

Motivational Orientation of the Methodical System of Teaching Mathematics in Secondary Schools

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Abstract—The article analyses the composition and structure of the motivationally oriented methodological system of teaching mathematics (purpose, content, methods, forms, and means of teaching), viewed through the prism of the student as the subject of the learning process. Particular attention is paid to the problem of methods of teaching mathematics, which are represented in the form of an ordered triad of attributes corresponding to the selected characteristics. A systematic analysis of possible options and their methodological interpretation enriched existing ideas about known methods and technologies of training, and significantly expanded their nomenclature by including previously unstudied combinations of characteristics. In addition, examples outlined in this article illustrate the possibilities of enhancing the motivational capacity of a particular method or technology in the real learning practice of teaching mathematics through more free goal-setting and varying the conditions of the problem situations. The authors recommend the implementation of different strategies according to their characteristics in teaching and learning mathematics in secondary schools.

Keywords—Education, methodological system, teaching of mathematics, teachers, lesson, students motivation, secondary school.

I. INTRODUCTION

WHEN considering the formation of motivation to learn mathematics as a methodical challenge, it is necessary to determine its role and place in the methodological system of teaching this discipline. Traditionally, this system includes the purposes, content and methodological support of the learning process, the components of which are strategies, methods, and resources of teaching. These components are closely interrelated with each other, which ensure the integrity of the whole system, expressed in the unity of the functions it realises. A methodological approach to the goals of teaching mathematics determines the general patterns of its functioning at different levels: the theoretical aspect, the educational aspect and real educational process.

The purposes of teaching mathematics are reflected in the corresponding content of mathematical education - the key elements of which are mathematical concepts, theorems, problems, rules and algorithms. In turn, the features of the subject content to be studied largely determine the nature of the teaching strategies used, as well as the forms of organisation of students' activities and the range of necessary

training facilities. It should be noted that in practice the subordination of goals, content and methods of instruction is not always unambiguous. At different stages the teaching of the role of the leading component can be assumed in the content of mathematical education or the methods [1]–[6].

II. THE STRUCTURE OF THE METHODOLOGICAL SYSTEM OF TEACHING MATHEMATICS

In taking a person centered approach, it is always advisable to construct the methodical system in a way which takes into account the personality characteristics of the student and also the motivation to learn mathematics [2], [5]–[9]. This is shown as a model in Fig. 1.

The main components of the methodological system are shown in the form of circles that embody the purposes, the content of the training, the methodological support of the learning process, and the personal (Int), competence (Com) and cognitive (Cog) characteristics of each student [5], [9]. The nature of the interaction of all the selected components of the system is determined by the correspondence between the subject knowledge, and the internal needs and capabilities of the student at any given stage of development.

With regard to the learning process, the learning objectives should have a personal meaning for the student, should correspond to the student's life views and should also be offered in a form that takes into account specific cognitive abilities. In Fig. 1, the fulfillment of these conditions (closely interrelated, but not identical to each other) is indicated as three pairs of oppositely directed vectors. The correlations of their absolute values determine the external normative learning. If the vectors originating from the centre of the circuit substantially exceed the oppositely directed vectors in absolute value, the teaching has a so-called "free" (spontaneous) character. Otherwise, externally set standards will dominate, which, as is known, often negatively affect the manifestation of cognitive initiative and creative independence. Today, both tendencies ("from the environment" and "from the student") must balance each other, meeting both the demands of society and also the needs and capabilities of the student (the "effective teaching zone"). Careful calibration and adjustment may be needed to ensure the teaching methods correspond to the stated goals.

The school course content with regard to algebra, for example, has been fine-tuned by authors of textbooks, methodologists and teachers to ensure the possibility of a constant generalisation of the mathematical content studied and to provide clear explanations on equations and inequalities within parameters. This is reflected in the widespread use in

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recent years of relevant assignments during the entrance examinations to many universities, as well as final examinations at schools. This has enhanced the subjective significance of this material for teachers and students and also led to the current trend towards the inclusion of material on equations and inequalities with parameters in the main content of school mathematical courses, with a reduction of emphasis on elements of mathematical analysis in the upper grades and aspects of trigonometry in the middle grades. On the other hand, the teaching practice was not quite ready to include this material in mathematical content. This, in particular, manifested itself in the diversity of the proposed methodological recommendations to teachers, often aimed at reducing the solution to a very uneconomical formal search of possible values of parameters. This "obscures" the meaning of the concept of the parameter for students.

