

# Modeling and analysis of a robust control of manufacturing systems: flow-quality approach

Lotfi Nabli, Achraf Jabeur Telmoudi, Radhi M'hiri

**Abstract**—This paper proposes a modeling method of the laws controlling manufacturing systems with temporal and non temporal constraints. A methodology of robust control construction generating the margins of passive and active robustness is being elaborated. Indeed, two paramount models are presented in this paper. The first utilizes the P-time Petri Nets which is used to manage the flow type disturbances. The second, the quality model, exploits the Intervals Constrained Petri Nets (ICPN) tool which allows the system to preserve its quality specificities. The redundancy of the robustness of the elementary parameters between passive and active is also used. The final model built allows the correlation of temporal and non temporal criteria by putting two paramount models in interaction. To do so, a set of definitions and theorems are employed and affirmed by applicator examples.

**Keywords**—Manufacturing systems control, flow, quality, robustness, redundancy, Petri Nets.

## I. INTRODUCTION

**T**he manufacturing systems are generally subject to disturbances which implicitly influence the prescribed output. A company is usually under the obligation to control the production and the development cycle of products in order to guarantee a certain product quality within a delay often determined by the customer. This requires a robust control of the system allowing the conservation of the system aptitudes. The definition of the parameters' conformity intervals of the system must always anticipate the phase of design of the target control law that will have to guarantee the respect of these specifications. The field of enquiry of the discrete events of manufacturing systems control is frequently met in the literature. Our interest will be focused on the comprehension of the robust control laws vs. the temporal and/or non temporal constraints. A certain number of works which are explicitly related to the study and the robust control design are numbered. As an example, we mention the works of Collart [9],[10] proposed a method of robust control vs. stay time constraints on which depends directly the conformity intervals of the product parameters quality. Besides, we quote the work of Bonhomme [2] in which he imposed the inter-product robustness field of enquiry so as to optimize the stay time of the various products manufactured in the same workshop. The work of thesis, Dhoubi [12] extended a method contributing to a robust and reactive control of manufacturing systems with non temporal constraints so as to react to disturbances of drifts

quality type. The design tool most frequently used to model the production systems with temporal constraints is the P-temporal Petri nets [2], [5], [6], [13], [16]. This tool seems unable to model the production systems' problem of robustness with non-temporal constraints. The Intervals Constrained Petri Nets (ICPN) tool [7], [8] presents a complement to the P-temporal Petri nets. It allows the modeling of any unspecified parameter in a manufacturing process. Indeed, this modeling tool is considered as a significant research way for the determination and evaluation of robustness [7], [11].

The objective of this article is to develop a method of constructing control laws allowing the interpretation of the total robustness type of the Manufacturing systems vs. temporal and non temporal disturbances. The subjacent idea is to define hybrid local models allowing the specified properties conservation of the subsystems by the exploitation of the redundancy of the robustness margins between passive and active. From these local models, we can generalize and ascertain the total robustness of the system.

As a first step, we present some usual definitions and notations related to the robustness of manufacturing systems, along with a reminder of the P-time Petri Nets and the Intervals Constrained Petri Nets. As a second step, we present the flow model and the quality model. After presenting both correlating models, we get to a third and last step where we present the final control laws model for manufacturing production systems.

## II. MODELING OF ROBUST CONTROL

### A. Definitions and notions of robustness

*Definition 1:* For a manufacturing system, robustness is defined as the aptitude of the system to preserve its specified properties against foreseen or unforeseen disturbances [1].

*Definition 2:* Passive robustness answers to the case when no modification is necessary to the control so that the specified properties are preserved in the presence of variations [1].

*Definition 3:* Active robustness corresponds to the case when the specified properties can be maintained, but at the cost of a total or partial calculation of control [1].

Indeed, robustness is the consequence of two intrinsic elements standing for the type of variations on the one hand and the definition of qualities necessary for the exit of the system on the other. To react to these disturbances, a system must be having decision criteria that enable it to take into account the concept of robustness. When the objectives are maintained without modification of the control, we speak about a passive robustness. The active robustness, however, translates the capacity of a system to ensure the performances at the price

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of a real time control modification. The determination of this robustness provides decision criteria for the calculation of a new control in case the margin of passive robustness is violated (Fig. 1).

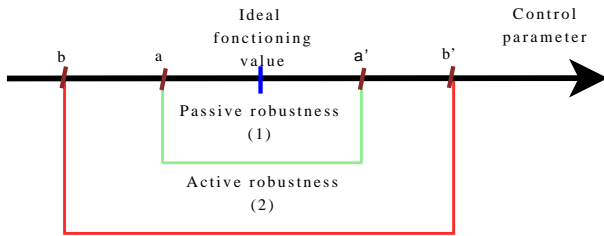


Fig. 1. Robustness margins

(1) The specified properties are guaranteed without any change of the control. The values  $a$  and  $a'$  correspond to the passive robustness.

(2) A control must be inventoried; dynamic margins are modified but the sequencing remains the same. The values  $b$  and  $b'$  correspond to the active robustness.

### B. Petri nets for the robustness control

The exploitation of modeling is adapted as an essential way of research for the determination of the robustness in the production systems. The study of the workshops with temporal or non temporal constraints contains a singular problem which occurs when one is in the presence of a synchronization mechanism. Since automata do not, by definition, represent in an explicit way the synchronization structures, we choose the Petri nets (P-time Petri Nets and Intervals Constrained Petri Nets) as a modeling tool. In fact, this tool is known as being a powerful tool of synchronization of modeling, parallelisms, conflicts and divisions of resources.

