Modelling of a Stress-StrainState of Screws of Transpedicular Spine Fixation System

Oleksandr Poliakov, Genadiy Olinichenko, Yevgen Pashkov, Vadym Kramar, Mykhaylo Kalinin

Abstract—For maintenance of a spine stability during the postoperative period a transpedicular fixing of its elements is often used. Usually the transpedicular systems are formed of rods which as a result form a design of the frame type, fastening by screws to vertebras. Such design should be rigid and perceive loadings operating from the spine without essential deformations. From the perfection point of view of known designs their stress-strain state as a whole, and each of elements, in particular is of interest. In this study the modeling of the transpedicular screw is performed and the estimation of its deformations taking into account interaction with a vertebra body having variable structure is made.

Keywords—Spine, screw, stress-strain state, transpedicular fixation system,vertebra

I. INTRODUCTION

DECOMPRESSION of spinal structures [1] and interbody stabilization [2] by one surgical access are a special interest in modern vertebrology owing to small trauma. New technological methods of modern interbody stabilization have appeared rather recently and still demand the further development for the purpose of improvement quality of the patient's life.

Posterior lumbar interbody fusion (PLIF) technology [3] represents a surgical method of bone or metal implant placing between adjacent vertebras (interbody). Indications for this procedure may include pain and spinal instability resulting from spondylolisthesis, degenerative disc disease or when a discectomy is performed to relieve nerve compression and the patient has associated low back pain.

Usually in the course of healing of bone fabrics for spine stabilization after PLIF use a system of instrumental fixation. By way of illustration on Fig.1 X-ray patterns of patient N after PLIF at two levels with instrumental fixing by system "Bridge" are presented.

There is a set of various instrumental systems of the spine fixing among which an important place occupy, so-called, transpedicular rod systems which feature is the way of

O. Poliakov is with the SevastopolNationalTechnicalUniversity, Laboratory of Biomechanics, Sevastopol, 99053 Ukraine (Corresponding Author Work Phone: +38-0692-435-161; e-mail: alex_polyakov@ inbox.ru).

G. Olinichenko is with the SevastopolCityHospital№9, Department of Surgical Arthrology, Sevastopol, 99006 Ukraine (e-mail: gena.mom@mail.ru).

Y. Pashkov is with the SevastopolNationalTechnicalUniversity, Sevastopol,99053Ukraine.

V. Kramar is with the SevastopolNationalTechnicalUniversity, Sevastopol, 99053Ukraine.

M. Kalinin is with the SevastopolNationalTechnicalUniversity, Laboratory of biomechanics, Sevastopol, 99053 Ukraine.

fastening by the screws established transpedicularly. In such cases the fixing system fastens to vertebras most strongly and is capable to counteract big enough external forces [4].
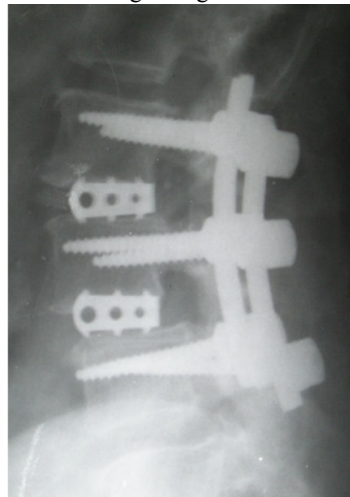

Fig. 1 X-ray patterns of patient N after PLIF in two levels with instrumental fixing by system "BRIDGE"

A maquette of established transpedicular system and the screws used for its fastening are presented in Fig. 2 (http://alfa-medica.ru/neiro.html).

From the point of view of perfection of known designs their stress-strain condition as a whole, and each of elements, in particular is interest. In this study the modeling of work of pedicle screw is performed and the estimation of its stresses and deformations taking into account interaction with a body of vertebra having variable structure is made.

II. PHYSICAL MODEL OF PEDICLE SCREW FUNCTIONING

The elementary closed force chain peculiar to functioning of transpedicular system can be presented as follows: the vertebra-1 – the pedicle screw-1 – the beam – the pedicle screw-2 – the vertebra-2 – the cages – the vertebra-1. Thus, according to many researchers, the beams perceives about 20 % of the loading operating from a person body to vertebra segment, the cages – about 80 % (Fig.3).

