

# Modeling HIV/AIDS Prevention by Defense

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**Abstract**—The functional response of an infective is the relationship between an infected individual's infection rate and the abundance of the number of susceptibles that one can potentially be infected. In this paper, we consider defensive attitudes for HIV prevention (primary prevention) while at the same time emphasizing on offensive attitudes that reduce infection for those infected (secondary prevention). We look at how defenses can protect an uninfected individual in the case where high risk groups such as commercial sex workers and those who deliberately go out to look for partners. We propose an infection cycle that begins with a search, then an encounter, a proposal and contact. The infection cycle illustrates the various steps an infected individual goes through to successfully infect a susceptible. For heterogeneous transmission of HIV, there will be no infection unless there is contact. The ability to avoid an encounter, detection, proposal and contact constitute defense.

**Keywords**—Functional response, Infection cycle, Prevention, Defences, SSS equation.

## I. INTRODUCTION

As a form of adaptation, many animals have developed defences against their predators in a bid to outwit them. To avoid being wiped out, prey reduce predation risk by disrupting the predation cycle characterized by the following: search, encounter, detect, attack and feed. It has been shown that prey defensive attitudes are essential for survival and that predator offensives increase their predation changes [1]. Prey defenses can be a stabilizing factor in predator-prey interactions. Predation can be a strong agent of natural selection as easily captured prey are eliminated, and prey with effective defenses (that are inherited or learnt) can dominate the population. We can draw some interesting parallels with the interaction between individuals infected with HIV and those who are not. Jeschke, Tollrian and Kop [1], [2], [3], [4], give a detailed analysis on the role of prey defences in reducing the predators attack efficiency and success rate. The dependance of prey defences and predator offenses on prey density is investigated. Their analysis closely parallels the HIV/AIDS pandemic that has ravaged societies. As regards HIV/AIDS, the uninfected individuals must have defensive attitudes that can protect them against diseases and reduce the risk of contracting the diseases. On the other hand, those that are infected must have offensive attitudes that reduce the chances of spreading the disease. We consider HIV/AIDS as an example, and argue that we can apply the predator-prey interactions to model the spread of HIV among human individuals where defences can be used to avoid infections. While it is true that predators look for prey for survival and infected individual can survive without sexual encounters, we argue in this paper that individuals such as commercial

sex workers have to have sex in order to survive. Lack of counseling services, declining health standards and loss of hope in the event of HIV contraction has led to individuals having discovered that they are HIV positive, opting for revenge and the 'the I will never die alone' attitude. It is also apparent that when infected individuals are on a revenge mission, the predator instincts will be controlling them. We can consider those with the virus to be preying on the susceptible population. In many mathematical models, the infected have been subdivided into classes of those that are under treatment and those with full blown AIDS, but they have one thing in common, they are still infectious. It is therefore important to emphasize more on the defensive attitudes for HIV prevention (primary prevention) while at the same time dealing with the offensive attitudes of those infected (secondary prevention).

Mathematical models have also been used to model the dynamics of HIV/AIDS. Incorporation of interventions in these models has attracted significant attention in recent years [5], [6]. The epidemiology of HIV/AIDS has moved beyond the virus and the risk factors associated with its transmission to a more detailed understanding of the mechanisms associated with the spread, distribution and impact of any intervention on the population. Consequently, the impact of health policies, such as poor access to care, delayed treatment or the use of screening for asymptomatic cases can be calculated [7]. In this paper we look at recent research advances in ecology and draw interesting parallels with the spread of HIV/AIDS in the human population. In this study, sexual encounters are responsible for the loss of susceptibles (the prey, individuals without the HIV). Just as the predator per capita feeding rate on the prey, i.e its functional response, provides the foundation for predator prey interaction, the sexual encounter rate of an infected individual plays a vital role in the spreading of HIV. While the predator's feeding rate represents the transfer of biomass between trophic levels [8], the rate of sexual encounters by an infected individuals represents transfer of the virus, resulting in the loss of susceptibles.