**Note:** in this work we use the RdP with inhibitors arcs.

We distinguish two classes of Petri nets agreed to model the robust control:

**The P-time Petri Nets for the study of the workshops with temporal constraints:** The theoretical bases of the P-time Petri Nets were elaborated by Khansa in his thesis [14]. Hi has shown that they represent a powerful and recognized formalism for modeling the respect obligation of setting times (synchronization under obligation) [15].

**Definition 4:** [15] P-time Petri Nets is a t-uple  $\langle P, T, \text{Pre}, \text{Post}, M_0, \text{IS} \rangle$ ; where  $\langle P, T, \text{Pre}, \text{Post}, M_0 \rangle$  is a marked Petri net provided with an initial marking  $M_0$  and  $\text{IS}$  is a definite application per:

$$\text{IS: } P \rightarrow (Q^+ \cup \{0\}) \times (Q^+ \cup \{-\infty, +\infty\})$$

$$p_i \rightarrow \text{IS}i = [\alpha_i, \beta_i]; \text{ where } 0 \leq \alpha_i \leq \beta_i$$

$\text{IS}i$  defines the static interval of sitting time of a mark in the place  $p_i$  ( $Q^+$  is the set of positive rational numbers). A mark in the place  $p_i$  takes part in the validation of output transitions only if it remained at least the duration  $\alpha_i$  in this place. It must leave the place  $p_i$  at the latest when its setting duration becomes  $\beta_i$ . If it cannot do so, we would say that the mark is 'dead' and won't take part in the validation of transitions.

### The Intervals Constrained Petri Nets (ICPN) for the study of the workshops with non temporal constraints:

The Intervals Constrained Petri Nets were introduced by [8] to amplify the field of application of P-time Petri Nets through the abstraction of the basic concepts on the parameter granted to places. Indeed, the same mathematical definition of the tool is almost inherited. Except that, the restriction of parameters associated to places with a positive rational is not justified any more like a guiding principle for all dimensions. For example, there is not any necessity to a variation of a temperature or a position. The definition of the Intervals Constrained Petri Nets is given in what follows:

**Definition 5:** [7] An ICPN is a t-uple  $\langle R, M, \text{IS}, D, \text{Val}, \text{Val}_0, X, X_0 \rangle$ ; where:

- $R$  is an unmarked PN,
- $M$  being an application associating token to places as:  $m$  is a vector indexed on the set of places  $P$   
Let  $m(p)$  be a place marking  
Let  $V$  be a non empty set of rational variables  
Let  $\mu V$  be a multiset defined on  $V$ ,
- $\text{IS}: P \rightarrow Q \cup \{-\infty, +\infty\} \times Q \cup \{-\infty, +\infty\}$  defines the intervals associated to places  
 $Q$  is the set of rational numbers  
 $p_i \rightarrow \text{IS}i = [\alpha_i, \beta_i]; \text{ where } \alpha_i \leq \beta_i$ ,
- $D: \{M(p) \times \{p\} | p \in P\} \rightarrow V$   
 $\forall i, 1 \leq i \leq n, n = \text{Card}(P)$   
Let  $k$  be a token,  $k \in m(p_i)$   
 $k \rightarrow q_i | \alpha_i \leq q_i \leq \beta_i$   
 $D$  associates a rational local parameter to each token in a place,
- $\text{Val}$  be an application:  $M(P) \rightarrow \mu V$   
 $(k \in m(p), p) \rightarrow v \in \mu V$ ; where  $k$  is a given token in  $p$   
 $\text{Val}$  associates a multi set of parameters to each token  
This multi-set is carried by the token thorough the net,
- $\text{Val}_0$  corresponds to initial values associated to tokens,
- $X$  defines the evolution of the local parameter associated to each token in a place  
 $X: V \rightarrow Q$   
 $v \rightarrow q \in Q$ ,
- $X_0$  is the vector of initial value of variables.

The significations of  $q$  and  $\text{Val}(k)$  are not fixed intentionally in order to provide a general model. With ICPN,  $X$  is not fixed mathematically. Nevertheless, it will be shown in the presented application, that some needed properties may be proved even if the  $q$  evolution is not taken into account.

**Definition 6:** [7] The state is defined by a quadruplet  $E = \langle M, D, \text{Val}, X \rangle$ ; where:

- $M$  assign a marking to the network,
- $D$  and  $X$  join to assign with each mark  $k$  in the place  $p_i$  rational number  $q_i(k)$ .

For more details on this modeling tool a reader may consult the references: [7], [8] and [12].

### C. Definition of a terminology

Before starting, we present a number of definitions. These ones are necessary to constitute a unified terminology for the issue of our study.

**Definition 7:** A basic production circuit  $C_p$  is defined as a whole of ordered machines influencing, directly or indirectly, by the variation of their production times, one of the specificities of the operating system.

**Definition 8:** A quality parameter is named explained variable if and only if it depends on variations of the other parameters measured upstream.

**Definition 9:** A quality parameter is named explanatory variable if and only if it takes part with other parameters, by its variation, in influencing an explained variable.

**Definition 10:** A quality parameter is named composed explained variable if and only if it depends at least on the variation of another explained variable measured upstream.

An explained variable can be explanatory for other downstream measured variables. If we are in an unquestionable environment, the relation between the explained variable and the explanatory ones would be determined by a mathematical formula. If we are in a doubtful environment, the relation between the explained variable and the explanatory ones would be determined by a fuzzy relation or by the use of the tools of statistical estimation (linear regression, nonlinear regression, ...).