The vertebra has a complex structure, presented by the firm enough external (cortical) layer and a soft internal body (spongy). In simplified model it is possible to consider their as the homogeneous materials characterized by different Young's modules and different Poisson coefficients.
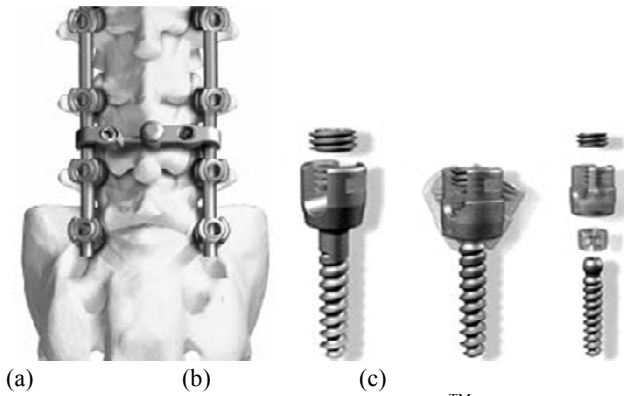
(a)   (b)   (c)

Fig. 2 A maquette of transpedicular system XIA^{TM} STRYKER (a); monoaxial pedicle screw XIA (b); polyaxial pedicle screw XIA (c)
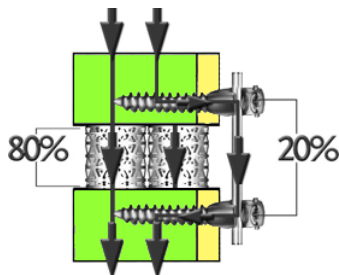

Fig. 3 The scheme of the loading distribution operating on the spine segment elements

The pedicle screw is fixed in the vertebra, interacting both with external, and with an internal body. Thus, it is possible to present in the form of rod having variable on length cross-section, being on the elastic basis, characterized by the variable stiffness with different coefficients of subgrade resistance. Thus the elastic basis coefficient of subgrade resistance changes stepwise at transition from one layer of vertebra to another. Besides, it is supposed that the vertebra is rigidly jammed (is conditionally motionless), and loading to the screw is transferred from the beam, being continuously distributed along the length of the local field at the end of the screw (Fig.4).

III. MATHEMATICAL MODEL OF PEDICLE SCREW FUNCTIONING

According to described above physical model we will consider that pedicle screw (beam) having variable section is based upon the elastic basis. With sufficient accuracy for the given problem we will neglect by warping of beam cross-sections in the process of deformation and also we will neglect a friction between a beam and the basis. It is necessary to consider the reaction of basis as a bilateral or, in another way, having possibility to accept different signs.

In that case the differential equation of a beam bend can be presented in a kind:

$$\frac{d^2}{dx^2}\left[EJ(x)\frac{d^2v(x)}{dx^2}\right] = p(x) - q(x), \qquad (1)$$

where $v(x)$– beam deflection; x – co-ordinate of cross-section connected with the axis of beam; $p(x)$– external loading enclosed to the beam; $q(x)$– reaction of the elastic basis; $J(x)$– axial moment of inertia of the beam cross-section; E – Young module of the beam material.
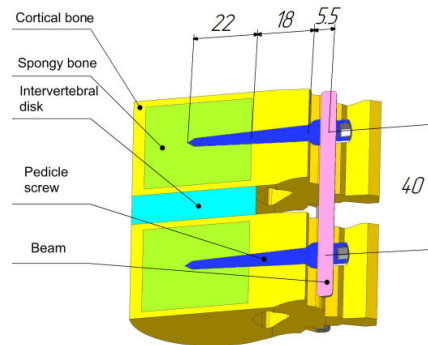

Fig. 4 Interaction of elements of transpedicular system with the spine segment

The differential equation (1) contains two unknown functions $v(x)$ and $q(x)$. For their definition to the equation (1) we will attach the additional condition expressing dependence between loading, enclosed to the basis and its upset, and also a condition about dense contact of the beam with basis.