We attempt to answer the following questions; how do defenses affect the number of susceptibles infected and the susceptible density (the infective's functional response - the number of susceptibles infected as a function of the susceptible individuals' density)? How well can such an approach be used to model behavior change in individuals? In an attempt to answer these questions we use the SSS (steady state satiation) equation derived in [3], as a functional response model to model how the defences of the susceptibles qualitatively affect functional response in the case of an HIV infected individual preying on the uninfected susceptibles. To the best of my knowledge, the idea of looking at functional responses, prey

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defenses and predator offenses in the context of HIV/AIDS is new. The author poses a very important question in epidemic modeling; what happens to a disease where treatment does not mean removal of infectives from the population but reduction in infectivity? A Type II response function could depict the growth of the infective population. In order to effectively control the rapid growth of infectives, the institution of measures that reduce the growth rate of the infective population seems to be the only solution.

## II. MODEL

As in [1], [3], the model is based on the disc equation and divides the infection cycle into four stages: search, encounter, proposal and contact. The stages are assumed to be mutually exclusive and are governed by the time spent in each stage and the conditional probability that the infective reaches a stage given that an individual has reached the previous stage. The infection cycle illustrates the steps an infective goes through to successfully infect a susceptible. We assume the following:

- the density of susceptibles is constant and the mixing is homogeneous.
- the desire to have a sexual contact depends on some level of satisfaction after a sexual contact. This can also be interpreted in the context of an intended objective being fulfilled. For example, a sex worker's satisfaction is a function of how much (in monetary terms) one has made for a particular day.
- the desire or determination of the infected to have sexual encounters does not change over time (in steady state). The choice of whom to have the encounters with may change though. The determination to have an encounter with a large number of individuals is equivalent to the hunger level of predators in an ecological setting.
- the probability that one proposes is unity i.e if an individual goes for a search and encounters potential prey, then without doubt a proposal will be made.
- only one susceptible and infective types are involved. This rules out the possibility several infections with different strains, which will make an infective susceptible to other strains. This is only true for closed communities with limited migration and emigration.
- the infection rate when there is a high density of susceptibles is limited by the handling time or satiation. By handling time, we mean the time spent with a victim. If an individual satiates after interacting with five individuals the the satiation parameter  $s = 0.2$ .
- we consider heterosexual transmission as the means by which the disease spreads.

We use the SSS equation to determine the effects of changing the characteristics (such as defenses) of the infectives and susceptibles on the infection rate. As given in [1], [3], searching stage, is the entry stage for an infected individual, with the probability that an individual will search (searching effort) being  $\alpha(N)$ , where  $N$  is the density of susceptibles (individuals per given area).

$$\alpha(N) \begin{cases} = 1 & \text{if the infective is not handling a victim,} \\ < 1 & \text{if the infective is handling a victim.} \end{cases}$$

$\alpha(N)$  is proportional to the level of intent  $h(N)$ , i.e the level at which one wants to make money in the case of sex workers with  $0 \leq h(N) \leq 1$ . In this case

$$\alpha(N) = \kappa h(N),$$

where  $\kappa$  is assumed to be unit. If there is no intent at all then  $h(N) = 0$  otherwise  $h(N) = 1$  if there is 100% intention to have a sexual encounter. As in [3], the differential equation

$$\frac{dh(N)}{dt} = h'(N) - sy(N),$$

where  $h'(N) = \frac{1-h(N)}{t_c}$  and  $t_c$  is the contact time, is important in determining the how intentions to have a contact change with time. The level of intent is proportional to the difference between the levels of satisfaction by intended actions and the assumed benefit of the action taken. At the steady state, the level of intent is given by

$$h(N) = 1 - cy(N), \text{ where } c = st_c. \quad (1)$$

We can define  $c$  as the processing time (depends on the level of satisfaction and the time it takes to do so). The rate at which the susceptibles are infected by an infective (the functional response) is given by

$$y(N) = \frac{aN}{1 + abN}, \quad (2)$$

where  $a$  is the success rate of the infective (area covered per given time),  $b$  the handling time and  $y(N)$  the number susceptibles infected per given time (rate of infection). This is popularly known as the Holling's (1959) disc equation.