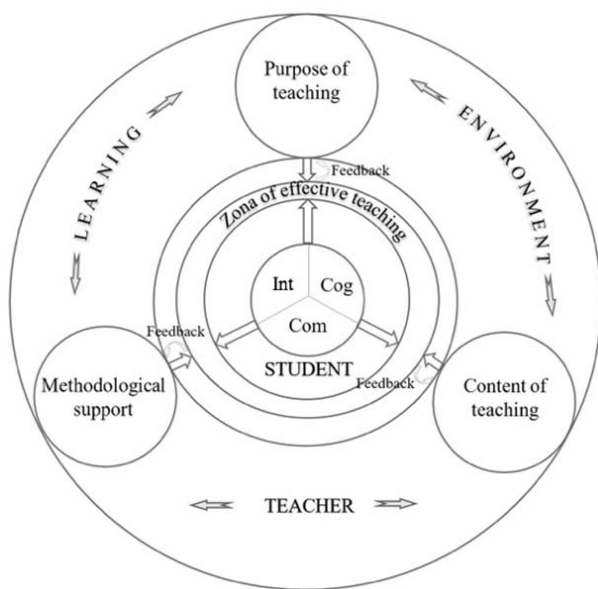


Fig. 1 The methodical system of teaching mathematics

Insufficient comprehension of the basic concepts and excessive formalisation of the solution of problems with parameters, as experience has shown, led to a negative attitude towards this material from a significant part of high school students. One of the reasons for this is the dominance of their imaginative substructures of thinking. Accordingly, it became necessary to develop a methodological approach that would stimulate cognitive activity by a more explicit reference to significant criteria for finding solutions to tasks with parameters. The optimal option was a graphical-geometric approach supported by analytical calculations as necessary.

The example shows how a change in one of the components of the methodological system of teaching mathematics necessarily leads to the transformation of its other components [5], [6]. And in the focus of all interactions between components, there must always be a subject of learning—a student whose personal characteristics set the orienting basis for their realisation.

III. THE PURPOSES OF MATHEMATICS EDUCATION

As is known, the ideal educational goal is the consciously planned image of the outcome of the educational process in relation to the actions and conditions that generate it. There is extensive literature on the different approaches to the presentation of the content and methods of setting educational goals. It is also important to consider motivational characteristics and to study the possibilities of accepting a particular goal by the subject of activity at various levels of consideration of mathematical education [1]–[6], [11], [12]. Here we can distinguish the level of social demand, the level of the teaching mathematics, the level of strategies and the level of technology. The nature of their interaction is realised, first of all, through the formulation and adoption of a set of educational objectives at the appropriate degree of abstraction. In particular, at the level of the real educational process, goals, concretised in the form of mathematical actions, are reproduced in the structure of the individual motivations of each student.

This process assumes a consistent realisation of the chain: awareness of the need, correlation of the need with the available opportunities (promotion of the goal of the action), choice of the mode of action, obtaining the result of the action and then checking the result [6], [11], [13]. The most clearly indicated chain of transitions is the analytical search for ways to solve mathematical problems and the proof of theorems. Here, firstly, the goal of each action naturally follows from the result of the previous action; secondly, at every stage, a range of sufficient data are produced for the fulfillment of the requirements of the theorem or problem; thirdly, careful selection is made from these data to enable explicit analysis and elucidation, and, fourthly, the degree of "convergence" of the conditions and the requirements of this given situation is constantly assessed. However, such an ideal path to motivation to learn mathematics is not always realised. In practice, in most cases there is a certain arbitrariness both in the choice of goals and in the directions for finding ways to achieve them, which can be because of both the innate objective features and complexity of the task and also the level of development of student motivation. Here the role of the teacher is to create the students' readiness to assume the role of the subject who will create a plan of solution by gradually "condensing" and rationally reconstructing the meaning of the initially unclear task [6].