As shown in [5], the differential equilibrium equation of the single-layered basis with two coefficients of subgrade resistance k and t, defining the work of the elastic basis on compression and on shift, looks like:

$$-2t\frac{d^2v(x)}{dx^2} + kv(x) = g(x)\psi(0), \qquad (2)$$

where

$$k = \frac{E_o\delta^*}{1-\nu_o^2}\int_0^h \psi'(y)^2\,dy\ ; \quad t = \frac{E_o\delta^*}{4(1+\nu_o)}\int_0^h \psi(y)^2\,dy\ , (3)$$

δ^*–conditional width of the beam; h – conditional thickness of the elastic basis; E_o, ν_o – Young module and Poisson coefficient of the elastic basis material, accordingly; the axis y is perpendicular to axes x and is directed towards the elastic basis from the beam axis.

For convenience a transversal distribution of a displacement function $\psi(y)$ can be chosen so that $\psi(0) = 1$. Thus generalized displacement $v(x)$ will represent the upset of the elastic basis surface, and the equation (1) will become:

$$-2t\frac{d^2v(x)}{dx^2} + kv(x) = g(x), \qquad (4)$$

where $g(x) = q(x)$–external loading on the basis.

Because the beam deflection coincides with the upset of the

elastic basis surfaces, the equation (1) and (4) can be considered in common:

$$\begin{cases} \dfrac{d^2}{dx^2}\left[EJ(x)\dfrac{d^2v(x)}{dx^2}\right] = p(x)-q(x) \\ \\ -2t\dfrac{d^2v(x)}{dx^2}+kv(x)=q(x) \end{cases} \qquad (5)$$

Excluding from system (5) function $q(x)$, we will receive the basic differential equation of a problem expressing dependence between loading, operating on a beam and its deflection:

$$\frac{d^2}{dx^2}\left[EJ(x)\frac{d^2v(x)}{dx^2}\right]-2t\frac{d^2v(x)}{dx^2}+kv(x)=p(x). \quad (6)$$

The function $\psi(y)$ in (3) it is possible to consider in a first approximation as linear and to accept in a kind: $\psi(y)=\dfrac{h-y}{h}$

then

$$t=\frac{E_0\delta^*}{12(1+\nu_0)}\ ;\ k=\frac{E_0\delta^*}{h(1-\nu_0^2)}. \qquad (7)$$

As the screw is represented in the form of the truncated cone, the conditions of its interface with the basis will differ from the similar conditions peculiar to the prismatic beam. Considering that diameter of the screw core at the length changes little, and also that the effort transferred by the screw to the basis is non-uniformly distributed on a surface, we will accept approximately $\delta^* = 0.5d$.

IV. EVALUATION OF THE FLEXURAL STRAINS OF SCREW

It is not possible to receive the exact solution of the equation (6). Moreover, the change account of diameter of the beam length leads to considerable complication of a solved problem without essential increase of the solution accuracy. In this connection we will consider the screw cylindrical, neglecting small change of diameter at length, having accepted its equal $d = 6\ mm$. The loading, distributed on a local site $p(x)$ we will lead to the beginning of the screw input in the vertebra body, having replaced it's by statically equivalent system presented by the main vector R and the main moment M.

The equation (6) can be written as

$$\frac{d^4v(x)}{dx^4}-2\beta^2\frac{d^2v(x)}{dx^2}+\alpha^4v(x)=0, \qquad (8)$$

where $\beta=\sqrt{\dfrac{t}{4EJ}}$; $J=\dfrac{\pi d^4}{32}$; $\alpha=\sqrt[4]{\dfrac{k}{EJ}}$.

As the coefficients α also β depend on properties of external and internal layers of the vertebra the equation (8) we will solve in two stages, on each of which the screw core we will represent in the form of the semi-infinite beam loaded on one end (an input to the vertebra body) by the concentrated force F_{ext} and the bending moment M_{ext} and free on other end. Such approach is justified, as screw deformations quickly fade and already on small enough distance from the input in the vertebra body are almost equal to zero.