**Definition 11:** A quality parameter forms a basic quality circuit  $C_q$  if and only if it is an explained variable.

**Definition 12:** The modular robustness is defined as the capacity to maintain locally the specific properties of a basic quality circuit in the presence of variations or uncertainties foreseen or unforeseen due to internal or external disturbances in order to preserve the total robustness of the production system.

We indicate by  $RM$  the modular robustness of a basic circuit of flow or quality type.

### III. MODELS FOR GENERATING ROBUSTNESS: MONO-CRITERIA APPROACH

The concerned systems are the manufacturing systems of flow-shop type. In this paragraph we are going to separately deal with the problems of flow robustness and quality robustness.

Let us consider a manufacturing system  $S$  constituted of  $n$  matter transformations resources. Each resource is characterized by the production elementary time ( $T_{m_i}$ ); with,  $i \in N$  and  $i \in \{1, 2, \dots, n\}$ . At the exit of each resource,  $n_i$  quality parameters will be measured.  $S$  consists of  $HCq$  and  $KCP$ ;  $H$  (respectively  $K$ ) presents the total number of  $Cq$  (respectively of  $Cp$ ). We define the time interval  $I_{p_i} = [a_{p_i}, b_{p_i}]$  (respectively  $I_{a_i} = [a_{a_i}, a_{p_i} \cup b_{p_i}, b_{a_i}]$ ) as the passive robustness margin (respectively of active robustness) relating to the production elementary time of  $R_i$ ; with,  $a_{p_i} \geq a_{a_i}$  and  $b_{p_i} \leq b_{a_i}$ . Seen that the time function is monotone increasing and that we are studying independently robustness flow and quality we can suggest that  $I_{a_i} = [b_{p_i}, b_{a_i}]$ . In the same way, we allot the interval  $I_{p_{i,j}} = [a_{p_{i,j}}, b_{p_{i,j}}]$  (respectively  $I_{a_{i,j}} = [a_{a_{i,j}}, a_{p_{i,j}}]$  and  $I_{a_{s_{i,j}}} = [b_{p_{i,j}}, b_{a_{i,j}}]$ ) as the passive robustness margin (respectively of active robustness margin) of the variable  $V_{i,j}$ ; where,  $V_{i,j}$  is the explanatory variable presenting the  $j^{th}$  parameter quality of

$i^{th}$  resource of  $S$ ; with,  $a_{p_{i,j}} \geq a_{a_{i,j}}$  and  $b_{p_{i,j}} \leq b_{a_{i,j}}$ ,  $j \in N$  and  $j \in \{1, 2, \dots, n_i\}$ .

**Hypothesis 13:**  $\forall i$  and  $j$ ,  $V_{i,j}(t)$  is increasing not defined.

**Hypothesis 14:** The variations of the temporal or non temporal variables do not exceed the active robustness margins.

The modeling of mono-criteria robust control laws relative to  $Cp_k$  (respectively to  $Cq_h$ ) allowing the redundancy of the robustness between passive and active can be presented in a pyramidal form;  $k \in N$  and  $k \in \{1, 2, \dots, K\}$ ,  $h \in N$  and  $h \in \{1, 2, \dots, H\}$ . The skeleton of such model is formulated by  $Nsp_k$  (respectively  $Nsp_h$ ) parallelism structures.

For the flow model: each structure is composed of a transition and two places modeling the passive and active robustness respectively (Figure 2).

$$Nsp_k = 2^{R_k} - 1 \quad (1)$$

Each  $Cp_k$  is made up by  $R_k$  resources; where  $r \in \{1, 2, \dots, R_k\}$ .

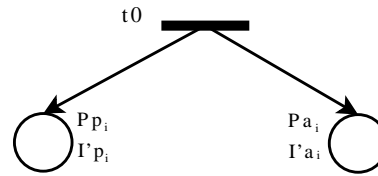


Fig. 2. Parallelism structure elements of flow model

The intervals allocated at these two places of the parallelism structure are:

- $I'p_i = [a_{p_i}, b_{p_i}]$ : interval allotted to the place  $Pp_i$  that models the passive robustness of  $R_i$ ,
- $I'a_i = [a_{p_i}, b_{a_i}]$ : interval allotted to the place  $Pa_i$  that models the active robustness of  $R_i$ .

For the quality model: each structure is composed of a transition and three places modeling the robustness active lower, passive and active higher respectively (Figure 3).

$$Nsp_h = \frac{3^{S_h} - 1}{2} \quad (2)$$

Each  $Cq_h$  is made up by  $S_h$  resources; where  $s \in \{1, 2, \dots, S_h\}$ .

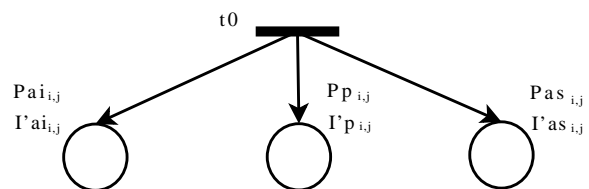


Fig. 3. Parallelism structure elements of quality model

The intervals allocated at these three places of the parallelism structure are:

- $I'ai_{i,j} = [a_{a_{i,j}}, a_{p_{i,j}}]$ : interval allotted to the place  $Pai_{i,j}$  that models the lower active robustness of  $V_{i,j}$ ,
- $I'p_{i,j} = [a_{a_{i,j}}, b_{p_{i,j}}]$ : interval allotted to the place  $Pp_{i,j}$  that models the passive robustness of  $V_{i,j}$ ,

- $I'as_{ij} = ]a_{ai,j}, b_{ai,j}]$  : interval allotted to the place  $Pas_{i,j}$  that models the higher active robustness of  $V_{i,j}$ .