Let us illustrate by an example. In recent decades, in the practice of teaching the school course of geometry, teachers often assign tasks based on ready-made drawings, which make it possible to significantly intensify the educational and research work of schoolchildren by obviating long and "tedious" constructions. However, despite all its obvious advantages, this orientation in many respects prevents the implementation of a full-fledged goal-setting process, substituting it with a purely operational approach. It is during the creation of the initial geometric configuration (and also in the process of additional constructions) that the student gets the opportunity to get used to its meaning, and to adapt perception of the studied content. The student's initial ideas

are then not suppressed—rather they are developed and refined in the course of an adequate analysis of the transformation of this configuration.

Accordingly, even for example at the initial stages of teaching geometry, the teacher needs to create conditions for the actualisation of the target component of objective actions by subordinating them to not the "logic of knowledge", carried out mainly by formal logical means, but to the "logic of thought" guided by the intuitive ideas of the student [14]. For example, in a simple proof of the Pythagorean Theorem, the textbook [15] considered the following chain of operations (Fig. 2):

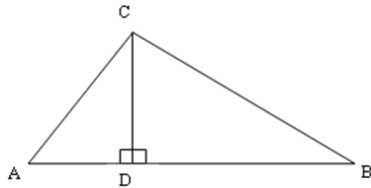


Fig. 2 The triangles ABC and ACD

1. Express $\cos \angle A$ from triangles ABC и ACD .
2. Equate the corresponding expressions.
3. From the resulting equality, find AC_2 .
4. Conduct similar actions for $\angle B$.
5. Add the resulting equalities and make the necessary transformations.

As can be seen from the above list, apparently seemingly simple and convincing proof (often used in the lessons) is not at all so for pupils, since each of its links, starting with the construction of the drawing, does not allow students to "grasp" the logic of its deployment. The process of goal-setting is ignored. At best, the individual experience only reflects the purely informative (operational) side of the proof and does not encourage intellectual development [16]. Strengthening students' motivational and target component in implementing the proof can be achieved by presenting it in the following form:

Problem 1. In the rectangular triangle ABC the length of the legs AC and BC are equal, respectively, to a and b . Define the hypotenuse of the triangle (Fig. 3).

Step 1: When solving this problem, students rely on the drawing of a right-angled triangle without height.

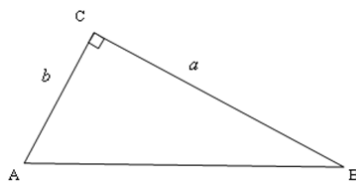


Fig. 3 The triangle ABC

Step 1: Based on this configuration, students can find the following expressions for $AB=c$:

$$c = \frac{b}{\cos A}; c = \frac{a}{\cos B}$$

An unsuccessful attempt to determine c from these relations (the angles A and B are unknown) actualises the need to redefine the task situation. In order to reach the goal of the next action, it is necessary to mobilise past experience (constructions with triangles) and to link the undefined elements to those known in some other way. In order to find this relationship, it is necessary to carry out an additional construction in such a way that as a result, as many new conditions and relations arise, and these conditions should be expressed in the simplest possible way. Further searching enables the discernment of various alternatives and the selection of the ones that best satisfy these criteria. The realisation of this choice marks the culmination moment of the act of goal formation, acting as a legitimate outcome of the transformation of the object of action (the initial geometric configuration). In this case, the transformation is the construction of the height of a rectangular triangle CD (Fig. 4), which results in three new segments, two new rectangular triangles and several new element relationships.

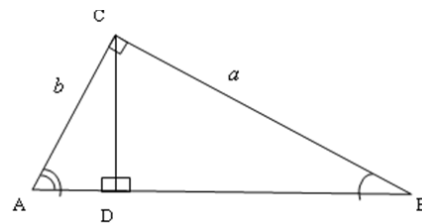


Fig. 4 The triangle ABC with the height CD

Thus, the second action will consist in constructing the height of the CD and deducing the simplest corollaries corresponding to the actualised need and satisfying the intended criteria for evaluating the result of the action.

$$\text{Step 2: } CD \perp AB; \cos A = \frac{AD}{b}; \cos B = \frac{DB}{a}$$

At this step, a new need arises naturally for the definition of the segments AD and DB , which is concretised in the goal of the next action: to link these segments to the elements of the task domain defined in the condition.

$$\text{Step 3: } AD + DB = c$$

Step 4: Finally, the fourth action has a synthetic character and consists in successive substitution in the formula of the results of the first and second actions.