Solutions of the equation (8) we will accept in a kind:

$$\begin{aligned} v_{1,2}(x) = {}& C_{1,5}e^{-\sqrt{\beta^2-\sqrt{-\alpha^4+\beta^4}}\,x} + C_{2,6}e^{\sqrt{\beta^2-\sqrt{-\alpha^4+\beta^4}}\,x} + \\ &+ C_{3,7}e^{-\sqrt{\beta^2+\sqrt{-\alpha^4+\beta^4}}\,x} + C_{4,8}e^{\sqrt{\beta^2+\sqrt{-\alpha^4+\beta^4}}\,x} \end{aligned} \qquad (9)$$

where $C_1,C_2,...,C_8$ - the constants defined from boundary conditions; α and β represent the step functions of a kind

$$\alpha=\alpha_1\left[1-H(x-a)\right]+\alpha_2 H(x-a),$$

$$\beta=\beta_1\left[1-H(x-a)\right]+\beta_2 H(x-a),$$

$$\alpha_i=\sqrt[4]{\frac{k_i}{EJ}}\ ,\ \beta_i=\sqrt{\frac{t_i}{4EJ}}\ ,\ k_i=\frac{E_{0i}\delta^*}{h(1-\nu_{0i}^2)}\ ,\ t_i=\frac{E_{0i}\delta^*}{12(1+\nu_{0i})}\ ,$$

$i=1,2$; $H(x)$ –a generalized Heaviside function; $a-$ a thickness of the cortical layer of vertebra.

Numerical values of parameters at the problem solving:
- the Young module of cortical bones: $E_{01}=1,6\cdot10^9\ Pa$;
- the Young module of spongy bones: $E_{02}=0,9\cdot10^9\ Pa$;
- the Young module of the screw material: $E=1,6\cdot10^{11}\ Pa$;
- the Poisson coefficients both the cortical and spongy bones: $\nu_{01}=\nu_{02}=0,3$. The parameters h and a are accepted by the equal: $h=0,03\ m$; $a=0,018\ m$.

Boundary conditions:

1) $x=0:\ EJ\dfrac{d^2v_1(x)}{dx^2}=M$; 2) $x=0:\ EJ\dfrac{d^3v_1(x)}{dx^3}=R$;

3) $x=\infty:\ v_1(x)=0$; 4) $x=\infty:\ \dfrac{dv_1(x)}{dx}=0$;

5) $x=a:\ v_1(x)=v_2(x)$; 6) $x=a:\ \dfrac{dv_1(x)}{dx}=\dfrac{dv_2(x)}{dx}$;

7) $x = \infty : v_2(x) = 0$; 8) $x = \infty : \dfrac{dv_2(x)}{dx} = 0$.

As a result the solving of the equation (8) is received in a following kind:

$$v(x) = v_1(x)[1 - H(x-a)] + v_2(x)H(x-a) \quad (10)$$

In the Fig. 5-9 the graphs of the parameters change characterizing the screw deformation, being under the influence of external loading are presented at various values M : M, $2M$, $3M$.

The analysis of the results of the problem solving allows to notice that deformations of the pedicle screw at action of rated loads on a vertebra segment are rather small. But pressure on its surface reach the big sizes (an order 40 - 60 MPa). Thus, as the screw represents the short core, essential are not only a normal flexural stresses, but also a shear stresses.

Big enough there are also pressure forces of the screw on a vertebra body that in practical cases can lead to its shaking at action of sign-variable loadings. In real surgical practice such cases take place, especially at work of screws in vertebras with pathologically changed structure.

Let's notice also that the greatest loading from the screw is perceived by the cortical layer of the vertebra and essentially increases together with increase of the external moment enclosed to the screw. Thus, from the point of view of a stress condition, the best is the screw head arrangement as it is possible more close to the vertebra body.
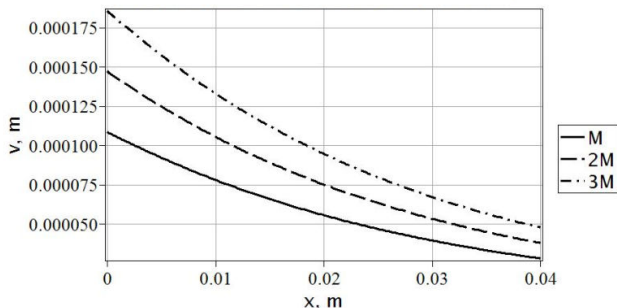

Fig. 5 Elastic line of the deformed screw axis

The same problem is solved by the finite elements method in the environment of CosmosDesignStar. 3D-models of the vertebrae segment fixed by transpedicular system have been thus used (Fig. 10).
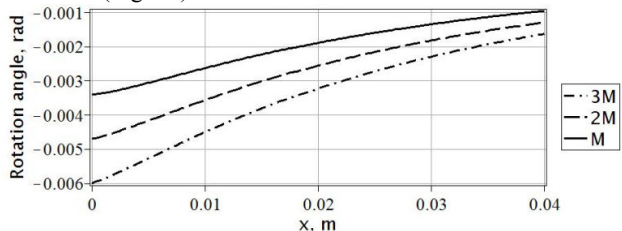