We indicate by:

$EP^-$  : set of places of the network modeling the passive robustness of the resources constituting the basic circuit

$EP^+$  : set of places of the network modeling the active robustness of the resources constituting the basic circuit.

**Definition 15:**  $Pc$  is named control course. It is defined as an oriented way that connects the marked transitions and places modeling the nature of the robustness, starting from the ones modeling the entry of a basic circuit towards those of the exit.

**Note:** We suggest, when a synchronization of the tokens remained in the places modeling a variable, that only the place which has the smallest upper limit allocated interval would be hypothetically considered marked. If, during the evolution of the network modeling  $Cp$  or  $Cq$ , all places of  $Pc$  are marked and relative transitions of exit are crossed,  $Pc$  would be called "marked".

We indicate by:

- $Pc_{k,p}$  : control course number  $p$  of  $Cp_k$  ;  $p \leq N$  and  $p \in \{1, \dots, 2^{R_k}\}$ ,
- $Pc_{h,q}$  : control course number  $q$  of  $Cq_h$  ;  $q \leq N$  and  $p \in \{1, \dots, 3^{S_h}\}$ ,
- $Pc_{P_{k,p}}$  : the subordinate set places of  $Pc_{k,p}$  and pertaining to  $EP^-$ ,
- $Pc_{A_{k,p}}$  : the subordinate set places of  $Pc_{k,p}$  and pertaining to  $EP^+$ ,
- $Pc_{P_{h,q}}$  : the subordinate set places of  $Pc_{h,q}$  and pertaining to  $EP^-$ ,
- $Pc_{A_{h,q}}$  : the subordinate set places of  $Pc_{h,q}$  and pertaining to  $EP^+$ .

**Lemma 16:** Let  $Pc_{k,p}$  (respectively  $Pc_{h,q}$ ) be a marked control course of a basic circuit  $Cp_k$  (respectively  $Cq_h$ ). If ( $Pc_{A_{k,p}} = \emptyset$  (respectively  $Pc_{A_{h,q}} = \emptyset$ )) then ( $Cp_k$  (respectively  $Cq_h$ ) is passively robust).

**Proof:** As long as:

- all resources of  $Cp_k$  (respectively explanatory variables of  $Cq_h$ ) have kept their specificities at the time of the operations of transformation while respecting the margins of passive robustness,
- the margin of passive robustness of each temporal variable (respectively non temporal variable) is selected at the beginning under total constraints translating the passive robustness of the explained variable of  $Cp_k$  (respectively explanatory variables of  $Cq_h$ ),

Definition 2 is applicable. ■

**Lemma 17:** Let  $Pc_{k,p}$  (respectively  $Pc_{h,q}$ ) be a marked control course of a basic circuit  $Cp_k$  (respectively  $Cq_h$ ). If ( $Pc_{A_{k,p}} \neq \emptyset$  (respectively  $Pc_{A_{h,q}} \neq \emptyset$ )) then ( $Cp_k$  (respectively  $Cq_h$ ) is actively robust).

**Proof:** As long as one (at least) of the explanatory variables of  $Pc_{k,p}$  (respectively  $Pc_{h,q}$ ) is actively robust whereas the others are passively robust, the specified properties of  $Pc_{k,p}$  (respectively  $Pc_{h,q}$ ), can only be maintained after a total or a partial calculation of control.

Definition 3 is applicable. ■

**Theorem 18:** There is only one control course  $Pc_k^-$  (respectively  $Pc_h^-$ ) reaching the passive robustness  $Cp_k$  (respectively of  $Cq_h$ ). The others reach the active robustness.

**Definition 19:**  $Pc_k^+$  (respectively  $Pc_h^+$ ) is named set of actively robust control courses. For a structure modeling  $Cp_k$  (respectively  $Cq_h$ ) constituted by  $K$  resources (respectively  $H$  explanatory variables), the set  $Pc_k^+$  (respectively  $Pc_h^+$ ) assemble  $NPc_k^+$  (respectively  $NPc_h^+$ ) courses with :

$$NPc_k^+ = 2^{R_k} - 1 \quad (3)$$

$$NPc_h^+ = 3^{S_h} - 1 \quad (4)$$

**Example 20:** Figure 4 illustrates the modeling of a modular robust control law in the pyramidal form related to a basic production circuit composed of two resources of which the variations of processing time in product influence the specific greatness  $Tce_1$ .

The principle of evolution of this model consists in ensuring, initially, the passive robustness of the resource.