$$c = b \cos A + a \cos B = \frac{b^2}{c} + \frac{a^2}{c} = \frac{a^2 + b^2}{c} \Rightarrow c^2 = a^2 + b^2$$

In the above proof, the logic of successive actions has a much greater motivational effect for students than in the previous case, since it relies on the realisation of full-value acts of goal-formation, each of which follows naturally from

the previous one. The key link in the chain of actions is the action of constructing the height of the triangle, which determines the basic search strategy, turning the abstract problem situation into a personally significant problem. In general, the resolution of such a problem (the choice of a "suitable" additional construction) requires considerable efforts from the students, which are caused by the variety of types of additional constructions reflected in school mathematical content, and also by the largely implicit opportunity to use them. Overcoming these difficulties requires tailored support by the teacher [17]. For example, for the teacher, when formulating the goal of the second action in the above example, the following questions can be proposed:

- 1) What additional constructions can you use here?
- 2) Which of them allows you to determine the largest number of known relationships?
- 3) Do these relationships include the values given in the condition?
- 4) What new values are involved in these relations?
- 5) Is it possible to relate these values with knowns on the basis of some other formulas?

Regular and systematic use of this approach enables students to gradually develop the criterion for evaluating the result of the choice of the goal of action. This is the basis of the process of goal-forming, which in the general case consists of identifying, for any given task, the maximum number of elements it contains and also the relations connecting them.

IV. CONTENT KNOWLEDGE IN TEACHING MATHEMATICS

The optimal deployment of school mathematical content in relation to motivation requires correlation both with the existing system of students' knowledge and also with students understanding of methods and strategies. These conditions are met through the implementation of effective interaction of situational and content-semantic motivational factors, taking into account both the specificity of the teaching material and the individual characteristics of the students [3], [5], [7], [9], [19]-[21].

In general, the motivationally oriented strategy for the deployment of mathematical content is implemented as follows. The starting point is the creation of a "motivational atmosphere" in the lesson by using particular techniques and means of stimulation which are defined both by the character of the content being studied and also by the dominant typological characteristics of students in a given class or group [8], [9], [12], [13]. So, for example, if among the clearly expressed typological characteristics of pupils of a particular class is the need for communication, then it is expedient to create a motivational atmosphere using the technique of joint search activity. The actualisation of the need for self-assertion is directly correlated with the creation of a competition situation. The need to achieve success is closely related to the resolution of problematic situations that are important for the students involved and the need for self-actualisation-with the creation of a situation of perspective that allows the subject of the activity to outline the possible range of consequences of its implementation. When mastering the subject-matter of

mathematical content, the formation of subject motivation is carried out in the following sequence: $M \Rightarrow C \Rightarrow H$, where M is the situational motivational setting, C is the content of the topic or section studied, and H is the system of the corresponding personal dispositions of the student. The core of this chain, the source of its "resource support" is the content of training, reflecting the effectiveness of mathematical activity [8], [12], [13], [23].

Let us consider the stages of mathematical activity. In particular, at the first "experimental-practical" stage, the students realise that the actualised system of their habitual representations does not correspond to the problem situation. Here the initial acceptance of the mathematical problem takes place. At the next - "intuitive-figurative" - stage, an attempt is made to determine possible directions for improving the system of mathematical knowledge, which is the "source" of the meaningful content. The third stage is that of verbalisation, whereby the initial intuitive ideas are "lived", and their "blurring" is overcome by translating it into a verbal and logical plan- the primary test of productivity. The next formal-logical- stage is key in the sense that what emerges is a fully fledged enunciation of the qualitatively new opportunities and general structure of the mathematical material being studied, and the acquisition of the leading motivational role contained in this content. Finally, at the final evaluation-reflexive stage, the overall meaning of the content is reconsidered, confirming the "commitments" declared at the previous stages and revealing the prospects for further expansion of the field of its functioning both in purely mathematical and applied terms.