Fig. 6 Distribution of the rotation angles of section on the screw length
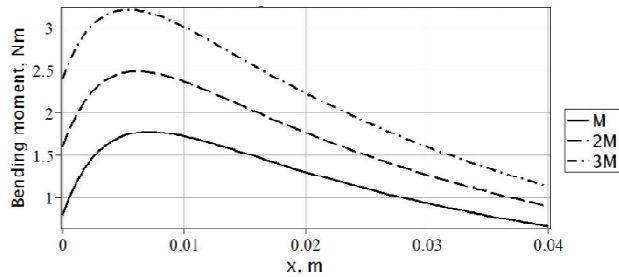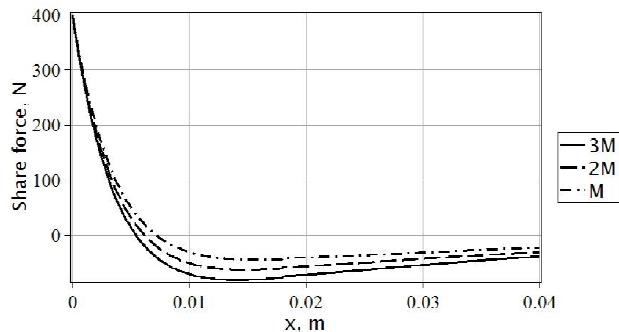

Fig. 7 Internal bending moment in screw section

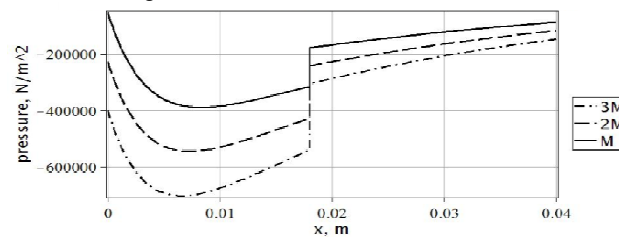Fig. 8 Internal shear force in screw section

Fig. 9 Distribution of the screw pressure to the vertebra body

The way of the model loading differed from accepted earlier at analytical modeling of the screw work. The bottom vertebra of the segment was considered conditionally motionless, and vertical loading was put to the top vertebra of the segment. Thus, the pedicle screw loading was carried out from the bar taking into account elastic properties of the intervertebral disk deformed under loading. Some results of modeling are presented in Fig.11. In particular are pictures of stresses distribution on the surfaces of elements of the vertebra segment model.
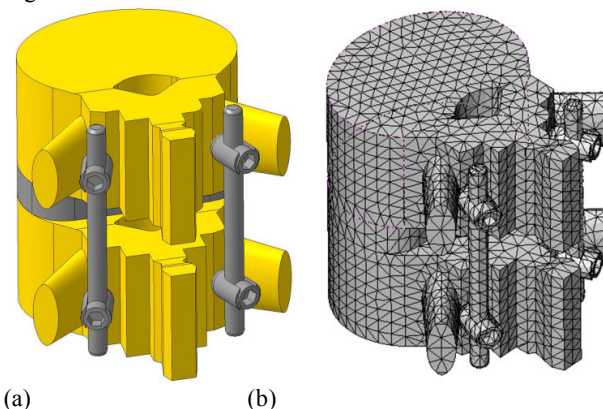