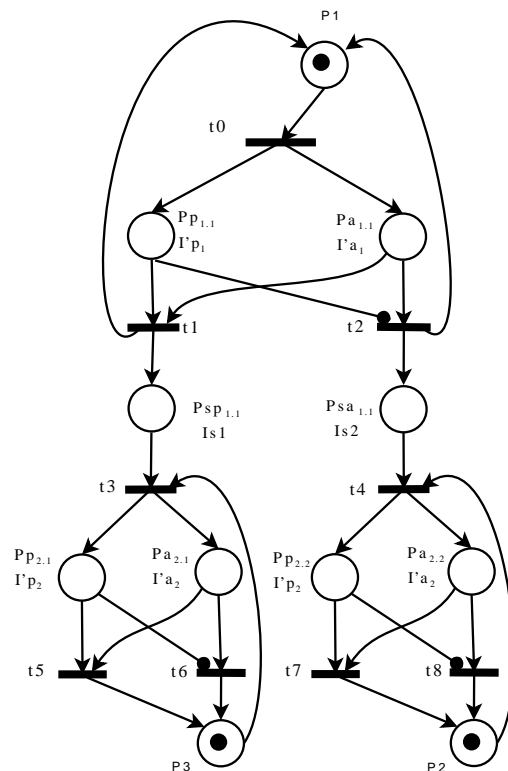


Fig. 4. Flow robustness redundancy (pyramidal structure)

Where:

- P1, P2 and P3: places modeling the passive robustness,
- Psp1,1 and Psa1,1 : places modeling the stock between two resources.

**Example 21:** Concerning the quality model, we illustrate, By the figure 5, a pyramidal modeling of a modular robust control law relating to a basic circuit of a quality parameter

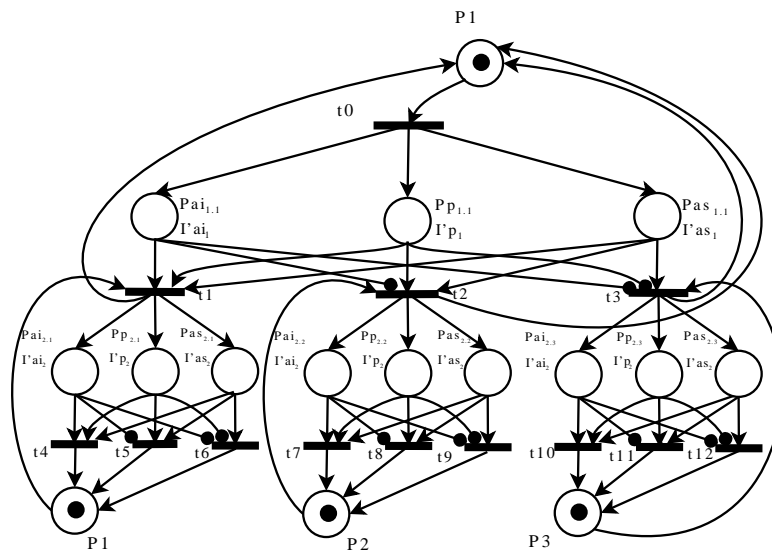


Fig. 5. Quality robustness redundancy (pyramidal structure)

of the type "explained variable" that depends on two quality parameters of the type "explanatory variables". The principle of evolution of this model consists in prioritizing the passive robustness of the resource.

**Note:** each interval corresponds to an index  $ij$  defined at the beginning.

This structure seems to be more complex when the number of basic circuit resources is large:  $R_k = 6$  (respectively  $Nsp_k = 32$ )

Figures 6 and 7 propose a parallel structure equivalent to the pyramidal structure.

The extension of modeling properties and principles developed, brings us to extricate the following lemmas:

**Lemma 22:** Let a manufacturing system  $S$  be constituted of  $KC_p$  (respectively  $HC_q$ ). If ( $\forall k \in \{1, 2, \dots, K\}$ ) (respectively  $\forall h \in \{1, 2, \dots, H\}$ ),  $Pc_{Ak,p} \neq \emptyset$  (respectively  $Pc_{Ah,q} \neq \emptyset$ ) then ( $S$  is passively robust).

**Proof:** Same reasoning as proof of lemma 16. ■

**Lemma 23:** Let a manufacturing system  $S$  be constituted of  $KC_p$  (respectively  $HC_q$ ). If (it  $\exists$  for  $\forall k \in \{1, 2, \dots, K\}$ ) (respectively  $\forall h \in \{1, 2, \dots, H\}$ ), at least  $Pc_{Ak,p} \neq \emptyset$  (respectively  $Pc_{Ah,q} \neq \emptyset$ ) then ( $S$  is actively robust).

**Proof:** Same reasoning as proof of lemma 17. ■

#### IV. DESIGN MODELS FOR TOTAL ROBUSTNESS

##### GENERATION: BI-CRITERIA APPROACH (QUALITY-FLOW)

###### A. Bi-criteria robustness of resource

Using the heritage principle, we propose a skeleton model formed by  $n_i+1$  entry places.

The First is followed by a parallelism structure including two places modeling the margins of passive and active temporal robustness. For the others, they are assiduous; each by a parallelism structure including three places each indicating the lower active robustness margin, the passive robustness margin and the higher robustness active margin respectively of each explanatory variable  $j$  of  $R_i$  (Figure 8).