Let us illustrate the implementation of the selected sequence of stages with the example of the topic "logarithms" [9]. To create the initial stimulating effect, a situation was chosen that contained the "Bar-Kokhba legend", which is a popular version of one of the known problems in information theory [22]. In 135 A.D., an uprising broke out in Judea against the domination of the Romans. The leader was Bar-Kokhba - "son of the star". The Roman army, superior in strength, laid siege to the fortress, which was defended heroically by a small garrison under the command of Bar-Kokhba. One night, Bar-Kokhba sent his scout to the Romans camp. However, the Romans seized the scout, subjecting him to cruel tortures and cutting out his tongue, before he escaped and returned to appear before his leader, but unable not to tell him about what he had learned about the enemy. To learn what he wanted to know, the wise Bar-Kokhba began to ask questions that could be answered either "yes" or "no", and the mutilated warrior either nodded or shook his head in response. In this way, Bar-Kokhba managed to find out from him all the information about the Roman army that he needed to defend the fortress.

Bar-Kokhba became the name of an early twentieth century game, which enjoyed extraordinary popularity. The essence of the game is that one of its participants must guess information from the other, but only by asking questions that allow only two answers: either "yes" or "no." Success involves a reasoned logarithmic approach to find the answer using the minimum possible number of questions. This is a particular example of a general approach to determining the smallest number of

questions necessary in a given situation.

At present, the functional approach is predominant in school, according to which the logarithmic function is defined as the inverse of the exponential function. At the same time, it loses its uniqueness for students, becoming only an "ordinary" representation of a wide class of elementary functions studied. Another common approach involves the introduction of the logarithm as the root of the exponential equation, after which the transition to the consideration of the logarithmic function is immediately carried out. Both variants, in our opinion, do not have sufficient motivational and semantic potential, since both the notion of the inverse function and the solution of the exponential equations are considered just before the introduction of the material about the logarithms and, accordingly, do not fully correspond with the past experience of high school students.

In the variant proposed by us, the logarithm operation appears as an intuitive assumption about the existence of two inverse operations for the operation of exponentiation. At the same time, an algebraic line is built in the minds of the students, the beginning of which is laid out in primary school. This line creates a kind of motivational canvas, linking together the educational and cognitive problems about the definition of certain operations through inverse algebraic (and also non-algebraic) operations. Such an approach naturally leads to the formulation of not one, but two main logarithmic identities that differ in the order of the operations performed:

$$\log_a(a^b) = b \quad a^{\log_a b} = b \quad (a > 0; a \neq 1; b > 0)$$

At the next stage, the terminology is refined, the definition of the logarithm is formulated, and some of its properties are derived. Here the original "algebraic" meaning of the logarithm as the result of the logarithm operation is supplemented by its understanding as a special form of the number record. This understanding, in particular, is facilitated by a large number of exercises for the calculation of expressions with logarithms and their comparison, some of which, due to students' ignorance of the properties of the logarithmic function, are solved on an empirically intuitive level. The core of the content of the topic is a functional block that introduces a graphic interpretation of logarithmic dependence and provides students with mathematical instruments for solving logarithmic equations and inequalities of varying degrees of difficulty.

Let us look at some methodical nuances having a certain value in the motivational and semantic sense.

The first nuance consists of the appropriate use of notation for the exponential function. The traditional notation ($f(x) = a^x$), as practice shows, does not have a pronounced "functionality" for students due to the non-standard arrangement of the argument (in the upper right corner). Accordingly, at the beginning it is more convenient to use another notation: $f(x) = \exp_a(x)$. Then the mutual invertibility of the logarithmic and exponential functions acquires a more "tangible" connotation for students:

$$\exp_a(\log_a x) = x; \log_a(\exp_a(x)) = x \quad (x > 0; a \neq 1; a > 0)$$

Another nuance concerns the estimation of the rate of change of the logarithmic function. In the current practice of teaching, the teacher often does not pay attention to the integrity of the figurative representation of students about the logarithmic function, presupposing the awareness of the qualitative features of its graphical representation. As a result, many schoolchildren adequately perceive only a fragment of the graph, located near the origin. With an increase in the argument, the graph begins to quickly "move away" from the abscissa axis. Therefore, it is advisable to pay special attention to the "almost parallel" nature of this graph of the OX axis for sufficiently large values of the argument. This idea of the graph helps, in particular, in the realisation that the value of any logarithmic function for sufficiently large values of the argument is less than the corresponding values of the power function with any positive exponent. The functional perspective on the material in question allows us to significantly expand its potential use in solving logarithmic equations and inequalities (continuity, monotonicity, evenness/oddness, etc.). In addition, the fulfillment of many tasks involves the application of research skills.

