International Journal of Mechanical, Industrial and Aerospace Sciences
ISSN: 2517-9950
Vol:4, No:3, 2010

# Modeling And Analysis of Simple Open Cycle Gas Turbine Using Graph Networks

Naresh Yadav, I.A. Khan, and Sandeep Grover

*Abstract*—This paper presents a unified approach based graph theory and system theory postulates for the modeling and analysis of Simple open cycle Gas turbine system. In the present paper, the simple open cycle gas turbine system has been modeled up to its subsystem level and system variables have been identified to develop the process subgraphs. The theorems and algorithms of the graph theory have been used to represent behavioural properties of the system like rate of heat and work transfers rates, pressure drops and temperature drops in the involved processes of the system. The processes have been represented as edges of the process subgraphs and their limits as the vertices of the process subgraphs. The system across variables and through variables has been used to develop terminal equations of the process subgraphs of the system. The set of equations developed for vertices and edges of network graph are used to solve the system for its process variables.

*Keywords*—Simple open cycle gas turbine, Graph theoretic approach, process subgraphs, gas turbines system modeling, system theory

## I. INTRODUCTION

Various efforts have been done in the past for developing the mathematical model of the simple gas turbine systems keeping into consideration the desired outcomes i.e. type of applications, work outputs and the optimized conditions for achieving maximum thermal efficiency and the component efficiencies with in the system. Irrespective to the type of application, during the modeling and analysis of simple cycle gas turbines, the parameters related to system elements like compressor, combustor and the turbines etc. are matched for their peak performances to achieve their maximum efficiency [1] in the integrated system. Therefore, these parameters are interlinked for the attainment of objectives like high thermal efficiency, low specific fuel consumption or high thrust [2] etc. based on the type of applications.

It is worth to mention here that once the preliminary design constraints like type of application, type of Turbo-machineries to be used, operating conditions and the business objectives, attention is focused on the thermodynamic evaluation [3] of the gas turbine components and subsequently the aero-thermal and mechanical design constraints [4] are resolved for peaking thermal efficiency. New and advanced materials are to be developed for matching better performance at high firing temperatures. Therefore, the thermodynamic modeling [5] of the simple gas turbine system is the most important issue in the design of gas turbine systems. Since such developments encourage the researchers to achieve higher thermal efficiencies within the realistic constraints [6], the advanced gas turbine systems have been modeled and analyzed [7] for power generation applications. The market leaders [8] and the premier consultants [9]- [10] in the area of gas turbines have provided an exhaustive database for the performance evaluation of gas turbine systems including the effect of its operating parameters on the gas turbine system performance.

Over the past few years, the simplicity in the algorithms of interdisciplinary approaches have attracted the researchers and scientists for system alternative representations and their subsequent solutions for better understanding, user friendly and precise decision making in the operating environment. The interactive software [11] for Simple Cycle gas Turbine performance for typical aircraft application has also been developed. Various thermal system elements have been modeled [12] using graph theoretic approach for thermodynamic evaluation of thermal- hydraulic systems. Similarly, efforts have be done for the formulation of several other engineering systems [13] and has been successfully implemented for the modeling and analysis of some typical engineering systems [14] like truss structures, gear trains etc.. The present work also aims at implementation of system theory postulates and graph network theory for formulation and analysis of simple open cycle gas turbine system with validation through first law and second law of thermodynamics.

## II. WORKING OF SIMPLE OPEN CYCLE GAS TURBINE

In a simple open cycle gas turbine, as shown in Figure 1, the ambient air at pressure $P_1$ and temperature, $T_1$ is sucked at the eye of the compressor. The ambient air is compressed in the compressor to a high pressure $P_2$ through an adiabatic process. Due to work of compression, the temperature of the air also increases to $T_2$ at the exit of the compressor. The compressed air at pressure $P_2$ and temperature $T_2$ is passed through a combustion chamber wherein a fuel of high calorific value is burnt along with the present air at almost constant pressure conditions, which results in a gas mixture with much higher heat content, thus leading to a very high temperature, $T_3$ at the exit of the combustor and subsequently at the entry to the Turbine. The gas mixture undergoes process of Polytropic or adiabatic expansion and gives work output $W_T$. A fraction of this work output is utilized to run the compressor in the cycle. The exhaust gases coming out of the turbine are lost to the ambient or the atmosphere.