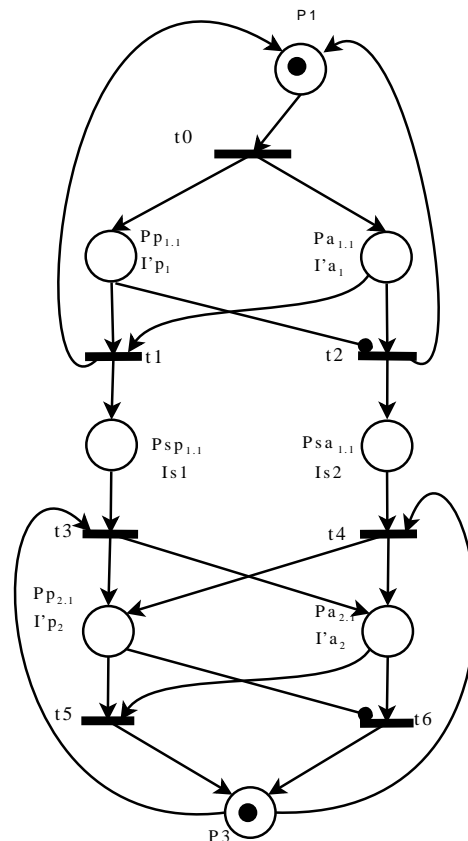


Fig. 6. Flow robustness redundancy (parallel structure)

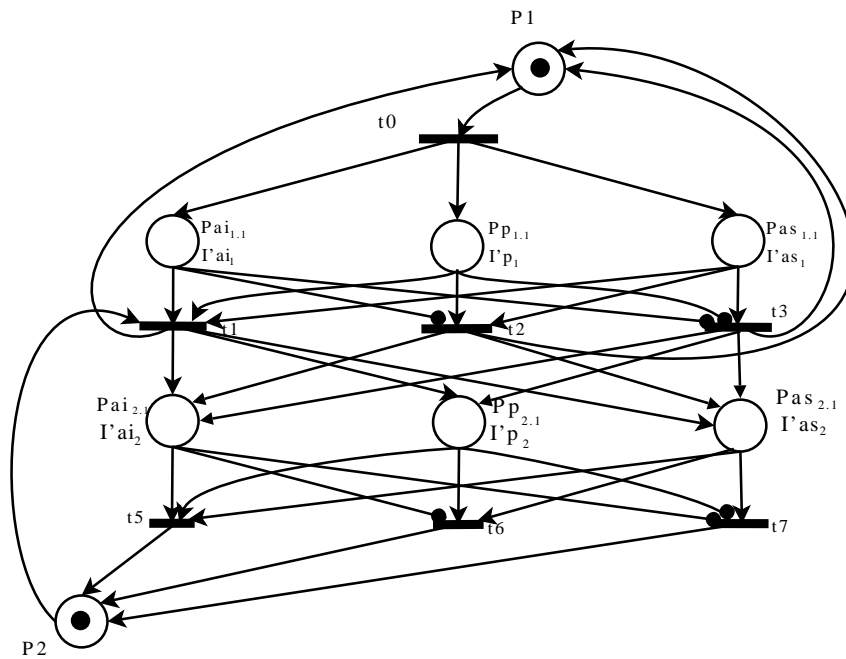


Fig. 7. Quality robustness redundancy (parallel structure)

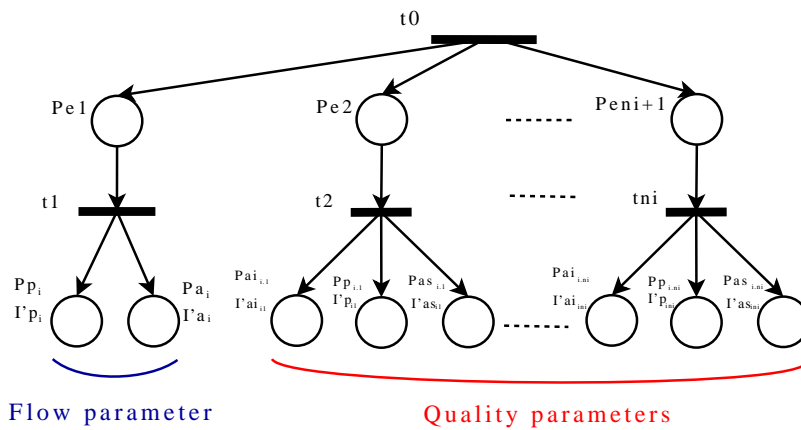


Fig. 8. Parallelism structure elements of a model flow-quality

Let  $S^o p(x)$  is the moment of  $x^{th}$  crossing of  $^o p$  and  $S p^o(x)$  the moment of  $x^{th}$  crossing of  $p^o$ .

It should be noted that  $S p^o$  (respectively  $^o p$ ) the output transition (respectively input transition) of the place  $p$ .

**Theorem 24:** A resource  $R_i$  is passively robust if and only if the two following conditions are satisfied:

Condition 1: in  $t - \varepsilon, t = \min \{ S P p_{i,j}^o(x) | j \{1, 2, \dots, n_i\} \}$ , all places modeling the passive robustness of  $R_i$  are marked.

$$\left. \begin{aligned} \text{card}(m(P p_i)) = 1 \\ \forall j, \text{card}(m(P p_{i,j})) = 1 \end{aligned} \right\} \Rightarrow$$

$$\text{card}(m(P p_i)) + \sum_{j=1}^{n_i} \text{card}(m(P p_{i,j})) = n_i + 1 \quad (5)$$

Where:  $\varepsilon \in R$  and  $\varepsilon \ll t$

Condition 2: in  $t = S P p_i^o(x) \in ]a_{p_i}, b_{p_i}]$  the relative exits transitions of the modeling passive robustness places are

crossed.

$$\forall j, S P p_i^o(x) \geq S P p_{i,j}^o(x) \quad (6)$$

*Proof:* The tokens remained in the places modeling really the temporal and qualitative state of only one product. This implies that a token can be passed to model the product state in another resource only when each parallelism structure contains at least a token able to be drawn (each place can comprise only one token).

