGA-II and SPEA2 to form NSGA-II/CAO and SPEA2/CAO. It was demonstrated that inclusion of the CAO improved the convergence property of the solution set obtained by these multiobjective evolutionary algorithms.

In this study, the EED problem is solved using the NSGA-II and its CAO-hybridized form, the NSGA-II/CAO. Standard performance metrics are then used to compare the solution sets obtained by using the two algorithms.

II. EED PROBLEM FORMULATION

The environmental/economic dispatch problem involves the simultaneous optimization of two competing objective func-

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tions, fuel cost and emission while satisfying equality and inequality constraints. The problem is formulated as follows.

A. Minimization of Fuel Cost

The objective function for total fuel cost $F(P_G)$ in dollars per hour can be represented by the quadratic function:

$$F(P_G) = \sum_{i=1}^{N} a_i P_{Gi}^2 + b_i P_{Gi} + c_i \tag{1}$$

where N: number of generators

a_i, b_i, c_i: fuel cost of generator, i

P_{Gi}: real power output of the ith generator

P_G: vector of real power outputs of generators and defined as

$$P_G = [P_{G_1}, P_{G_2}, \ldots, P_{G_N}]^T.$$

B. Minimization of Emission

The total emission $E(P_G)$ in tons per hour of atmospheric pollution such as sulphur oxides (SO_2)and nitrogen oxides NO_x caused by the operation of fossil-fueled thermal generation can be expressed [6] as

$$E(P_G) = \sum_{i=1}^{N} 10^{-2}(d_i P_{Gi}^2 + e_i P_{Gi} + f_i) + g_i exp(h_i P_{Gi}) \tag{2}$$

where d_i, e_i, f_i, g_i and h_i are coefficients of the ith generator emission characteristics.

C. Constraints

The optimization problem is bounded by the following constraints:

1) Power Balance Constraints: There must exist a balance between the total electric power generation, the total load on demand as well as the real power loss in transmission lines. This is given by

$$\sum_{i=1}^{N} P_{Gi} - P_D - P_L = 0 \tag{3}$$

where P_D is the total load (MW) and P_L is the transmission loss (MW). P_L is determined by solving the load flow problem using the Newton-Raphson method.

2) Power Generation Limits: The power generated P_{Gi} by each generator should lie between its minimum and maximum limits i.e.

$$P_{Gimin} \leq P_{Gi} \leq P_{Gimax} \tag{4}$$

where P_{Gimin} is the minimum power generated and P_{Gimax} is the maximum power generated

D. Formulation

The multiobjective environmental/economic dispatch problem is therefore formulated as

$$\begin{aligned} minimize \quad & [F(P_G), E(P_G)] \\ subject\ to: \quad & g(P_G) = 0 \\ & h(P_G) \leq 0, \end{aligned} \tag{5}$$

where g is the equality constraint representing the power balance, while h is the inequality constraint representing the generation capacity.

III. NSGA-II AND NSGA-II/CAO

In this section, we describe briefly the NSGA-II algorithm and the component hybridized with it, the CAO.

A. The Non-Dominated Sorting Genetic Algorithm-II (NSGA-II)

NSGA-II [12] is an elitist MOEA that uses a fast non-dominated sorting as well as an efficient crowding-distance assignment approach. It involves the creation of an initial random population of solutions P_0 of size N which is sorted into different levels of non-domination. Each solution is assigned fitness equal to its non-domination level where level 1 is the best level. Binary tournament selector with a crowded tournament operator, recombination and mutation operators are then used to create an offspring population Q of size N. The parent and offspring populations are combined and a non-dominated sorting is performed. Thereafter, solutions from better non-dominated sets are selected to propagate to the next generation, one set at a time until the population is filled. If the available population slots are not adequate to accommodate all solutions of a non-dominated set, a crowding strategy is used to identify solutions which reside in a less-crowded area.

B. The Convergence Accelerator Operator

The Convergence Accelerator Operator (CAO) consists of two steps; the first step is a deterministic local improvement procedure in the objective space. This is the component responsible for speeding up convergence. It achieves this by steering objective values obtained by the NSGA-II towards an improved Pareto front. The new improved values for the objectives are determined by linearly interpolating a new value for each objective, between its current value and the next best value(s) achieved for that objective within the population.

The second component of the CAO consists of a neural network (NN) trained to map the new solutions generated in objective space by the first phase of the convergence accelerator back to the corresponding decision variable vectors. This is achieved by training a radial basis function (RBF) NN, using exact objective vectors as inputs and their corresponding decision variable vectors as outputs, to approximate a mapping function from the objective space to the decision space. The training data is the exact data resulting from the objective function values derived within one cycle of a MOEA (in this case, the NSGA-II).