(a)   (b)
Fig. 10 The model of the vertebra segment fixed by transpedicular system: (a) – a general view; (b) –a finite-elements model
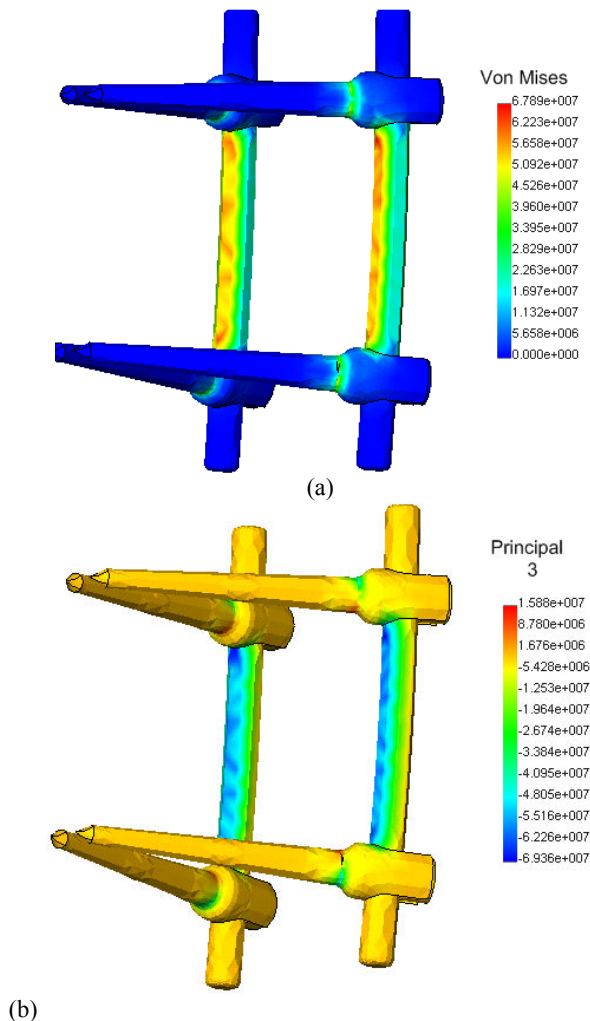
(a)

(b)

Fig. 11 Illustrations of stresses distribution in the elements of the transpedicular system: (a) –Von Mises stresses; (b) – principal stresses

The analysis of the pedicle screw stress-strain condition allows to notice that the maximum stresses on its surface at action of rated loads in size 1000-3000 *N*, enclosed to the top vertebra, reach 40-60 *MPa*. Thus, the results of the screw work modeling received by two various methods appear close enough that testifies to adequacy of the model described above.

At the same time, it is necessary to notice that in the first case (analytical modeling) the area of the greatest stresses in the screw is displaced on small size in a direction to the vertebra body, and in the second case (finite-element modeling) –is located in a transitive zone of the screw in which diameter of the screw changes in steps. It is possible to assume that this distinction is caused by the stresses concentration in the transitive zone which effect in the first case has not been considered for the objective reasons.

The experience of the various designs of transpedicular systems using testifies that pedicle screws collapse in the field of close to their input in the vertebra body. As a rule, it is the

fatigue failures, caused, except other, by the design lacks, by the poor of manufacturing quality, by the irrational installation of screws, by the loadings exceeding standard arising in the course of ability to live of patients.

V. CONCLUSIONS

The problem solving consisting in the estimation of the pedicle screw stress-strain condition under the influence of external loadings by two various methods has allowed to receive close enough results, allowing to explain the reasons of screws destructions and infringement of the functional properties of transpedicular systems as a whole. Thus, these methods can be used for the analysis of the stress-strain condition of various screw designs, to estimate influence of new constructive elements of screws on character of their deformations, for creation of the advanced designs of transpedicular systems, including new designs of dynamic systems.

REFERENCES

[1] N. Shealy, and V. Borgmeyer, "Decompression, reduction, and stabilization of the lumbar spine: acost-effective treatment for lumbosacral pain," *American Journal of Pain Management*, vol. 7, no. 2, pp. 663-665, April 1997.
[2] T. Lund, T. R. Oxland, B. Jost, P. Cripton, S. Grassmann,C. Etter, and L.-P. Nolte, "Interbody cage stabilisation in the lumbar spine," *Journal of Bone and Joint Surgery,* vol. 80-b, no. 2, pp. 351-359, March 1998.
[3] R. Schmid, D. Krappinger, P. Seykora, M. Blauth, and A. Kathrein, "PLIF in tthoracolumbar trauma: technique and radiological results," *European Spine Journal*, vol. 19, no. 7, pp. 1079-1086, July 2010.
[4] V.A. Radchenko, and N.O. Korzh, *Practical work on stabilization of thoracic and lumbar departments of a backbone,*Kharkov: Prapor, 2004.
[5] V.Z. Vlasov, and N.N. Leontyev, *Beams, plates and covers on the elastic basis,* Moscow: SPHof the Phys. and Math. Lit., 1980.