In addition, as long as the passive robustness margin of all the explanatory and temporal variables are selected at the beginning under total constraints translating the passive robustness of each resource. A resource can be passively robust if and only if the margins of passive robustness of all variables are respected.

Definition 2 is applicable. ■

*Lemma 25:* Let a resource  $R_i$  of  $S$ . If (at least one of the passive robustness margins of qualitative or temporal variables were not respected were not respected) then ( $R_i$  is actively robust ).

*Proof:* If one of the margins of passive robustness were not respected (the active robustness margin is respected), the specified properties of  $R_i$  would be maintained only after a total or a partial calculation of control.

Definition 3 is applicable. ■

In what follows, we present an applicative example of this theorem. Indeed, a modeling, by ICPN, translating the concepts of the theorem and lemma will be presented.

*Example 26:* Let a resource  $R1$  of a production system  $S$ . It is characterized by the temporal variable, which presents the production elementary time of  $Tm_1$ , and two qualitative explanatory variables  $V_{1,1}$  and  $V_{1,2}$ . We illustrate, by the figure 9, a bi-criteria robust control law (flow-quality) of the resource  $R1$ .

Where:

$Ci1, Ci2$  : Places of information exchange, present a communication channels between the places modeling the qualitative variables of  $R1$  and those specifying the temporal aspect of  $R1$ .

*Hypothesis 27:* We suppose that for each parallelism structure relative to a qualitative explanatory variable, one of the exit transitions is passable for the  $x^{th}$  time before or at the  $SPp_i^o(x)$  date.

### B. Bi-criteria robustness of Systems

*Lemma 28:* let a production system  $S$  regrouping  $n$  resources. If ( $\forall i \in \{1, 2, \dots, n\}$ ,  $R_i$  is passively robust ), then ( $S$  is passively robust ).

*Proof:* Same reasoning as proof of lemma 16. ■

*Lemma 29:* Let a production system  $S$  regrouping  $n$  resources. If (it  $\exists$  for  $i \in \{1, 2, \dots, n\}$ , at least  $R_i$  is actively robust ), then ( $S$  is actively robust ).

*Proof:* Same reasoning as proof of lemma 17. ■

For the design and the modeling of a robust control law of a complete system, the following ordered stages are proposed: to define the variables flow-quality specifying the state of each resource  $\Rightarrow$  to determine the basic production circuits  $\Rightarrow$  to determine the basic quality circuits  $\Rightarrow$  to model the resources  $\Rightarrow$  to attach the structures modeling the resources, all in respecting the product passage chronological order  $\Rightarrow$  to model the basic quality circuits by attaching the structures of the qualitative parameters forming each circuit.

*Example 30:* Let a flow-shop production system  $S$  regrouping 3 resources. It allows the production of only one type of product. The product passes in an ordered way by the various resources:  $R1 \rightarrow R2 \rightarrow R3$ . With:  $Cp1$  depends of  $Tm_1, Tm_2$  and  $Tm_3$ ;  $V_{2,2} = f(V_{1,1}, V_{2,1})$  form  $Cq_1$ ;  $V_{3,2} = f(V_{2,2}, V_{3,1})$  form  $Cq_2$ .

Where:  $Tm_i$  : production elementary time of  $R_i$ ;  $V_{i,1}$  : explanatory variable of  $R_i$ ;  $V_{1,2}$  : explanatory variable of  $R1$ ;  $V_{2,2}$  : explained variable measured at the end of the transformation activity relative to  $R2$ ;  $V_{3,2}$  : explained variable measured at the end of the transformation activity relative to

$R3$ . We illustrate, by the figure 10, a modeling by ICPN a bi-criteria robust control law (flow-quality) of  $S$ ; where Pt1, Pt2: places allowing the connection of the structures.  $Pa_{2,2}$  (respectively  $Pa_{3,2}$  ) model the active robustness of  $V_{2,2}$  (respectively  $V_{3,2}$  ).

### V. CONCLUSION

In this paper, a methodology of design and modeling of control laws is adopted. We modelled, by the use of the Intervals Constrained Petri Nets (ICPN) tool which presents a functional abstraction of the P-time Petri Nets; constraints subjected on flow and quality parameters while integrating the margins of passive and active robustness. The goal is to satisfy qualitative and quantitative needs of the market.

The redundancy of the local robustness between passive and active brings us, firstly, to define the ways ensuring the observation of the mono criteria modular robustness type of the basic circuits. Then, while following the same principle, we established the resource model. Finally, we developed the final model of a whole production system. Throughout this paper, applicative examples were used for illustration.

By this proposal, we hope to evaluate the robustness of the manufacturing systems by monitoring the control law parameters. We tend by the distribution of the margins of passive and active robustness to define detection thresholds.