More time should be devoted to various applications of logarithms, both in mathematics and in the study of phenomena and operative processes. Here schoolchildren get the opportunity to rethink the material studied, extrapolating beyond the school based normative framework. At the same time, the semantic paradigm is expanding, by demonstrating the "general cultural" value of logarithms, which are regarded as a kind of "transformer" of functional dependencies characterising various processes in many areas of human life activity.

V. METHODS OF TEACHING MATHEMATICS

The method of teaching is understood to mean an approved and systematically functioning structure of interaction between teacher activities (teaching), student activities (learning), and the content of teaching - consciously realised for the purpose of systematic improvement of students' individual experience and their specific personal qualities [1], [4]-[6], [10]. One of the main criteria characterising the value of any method in specific conditions is its potential to actualise the cognitive, enquiring, emotional-aesthetic and practical needs of students, so that they actively think, learn and develop using best knowledge of mathematical content and methodologies. This potential, in turn, depends on the degree of correspondence between the methods of teaching, taking into account the specifics of the mathematical content on the basis of objective unity of the goals of these activities [2], [9], [18], [24].

The motivational characteristic of the training method can be represented in the form of an ordered triad of characteristics: the dominant nature of goal-forming (external - A_1 , mixed - A_2 or internal - A_3); orientation toward a certain degree of correlation of the various forms of mathematical material representation corresponding to a particular cognitive

structure of thinking (minor - I_1 , medium - I_2 or high - I_3), and also to a certain level of generalisation of the assimilated content (low - G_1 , medium - G_2 or high - G_3). These parameters can be used as benchmarks for describing various strategies for teaching mathematics at all levels of its organisation. Their more detailed description is presented in Table I.

The use of a particular method in a given situation should be decided from the perspective of the entire system of

methods of teaching the topic, taking into account the individual and typological characteristics of students and also the position of the studied material in the structure of this topic, section or course. At the same time, in constructing a motivational plan it is very important to combine various teaching methods through preliminary analysis of the contents of the topic being studied and then correlating this with the indicators of students' mathematical competence and need and potential for development [6], [25].

TABLE I
THE TEACHING METHODS

	A	I	G
1	Direct instruction: the purpose "descends" from teacher	empirical/visual approaches to study content	Performing actions using a template or a specific algorithm
2	Work is being done to adopt the purpose by students	Appropriate conversion of content within the available cognitive range	Orientation to the variation application of general algorithms, backed up by guiding questions and instructions from the teacher
3	The purpose is realised by students in the course of partially independent solution of the problem situation	Organisation of problem research on the basis of a multilateral analysis of the situation	Orientation on formed general and special skills of students

Whatever method is chosen by the teacher, it must naturally "superimpose" on the indicated sequence of states. Let us illustrate the last point by the example of the lesson: "The solution of quadratic equations".

The preliminary stage is the elementary phase for the formation of the electoral relationship, which is conditioned mainly by external circumstances that attract a student's attention of a person, giving pleasure and a sense of achievement, and stimulating curiosity from individually or jointly performed activities. In particular, an active "entry" into the lesson can be realised by creating a "competition situation" in the conditions of free choice of the questions.

Several different equations are written on the board.

$$\begin{array}{ll}
 x^2 = 3; & x^2 - 10x + 25 = 0; \\
 x^2 - 7x = 0; & 9x^2 - 49 = 0; \\
 9x^2 + 11 = 0; & x^2 + 4x + 4 = 0; \\
 7x^2 - 1 = 7(x^2 + x); & 3x^2 - 4x + 4 = 0. \\
 & 9x^2 - 3 = 0;
 \end{array}$$

Pupils are encouraged to choose two equations from the above set and solve them in the most economical way. The requirement for and the opportunity of choice means students feel included, which in turn improves the overall positive emotional mood of the class. At the next stage, the students move to a new motivational state by analysing the current situation and "grafting" it into a new context. The main means of achieving this state is the discussion of the following questions:

- 1) Are all the proposed equations quadratic?
- 2) Which of them have solutions?
- 3) How to determine the number of roots of a quadratic equation?
- 4) What methods of solving quadratic equations that you know can be used here?
- 5) Which of the equations should be solved on the basis of the general formula? (justify the answer);
- 6) How can the optimal way of solving it be determined by

the form of the equation?