A description of NSGA-II hybridized with the CAO is presented in Listing 1.

```
-Generate random population P_0, size N
-Evaluate objective values
For i=1 to Gen
 -Assign rank to P_{i-1}
 -Determine crowding distance for each
 solution in P_{i-1}
 -Generate offspring population Q,size N
   Binary tournament selection
   Recombination
   Mutation
   Evaluate objective values for the
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        offspring population Q
-Combine P_{i-1} and Q
-Assign rank to the combined population
-Determine crowding distance for each
solution in the combined population
-Select N solutions to form P_i
Apply CAO:
-Component:Neural Network training
   -Initialize an RBF NN and train
   it with P_i
   Input: Objective vectors of P_i
   Output: Decision vectors of P_i
-Component: Objective space local
improvement of P_i
-Component: Objective space to decision
space predictions
-Component: Correction Step
-Update P_i
End Loop
```

Listing 1: The NSGA-II/CAO procedure [11]

IV. IMPLEMENTATIONS AND SETTINGS

In this paper, the feasibility of obtained non-dominated solutions is ensured by using a constrained tournament selection operator based on the constrain-domination principle. A solution $x^{(i)}$ is said to constrain-dominate a solution $x^{(j)}$ if any of the following conditions are true [13]:

- Solution $x^{(i)}$ is feasible and solution $x^{(j)}$ is not.
- Solution $x^{(i)}$ and $x^{(j)}$ are both infeasible, but solution $x^{(i)}$ has a smaller constraint violation.
- Solution $x^{(i)}$ and $x^{(j)}$ are feasible and solution $x^{(i)}$ dominates solution $x^{(j)}$ in the usual sense.

A procedure is also incorporated to ensure feasibility of non-dominated solutions produced by the CAO.

The solution of real-world multiobjective optimization problems also requires that the approximation set be achieved within an acceptable amount of time and a limited budget of objective function evaluations. Hence, in this study, for fair comparison of performance of the NSGA-II and the NSGA-II/CAO, a fixed computational budget was used. On all ten runs for each technique, the population size was fixed at 100. For the NSGA-II, the maximum number of generations was set at 300. In the case of the NSGA-II/CAO where invocation of the CAO leads to another round of function evaluations (since it is here called during each generation), the number of generations was reduced to 150 so that the number of function evaluations remains the same as for NSGA-II.

Crossover and mutation probabilities were chosen as 0.9 and 0.167, respectively. The distribution indices for crossover and mutation operators were each set at 20 in all optimization runs. A fixed interpolation factor of 10 was used in the first phase of the CAO while the spread factor for the RBF neural network was set at 0.0166. Several runs were carried out to set the parameters of each technique in order to get the best results for comparison. Feasibility checks were also carried out on the non-dominated solutions obtained to ensure that

TABLE I
GENERATOR FUEL COST AND EMISSION COEFFICIENTS

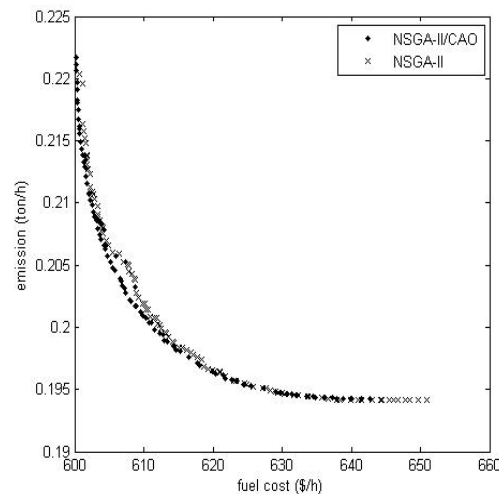|		G_1	G_2	G_3	G_4	G_5	G_6
cost	a	100	120	40	60	40	100
	b	200	150	180	100	180	150
	c	10	10	20	10	20	10
emission	d	6.490	5.638	4.586	3.380	4.586	5.151
	e	-5.554	-6.047	-5.094	-3.550	-5.094	-5.555
	f	4.091	2.543	4.258	5.326	4.258	6.131
	g	2.0E-4	5.0E-4	1.0E-6	2.0E-3	1.0E-6	1.0E-5
	h	2.857	3.333	8.000	2.000	8.000	6.667

Fig. 1.   Comparison of Pareto-optimal fronts, Case 1

they satisfy the system constraints. All techniques used in this study were implemented using MATLAB language on an Intel Core Duo processor running at 2.33GHz.

V. RESULTS AND DISCUSSION

In this study, we apply the two techniques to the standard IEEE six-generator 30-bus test system to assess their performance in solving the EED problem. The line data and bus data are as given in [6]. The values of generator fuel cost and emission coefficients are given in Table I.