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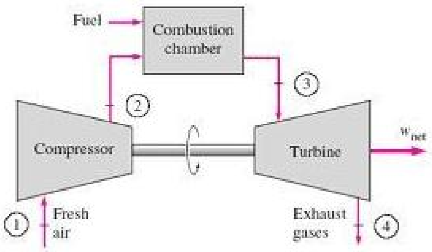



Fig. 1. Simple cycle gas turbine

Since, this is a direct heat loss, it is desirable to keep the temperature and thus heat content of the exhaust of the turbine to a minimum for efficient open cycle gas turbine system. In open cycle gas turbine, for every stage of heat or work transfer, various losses are also involved with in the system. In actual open cycle gas turbine, the compression process in the compressor and the expansion process in the turbine are never ideal to be called as adiabatic processes but in fact are better represented as polytropic. More work is to be done on the compressor to drive it and a lesser work output is achieved from the turbine as a result of polytropic expansion. Similarly, in the combustor, the mixing of two streams i.e. hot compressed air and gaseous fuel do not take place at constant pressure. As a result of the above major accountable losses, the thermal efficiency as well as specific work output is affected significantly.

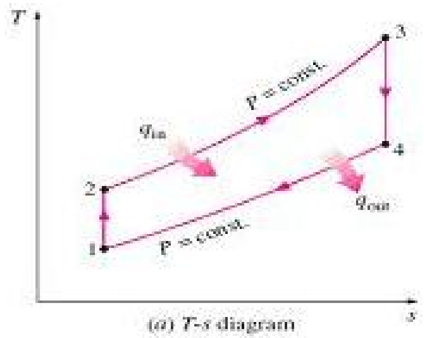



Fig. 2. T-S Diagram for ideal simple cycle gas turbine

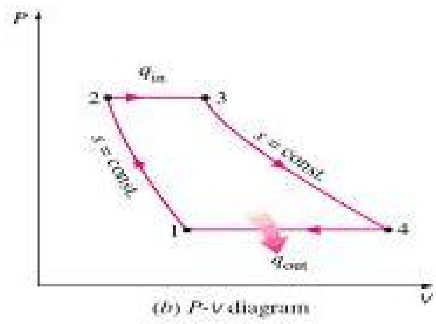



Fig. 3. P-V Diagram for ideal simple cycle gas turbine

In general, the PV- and T-S diagrams ,as shown in Figure 2 and Figure 3 respectively, are analyzed directly for the simple open cycle gas turbines to calculate the various output performance parameters along with system variables at the intermediate points i.e. entry and exit to the compressor, combustor and the turbines. The governing equations based on thermodynamic processes and laws of thermodynamics are used directly to solve the system problems. For a set of modified variables, the sensitivity analysis for the system performance is generally not the issue. However, the governing equations are re-iterated to reach to a new solution. The results thus obtained are generally analyzed and compared to reach to a conclusion regarding specific work output, overall thermal efficiency. The set of various thermodynamic relations being used to solve the systems are given in equation (1).
Compressor:

$$\frac{T_2}{T_1} = (\frac{P_2}{P_1})^{\frac{\gamma-1}{\gamma}} = (\frac{v_1}{v_2})^{\frac{\gamma-1}{\gamma}}$$

Combustion Chamber:

$$P_3 = P_2$$

Turbine :

$$\frac{T_4}{T_3} = (\frac{P_4}{P_3})^{\frac{\kappa-1}{\kappa}} \quad (1)$$

## III. GRAPH THEORETIC APPROACH

As stated earlier, the System theory as well as graph Theoretic approach has been successfully applied to the dynamic analysis of electrical networks synthesis, power systems as well as other fields of engineering. Graph theory is a useful representation because on one hand the elements of the graph can be defined in order to have one-to-one correspondence with many kinds of other systems. On the other hand, the theorems and the algorithms of the graph theory allow one also to represent behavioural properties of the system, such as rate of work transfer, rate of heat transfer, the pressure and temperature drops in the edges of the graph and the absolute pressures and temperatures at the vertices of the graph. In the graph theoretic approach, the system is usually represented as a diagram with nodes, lines and words or the numbers which assign values to some or all nodes or lines.

### A. System Elements And Graphs

The elements of the diagram are the syntax symbols and the diagram itself is considered to be a sentence which describes the system. A graph is defined by an ordered pair G= V, E where 'V' is the vertex set and 'E' the edge set and every edge is defined by its two end vertices. If each edge in the graph has a direction, the graph is known as a directed graph or the Digraph. If the digraph is a network graph, each edge and the vertex have properties of flow and the potential in the system respectively. For example, in case of simple open cycle gas turbine, the digraph edges represents the Pressure drops and the temperature drops in the flow path along with energy drops whereas the vertices represents the absolute pressures and temperatures as required to define the state of the system.