- 7) Choose any equation from among those that you have not solved and solve it verbally based on the selected characteristics.

It is important to note that since in practice the students choose basically different equations that are solved by their preferred methods, many of them have a "sense of their own validation" expressed in their desire to take an active part in resolving problematic situations arising during the discussion [21], [24], [26]. Logical completion of this stage is carried out by a multi-faceted analysis of one of the equations:

$$x^2 + 4x + 4 = 1$$

Here the teacher asks the students to explain why this equation was chosen (it can be solved in several ways at once), and then to implement the solutions they know. Comparing different methods, schoolchildren choose those they consider to be the most natural and rational.

At this stage, the factor determining the dynamics of motivational processes is the result of the solution of the task, which recalibrates the motivational state to the optimal level. In this lesson the content of this stage involves solving the equations after their preliminary transformation. The teacher, returning to the original set of equations, suggests that students determine the form of the equation: $7x^2 - 1 = 7(x^2 + x)$, and then check the answer. During the check, students come to see that the initial form of the equation does not always correspond to its internal structure, which is revealed during the transformation of both parts of this equation (in the above equation, after elimination components of the form: ax^2 this equation becomes linear). Such an unexpected result for students sets a new benchmark for the manifestation of their cognitive activity, consisting in the need to bring the initially complex mathematical construction to the simplest and most convenient for analysis. This direction finds its realisation when performing the task: "Choose me". It consists in choosing from among the proposed equations, each of which

is estimated by a certain number of points, several equations for independent solution so that each student has a minimum of four points.

$$x^2 - 22x + 25 = 2x^2 - 20x + 1$$

(1 point)

$$(x + 1)^2 = 3(x + 7)$$

(1 point)

$$(x + 4)^2 = 2(4x + 11)$$

(1 point)

$$11x + 26 = (x + 5)^2 + (x - 2)^2 - (x - 1)(x + 1)$$

(3 points)

$$2(x - 2)(x + 2) = (x + 1)^2 - 3$$

(2 points)

The above examples show how $\{A_1 I_1 G_1\}$ lesson structure can optimise motivational learning $\{A_2 I_2 G_2\}$; $\{A_3 I_2 G_2\}$; $\{A_2 I_2 G_3\}$ and $\{A_2 I_3 G_2\}$ through greater variation the conditions of the given task. This partial transformation is provided by the use of differentiated work, the distribution of roles in groups and the drawing up of multilevel tasks and exercises.

VI. CONCLUSION

Thus, within the framework of the theoretical study, a motivationally oriented methodological system for teaching mathematics was created, which resulted from adding into the traditional set of methodological components (purpose, content, methods, forms and means of teaching) the student's personality as a subject of the education process, together with the totality of respective needs and motivational features.

The implementation of such an approach necessitated a rethinking in the motivational plan of the role and significance of the target, content and technological components of the methodological system, as well as the nature of their interaction.

Particular attention was paid to the problem of methods and technologies for teaching mathematics, the motivational significance of which depends on the degree to which they correspond to the methods of educational activity of students. This thesis found its expression in the representation of the method as an ordered triad of characteristics that correspond to the components of the human motivational sphere. Systematic analysis of possible options and their methodological interpretation increased understanding and application of scientific ideas about known methods and technologies of education. It also significantly expanded student nomenclature by including hitherto unstudied methodologies. In addition it demonstrated the possibilities of strengthening the "motivational capacity" of any given method in the day-to-day practice of teaching mathematics.

The proposed pedagogical decisions were implemented in the development of the content of individual topics of the school course of mathematics, facultative courses and seminars for students, and were reflected in the manuals and methodological recommended by the authors. Of course, this theoretical study cannot claim to be an exhaustive description of all the features of the educational motivation of

schoolchildren in the lessons concerning mathematics. Many pedagogical problems remain, including the development of:

- 1) effective means of "motivational monitoring";
- 2) measurable criteria for assessing the "motivational potential" of the mathematics school textbooks and manuals;
- 3) a multifaceted study of the impact of computer based support in students' mastery of mathematical activity;
- 4) a better understanding of the effect on educational motivation of the practice of intersubject connections between mathematics and related subjects.

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