Two different cases of problem complexity are considered as follows:

- Case 1: The system is considered as lossless. The problem constraints are the power balance constraint without P_{loss} and the generation capacity constraint.
- Case 2: P_{loss} is considered in the power balance constraint. The generation capacity constraint is also considered.

Case 1: Both NSGA-II and NSGA-II/CAO have been applied to the EED problem and both objectives were treated simultaneously as competing objectives. The Pareto-optimal fronts for the best optimization run due to each technique are shown in Fig. 1.
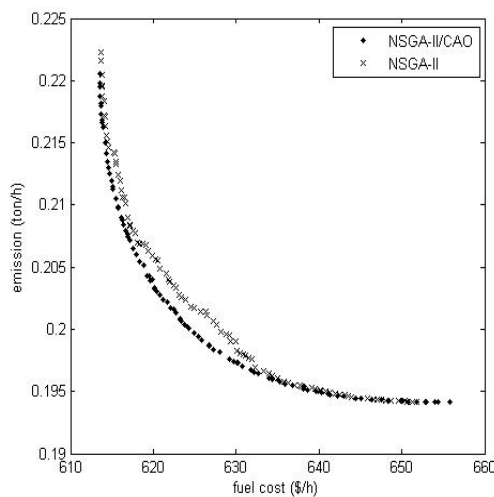
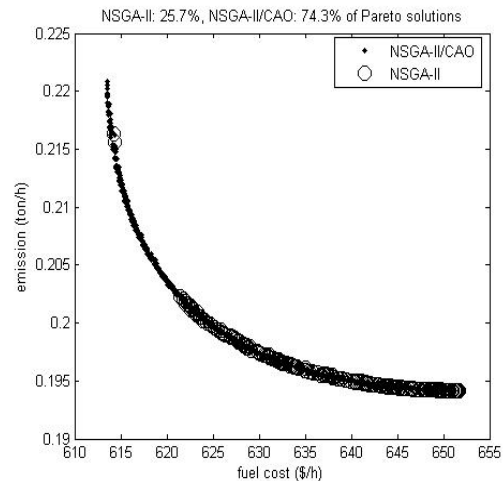Fig. 2.   Comparison of Pareto-optimal fronts, Case 2

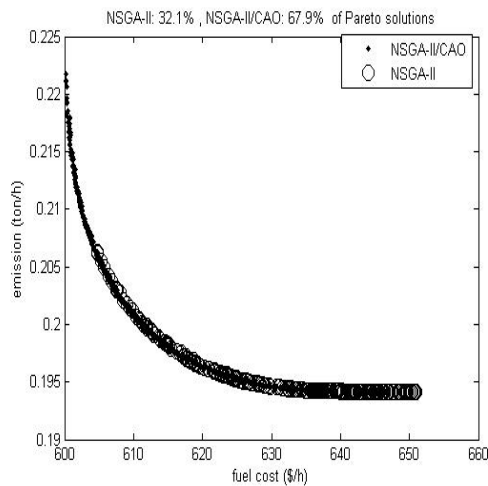Fig. 4.   Comparison of Pareto-optimal fronts, Case 2

Fig. 3.   Comparison of Pareto-optimal fronts, Case 1

It is clear that the obtained fronts have good diversity characteristics. Table II shows the best solutions obtained out of ten runs by both NSGA-II and NSGA-II/CAO. NSGA-II/CAO yields better fuel cost than NSGA-II while both methods yield the same value for emission.

Case 2: With system losses considered in the problem formulation, the Pareto fronts for the best optimization runs for both NSGA-II and NSGA-II/CAO are shown in Fig. 2. Table III shows the best solutions obtained out of ten runs by both NSGA-II and NSGA-II/CAO. The results also indicate that NSGA-II/CAO yields a solution with better fuel cost than NSGA-II.

VI.   PERFORMANCE METRICS

The major difficulty of multiobjective optimization assessment is that the output of the optimization process is not a single solution but a set of solutions representing an approximation of the Pareto front [14]. To evaluate the performances of different multiobjective metaheuristics, one needs to compare sets of solutions forming the non-dominated sets. The stochastic nature of evolutionary algorithms also makes it necessary to perform several runs to assess their performance.

Various metrics to assess the quality of the non-dominated sets in terms of their convergence and diversity characteristics have been suggested in the literature [15],[16],[17]. Convergence-based quality indicators include set coverage (also called C-metric), error ratio and generational distance metrics. Diversity indicators include metrics determining the distance between outer non-dominated solutions (also called the extent of Pareto front), spacing and entropy.

Table IV shows the comparison of results between NSGA-II and NSGA-II/CAO with respect to the set coverage metric. It can be seen that in Case 1, an average of 66.1% of non-dominated solutions returned by NSGA-II are covered by those of NSGA-II/CAO while only 16.4% of NSGA-II/CAO solutions are covered by NSGA-II solutions. From these values, it can be seen that solutions obtained by NSGA-II/CAO have better convergence properties.