We now consider the various strategies, being implemented in dealing with the HIV/AIDS pandemic. The various strategies affect the infection cycle. An individual's risk of acquiring HIV is to a large extent determined by an individuals attitude and his role during sexual intercourse [9]. If  $\eta$  is the transmission probability per contact, the probability that a susceptible individual will not be infected by a single contact with an infected individual is  $1 - \eta$ . So the probability that infection is avoided when  $n$  contacts have been made is  $(1 - \eta)^n$ . Thus the transmission probability per partner [10] is

$$\phi = 1 - (1 - \eta)^n.$$

The rate of disease transmission is in direct proportion to the proportion of infected sexual partners  $q$  an uninfected individuals comes meets. To model the role of condom use in HIV/AIDS prevention, we assume that condoms have an efficacy  $e_c$  and compliance  $c_c$  ( $0 \leq e_c, c_c \leq 1$ ) so that the product  $p = e_c c_c$  measures the level of protection against HIV by the use of condoms.  $p$  is defined as the condom induced preventability [11]. Thus  $(1 - p)$  measures the condom induced preventability failure. Where condoms are being used as a means preventing infection, one can take the disease transmission efficiency to be,

$$\epsilon = q(1 - p)\phi. \quad (3)$$

This explicit form of the disease transmission efficiency, allows us to account for condom efficacy and compliance, and

the possibility of a sexual contact not resulting in an infection. Increasing condom use and reducing the number of sexual partners reduce the success rate  $a$ .

The success rate

$$a = \beta\gamma\delta\epsilon$$

is a product of  $\beta$ , the encounter rate between an uninfected individual and an infected individual,  $\gamma$  the probability that an infected individual encounters an uninfected individual,  $\delta$  the probability that the encounter results in a proposal and  $\epsilon$  the disease transmission efficiency. The disease transmission efficiency can be taken to be the probability that a successful proposal results in a sexual encounter that results in an infection. Proposals are usually based on the derived benefits of the action. For example, for an infected woman who needs support,  $\delta$  would be the probability that the woman in question detects the availability of the support. Handling time depends on the time spent proposing per an uninfected individual,  $t_p$ , the time spent in sexual encounters per an uninfected individual,  $t_c$  and the disease transmission efficiency,  $\epsilon$ . We thus have,

$$b = \frac{t_p}{\epsilon} + t_c.$$

Short handling times are likely to increase the spread of the disease as a new search will then begin. Delayed sexual encounters by the youth can be interpreted as increasing the handling time in the case of a successful proposal. The handling time, also depends on the capacity of an infected individual to handle a given number of relationships (in the case were multiple relationship occur). A rich infected individual is more likely to handle many relationships due to the availability of the means to do so. The processing time determines how soon the next search will come. For  $c = 0$ , there will be no satiation.

Studies have shown that infected individuals and those in serodiscordant relationships reduce their risk behavior following testing. In developing countries, especially in Southern Africa, testing and counseling may carry greater impact in HIV prevention [12]. In situations where the infected is on a revenge mission, knowledge of one's HIV status may increase the searching effort,  $\alpha(N)$ , by decreasing the processing time in a bid to reach as much prey as possible in the shortest possible time, as is the objective in revenge or money making. We can now redefine the processing time,

$$C = \sigma \cdot c,$$

where  $\sigma$  is a parameter that measures the level of awareness on the processing time and  $\sigma > 0$ . An increase in the value of  $\sigma$  corresponds to an increase in the processing time.

So (1) becomes

$$h(N) = 1 - Cy(N). \quad (4)$$

Incorporating  $h(N)$  in (2) gives

$$y(N) = \frac{[1 - Cy(N)]aN}{1 + [1 - Cy(N)]abN}, \quad (5)$$

From (5), solving for  $y(N)$  gives the SSS equation below:

$$y(N) \begin{cases} \frac{1 - a(b+C)N - \sqrt{1 + a(2(b+C) + a(b-C)^2 N)N}}{2abCN}, & \text{if } a, b, C, N > 0, \\ \frac{aN}{1 + abN}, & \text{if } b > 0, C = 0, \\ \frac{aN}{1 + aCN}, & \text{if } b = 0, C > 0, \\ aN, & \text{if } b = C = 0, \\ 0, & \text{if } a = 0 \text{ or } N = 0. \end{cases}$$

This equation is the one used in [1], [2], [3], [4], [13], but imbedded in the parameters  $a, b$  and  $C$  are the various intervention strategies that are being implemented to curb the HIV/AIDS spread.