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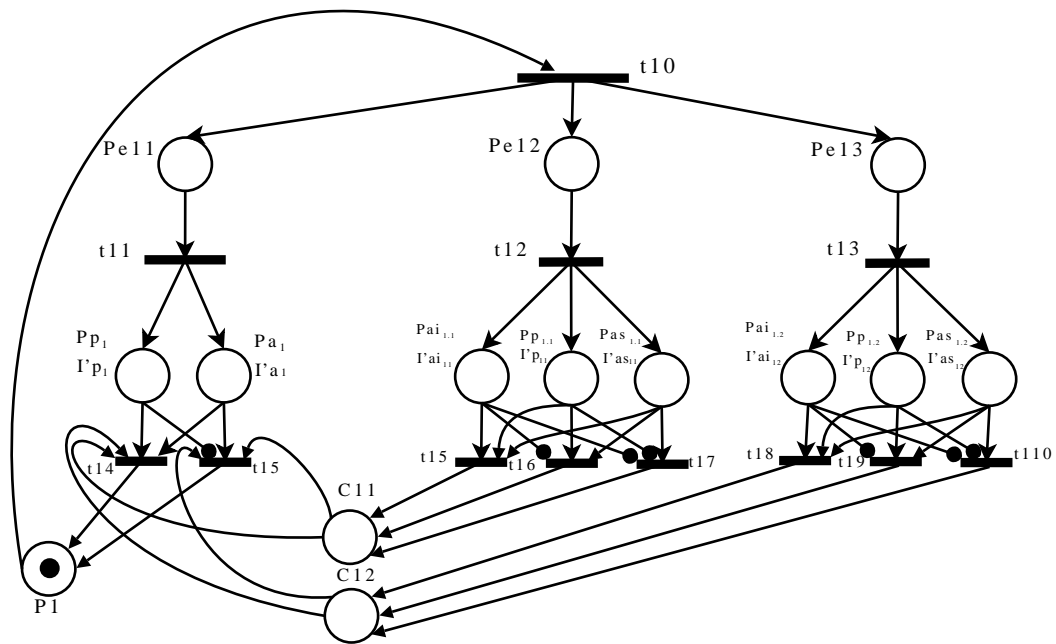


Fig. 9. Model of a bi-criteria robust control law (flow-quality) of the resource R1

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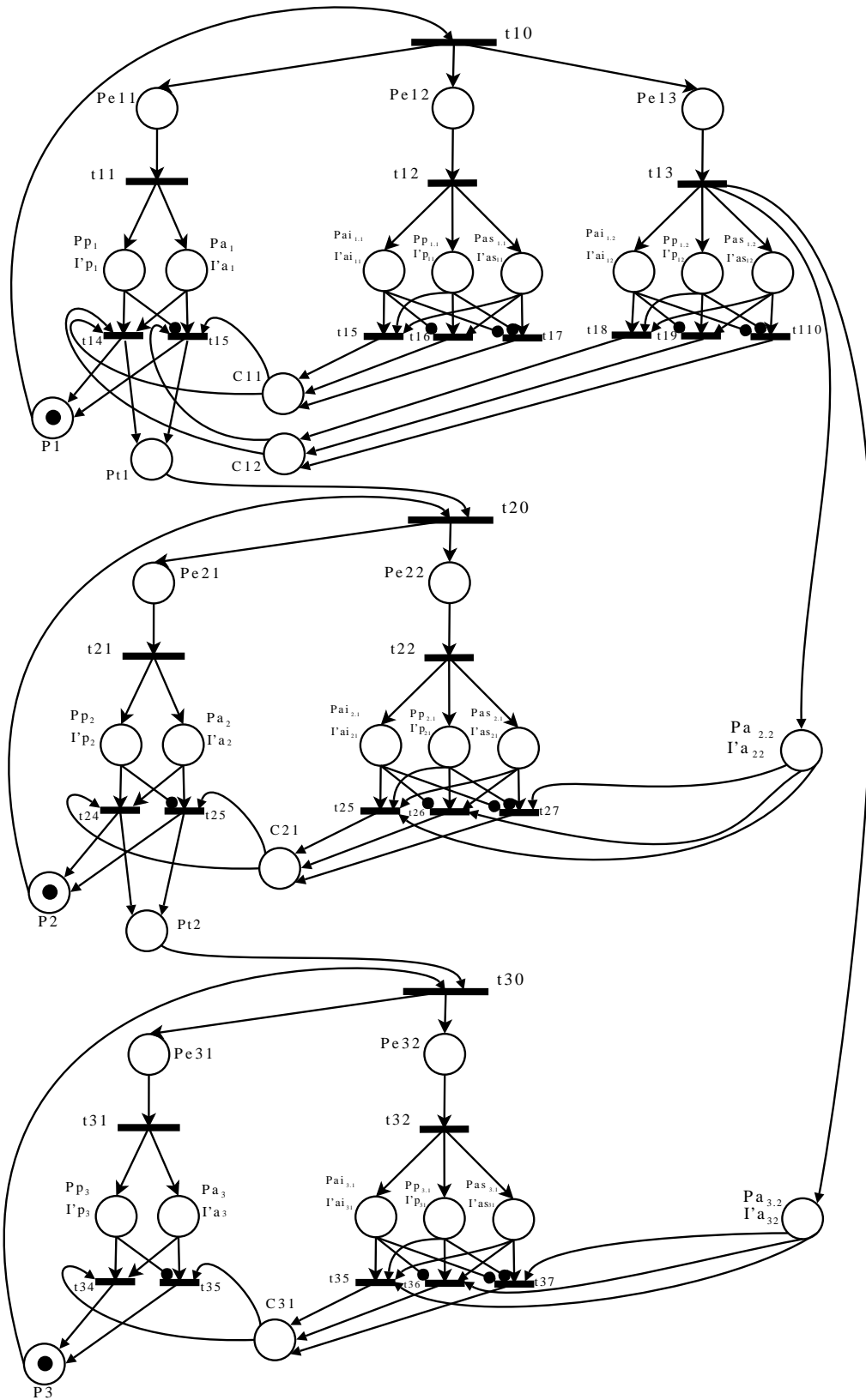


Fig. 10. Model of a bi-criteria robust control law (flow-quality) of  $S$