Table V shows the comparison of results for the metric determining the extent of Pareto fronts obtained by NSGA-II and NSGA-II/CAO. In each of Case 1 and Case 2, NSGA-II/CAO has a larger average value for this metric than NSGA-II, although the standard deviation for the latter is lower.

Another metric which simultaneously compares non-dominated solutions from different MOEA techniques was proposed in [6]. This metric determines each technique's contribution of non-dominated solutions to a reference Pareto set which is an elite set extracted from a combination of all non-dominated solutions obtained by each MOEA technique over all optimization runs. Figures 3 and 4 show the contributions of both the NSGA-II and NSGA-II/CAO to the reference Pareto set in each of Case1 and Case 2.

TABLE II
BEST SOLUTIONS OUT OF TEN RUNS FOR COST AND EMISSION OF NSGA-II AND NSGA-II/CAO, CASE 1

	NSGA-II		NSGA-II/CAO	
	Fuel Cost	Emission	Fuel Cost	Emission
P_{G_1}	0.0786	0.3988	0.1228	0.4123
P_{G_2}	0.2885	0.4573	0.2837	0.4627
P_{G_3}	0.5090	0.5330	0.5137	0.5182
P_{G_4}	0.9654	0.3958	1.0152	0.3946
P_{G_5}	0.6055	0.5492	0.5234	0.5598
P_{G_6}	0.3870	0.5007	0.3753	0.5032
Cost	**600.7422**	636.7316	**600.2056**	641.4809
Emission	0.2204	**0.1942**	0.2217	**0.1942**

TABLE III
BEST SOLUTIONS OUT OF TEN RUNS FOR COST AND EMISSION OF NSGA-II AND NSGA-II/CAO, CASE 2

	NSGA-II		NSGA-II/CAO	
	Fuel Cost	Emission	Fuel Cost	Emission
P_{G_1}	0.0919	0.3958	0.1055	0.4117
P_{G_2}	0.3229	0.4578	0.2931	0.4622
P_{G_3}	0.6015	0.5641	0.5377	0.5624
P_{G_4}	1.0081	0.4148	0.9940	0.4045
P_{G_5}	0.5225	0.5540	0.5701	0.5510
P_{G_6}	0.3466	0.5079	0.3931	0.5041
Cost	613.6759	648.7090	613.5488	650.7343
Emission	0.2223	0.1942	0.2205	0.1942

TABLE IV
RESULTS OF THE SET COVERAGE METRIC (A= NSGA-II/CAO, B= NSGA-II)

Set Coverage	Case 1		Case 2	
(%)	C(A,B)	C(B,A)	C(A,B)	C(B,A)
Best	84	46	93	24
Worst	41	9	38	4
Average	66.1	16.4	71.8	11.8
Median	70	13	73.5	8.5
Std.Dev.	13.0337	10.9057	16.0125	7.3907

TABLE VI
NORMALIZED DISTANCE METRIC

	Normalized Distance	
	Case 1	Case 2
NSGA-II	0.9112	0.9817
NSGA-II/CAO	0.9870	0.9873

The results indicate that NSGA-II/CAO was able to find more solutions that are members of the reference Pareto set than NSGA-II. This is in agreement with results for the set coverage metric. The inclusion of the CAO thus definitely improves the convergence of obtained solutions towards the true Pareto front.

The normalized distance between the outer non-dominated solutions contributed by each technique to the reference Pareto set is shown in Table VI. This measure gave relatively larger values for NSGA-II/CAO. It can be concluded that inclusion of the CAO in the algorithm does not cause a deterioration of the diversity of the non-dominated solutions.

VII. CONCLUSION

In this study, an evolutionary algorithm, the NSGA-II and a hybrid evolutionary algorithm, the NSGA-II/CAO have been successfully applied to solve the environmental/economic power dispatch problem and a comparison made between them. The techniques have been compared with each other on the basis of their convergence to the Pareto-optimal front and the diversity of the solutions. The results indicate that NSGA-II/CAO possesses better convergence properties than NSGA-II. Moreover, hybridization with the CAO preserves the diversity of solutions in the non-dominated set.

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TABLE V
RESULTS OF THE SET COVERAGE METRIC (A= NSGA-II/CAO, B= NSGA-II)

Extent	Case 1		Case 2	
	NSGA-II	NSGA-II/CAO	NSGA-II	NSGA-II/CAO
Best	50.1690	52.6401	38.0628	46.2683
Worst	43.7793	42.4924	34.7306	38.2821
Average	47.4560	47.6583	36.1481	41.2588
Median	47.5161	48.2772	36.2031	40.4462
Std.Dev.	1.7969	3.3925	1.2582	2.7007

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