### III. SURVIVAL OF SUSCEPTIBLES

Susceptibles can survive the HIV/AIDS scourge if their defensive attitudes decrease the success rate of the infectives. Educational campaigns in the fight against HIV are aimed at preventing new searches, increase the handling time and preventing new infections on proposals that have been successful. The ability of the susceptibles to handle life issues (e.g women empowerment, education, sexual risk-reduction skills, strong marriages etc.) increase the handling time. It also increases the processing time  $C$ , thus reducing the infected individual's desire to go for the next search. Predator avoidance strategies are important for survival because they prevent encounters and detection that lead to proposals. Abstinence has always been regarded as the best strategy to avoid infections.

Protection of children in their most vulnerable state is of primary importance to prevent and minimize encounters with the infected. This protection may be in the form of sex education. Uninfected individuals should by all means avoid areas where the high risk group frequently visit in a bid to minimize encounters with the infected. While it is difficult to prevent detection and encounters, it is not difficult to prevent the success of a proposal or a contact. In the event that a successful proposal results in sexual encounters, protective behavior will prevent infection i.e the use of condoms. Sometimes intimidation of potential predators can also prevent infections. Naturally, some prey can survive by intimidation [14]. One can also argue that increasing the number of activities, in the form of exercises and social grouping can reduce the time spent in sexual activities. The underlying principle is that, one can not be infected unless their is contact, in the case of heterosexual transmission. Survival of susceptibles largely depends on the reduction of the infection rate.

A comparison of the disc and SSS equation are given in Fig. 1. The disk equation assumes that every proposal is successful and the infective does not become satiated. This has been deemed unrealistic as species behave adaptively and their density is important[3], [15]. We now choose hypothetical parameter values for illustrative purposes and no significance should be attached to these values.

Condom use has been advocated for in recent years as a means to reduce new infections. We observe in Fig. 2 below that increased condom preventability is accompanied by a decline

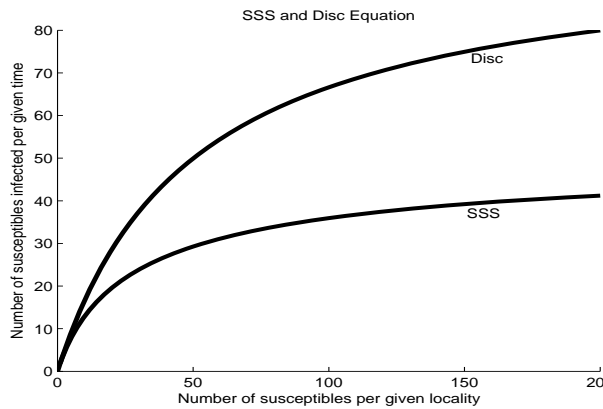


Fig. 1. The infection rate as depicted by the disc and SSS equations for the following parameter values:  $c = 0.02$ ,  $\beta = 20$ ,  $\gamma = 0.1$ ,  $\delta = 0.95$ ,  $q = 5$ ,  $p = 0.3$ ,  $\phi = 0.3$ ,  $b = 0.01$

in the number infectives. This is so because increasing  $p$  reduces the disease transmission efficiency  $\epsilon$ . For example, if  $p = 0.9$ ,  $\epsilon = 0.15$  and if  $p = 0.7$ ,  $\epsilon = 0.45$ .

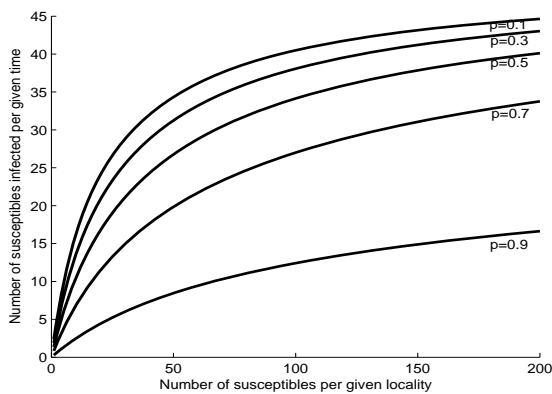


Fig. 2. The infection rate as depicted by the SSS equations for the following parameter values:  $c = 0.02$ ,  $\beta = 20$ ,  $\gamma = 0.1$ ,  $\delta = 0.95$ ,  $q = 5$ ,  $\phi = 0.3$ ,  $b = 0.01$  with  $p$  being varied.

A contrasting result is obtained when the transmission probability is increased. Increasing  $\phi$  leads to an increase in the disease transmission efficiency  $\epsilon$ .

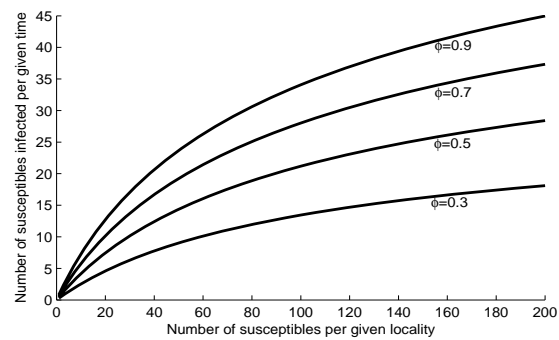


Fig. 3. The infection rate as depicted by the SSS equations for the following parameter values:  $p = 0.9$ ,  $\beta = 20$ ,  $\gamma = 0.1$ ,  $\delta = 0.95$ ,  $q = 5$ ,  $b = 0.01$  with  $\phi$  being varied.

The level of awareness  $\sigma$  about a disease is critical if desired levels success in disease prevention are to be realized. We vary the level of awareness in the SSS equation and the results are graphically presented below.

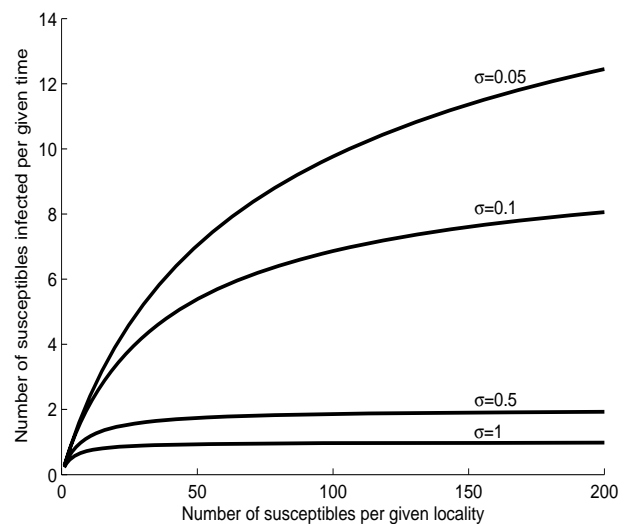


Fig. 4. The infection rate as depicted by the SSS equations for the following parameter values:  $p = 0.9$ ,  $\beta = 20$ ,  $\gamma = 0.1$ ,  $\delta = 0.95$ ,  $q = 5$ ,  $\phi = 0.3$ ,  $b = 0.01$  with  $\sigma$  being varied.

#### IV. DISCUSSION AND CONCLUSION

Prey defensive attitudes diminish the population that pre-dates on the prey. HIV/AIDS is a disease that is preventable through behavioral change. To develop appropriate HIV prevention interventions strategies, a lot of factors have to be considered. These include risk-related cognitive and attitude factors (incorrect beliefs about the disease, weak intentions to change behavior, negative attitudes towards condoms, poorly perceived self efficacy), poor risk reduction skills ( correct condom use, limited power to negotiate safe sex) and poor social support structures [12]. The spread of HIV/AIDS, especially

in Southern Africa has been attributed to the complex social networks, diverse cultures and cultural practices, poverty, substance abuse, migration etc [16]. The main focus in the fight against the disease has been on prevention through behavioral change with the aim of preventing new infections. One way in which those that are not infected can avoid infection, is to develop defense mechanisms that ensure survival. Studies have shown that predators prefer to hunt small-brained animals [17]. One can argue the need to have a strong resolve, will and mind in order to survive the HIV/AIDS pandemic, especially in an environment where one in every three are infected (the case of Botswana, Swaziland and Lesotho in Southern Africa) [18]. To have significant success in HIV prevention, individuals must avoid and defend themselves from infection. By defence, we mean any characteristic that reduces the likelihood of one getting infected.

The HIV virus has been characterized by rapid evolution and high rates of genetic mutation. It is known that virus evolve rapidly in response to survival threats such as the anti-retroviral drugs (ARVs) and as such the fight against HIV using pharmaceutical products will always remain elusive. This has made scientific progress in the fight against the virus very slow. At the mean time, the possibility of achieving mutual coexistence between HIV and humans should be supported. The advent of ARVs means humans must adapt to the presence of HIV in the population, while making sure to avoid infection by sound defensive mechanisms.

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