In the present paper, special graph theory representations have been described and their embedded properties have been used to model and analyze the Simple open cycle gas turbine. Central to understanding these graphs are particular type of graph called a tree. A tree is a connected graph with no circuits or loops. The relation between the number of vertices 'v' and edges 'e' of a tree are fixed as given by equation (2).

i.e.

$$e(T) \quad = \quad v(T) - 1 \tag{2}$$

and there is only one and only one path between any two different vertices. A spanning tree is a subgraph of graph 'G' witch is a tree and which includes all the vertices of 'G' but only a subset of the edges. The edges of the tree are called branches, and the edges not in the spanning tree are called chords.

### B. Graph Representation Conventions

For convenience, this paper uses a line type attribute for the representation of flow paths or the thermodynamic processes. A solid line has been represented for an edge with an unknown value of the flow or the potential drops in terms or pressures and temperatures. A dashed line is a chord which is an edge not included in the spanning tree but forms the graph. When a graph G is represented, it is possible to tell which edges or the branches are incident at which nodes or the vertices and what the orientations relative to the vertices are. The most convenient way in which this incident information can be given is in a matrix form. For a graph 'G' with 'n' vertices and 'b' edges or branches, the complete incident matrix $A = a_{hk}$ is a rectangular matrix of order 'n x b' whose elements have the values given by equation (3).

$$a_{hk} = \begin{cases} 1 & \text{if edge k is associated with vertex h and} \\ & \text{oriented away from vertex h} \\ -1 & \text{if edge k is associated with vertex h and} \\ & \text{oriented towards vertex h} \\ 0 & \text{k is not associated with vertex h} \end{cases} \tag{3}$$

Any one row of the complete incident matrix can be obtained by the algebraic manipulation of other rows, indicating that the rows are not independent. Thus, by eliminating one of the rows, the incident matrix is reduced to a reduced incident matrix with rank as 'n-1' and its size as 'n-1 x b'. This reduced incident matrix is generally used to solve the system equations through mathematical relationships within the elements. For example, a simple digraph as represented in Figure 4 is considered.

The complete incident matrix [A] and its reduced forms $[A_r]$ are represented in equation (4) and equation (5) respectively.

$$[A] = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 0 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & 1 \end{pmatrix} \end{array} \tag{4}$$

$$[A_r] = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & -1 & 1 & 0 & 0 \end{pmatrix} \end{array} \tag{5}$$

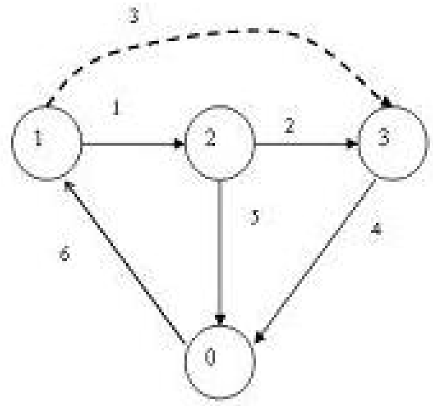



Fig. 4. A simple network graph

In general, on the basis of measurement techniques adopted, two different kinds of system variables are considered in order to analyze thermodynamic systems. These are system through variables and the system across variables. A system through variable acts or flows through the element whereas a system across variable is measured as a change across any two points in the system. A process in an element or the subsystem of the system involving the system is generally characterized by the empirical relations. Denoting the system across variables by '$\Delta x$' and the system through variables by 'y', these mathematical relations can be described by equation (6) as:

$$\begin{aligned} y = f(\Delta x) & \quad \text{or} \\ \Delta x = f(y) & \quad \text{or} \\ f(\Delta x, y) = 0 & \end{aligned} \tag{6}$$

### C. System Theory Postulates

When a system consists of 'n' subsystems or 'n' elements, each modeled with a single edge and a single equation, then according to system theory:

- **Postulate 1 :** The system consisting of 'n' subsystems or the 'n' terminal elements, the system can be represented by 'n-1' terminal equations in 'n-1' pair of system variables '$\Delta xi$' and '$y_i$'. The reduced incident matrix $[A_r]$ as stated above contains all the information related to 'n' system variables with 'n-1' terminal equations. As per the embedded property of the reduced incident matrix any set of 'n-1' vertices can be used to describe the system terminal equations completely.
- **Postulate 2 :** The oriented sum of all the across variable measurements along the edges in a given circuit is zero at any instant of time.
- **Postulate 3:**The oriented sum of all the through variable measurements on a given vertex is zero at any instant of time. In cases of thermal systems, these Postulates become the statements for thermodynamic compatibility and equilibrium respectively. The oriented sum in the Postulates means that the system variable measurement is positive if the directions of the edges are in line with the

conventions of the reduced incident matrix stated above. According to Postulate 3, the vertex equations may be stated by equation (7)as:

$$[A_r].[y] = 0 \tag{7}$$

where notations have their usual meanings as mentioned above. If 'x' is a set of all system across variables measured at the vertices 'I' relative to reference vertex then, according to fundamental property of the reduced incident matrix $[A_r]$, the system across variable associated with the edges of the graph 'G' can be represented by equation (8).

$$\Delta x = [A_r]^T.[x] \tag{8}$$

Then above equation can be considered as a statement of the Postulate 2.

## IV. METHODOLOGY

For such thermodynamic systems, the process graphs are generally represented which define the behaviour of the thermodynamic system in respect of all kind of flows involved in the process it is undergoing. The energy flows and energy transformations can be represented in a unified manner along with their interactions. This is achieved by modeling the internal structure of the system as well as its sub-structure (i.e. subsystems) using graph theory and the mathematical relationships describing the behaviour of various elements of the systems in terms of set of system measurable variables. Following steps are generally followed for the analysis of such systems:

1) Identify the system variables for the simple open cycle gas turbine which are sufficient to describe the physical processes in the system.
2) In order to characterize such thermodynamic systems; identify the subsystems or the elements up to the level of the interaction stage with in the system. Develop the terminal graphs which are self explanatory in terms of system through and across variables.
3) Represent the energy transfers and energy transformations in the system elements in terms of their terminal graphs with respect to the reference. Generally, in such cases, the ambient conditions or the surrounding is considered to be the reference for all kind of energy transfer rates.
4) Using linear graph theory, develop the associated terminal graphs for the individual system through and system across variables which may be termed as process subgraphs for future reference.
5) verify the circuit Postulates and the vertex Postulates for the associated terminal graphs i.e. process subgraphs. Obtain the incident and reduced incidents matrices as based on the principles linear graph theory.
6) Obtain the significant effects modeling related to work and heat transfers i.e. WEMs (Work effect Models) and EFMs (Energy Flow Models) in the system and at the subsystem level, if any using the laws of thermodynamics as applied to steady flow processes.
7) Obtain and solve the set of equations for the vertices as well as on the edges of the process subgraphs simultaneously for the set of unknown variables.
8) Calculate the dissipation along all the elements of the simple open cycle gas turbine using governing equation based on network theory.
9) The results thus obtained are to be verified using conventional calculation procedures.

## V. SIMPLE OPEN CYCLE GAS TURBINE MODELING

### A. System Variables

In case of a simple open cycle gas turbine, the three main elements i.e. compressor, combustor and the turbine connected in series along the flow constitute the system. Since all the processes related to work and energy transfers are associated with these elements only, hence these elements may be considered to be the significant sub-systems of the system. Even though, these sub-systems are connected to each other with a set of insulated piping system which are responsible for intermediate flow and energy transfers, yet in order to consider the preliminary modeling stage their effect may be neglected at present. While modeling the entire simple open cycle gas turbine system in terms of system across variables and system through variables responsible for complete description of the system, it has summarily been analyzed that for the work effect related to mass flow processes, the pressure of the air flow represents itself the system across variable and the volume or mass flow rate as the system through variable. While for the energy flow processes in the system, the temperature represents itself as the system across variable and the energy flow rate as the system across variable with in the system. The ambient or the surrounding is considered as reference for calculating the changes in the across variable measurements with in the system.

### B. Terminal Elements of The System

While modeling the simple open cycle gas turbine system, the system is represented into its elements and its terminals in terms of rate of heat and work transfers over the sub-systems and their terminals. In the present system, various terminals have been represented across the different system elements or the sub-systems and the directed edges have been shown for energy transfer rates at various subsystems. The various work and heat input rates with in the system are treated as energy sources and sinks. The standard conventions have been followed for the rate of heat and work transfers in the system i.e. heat input to the system and work output from the system as positive value while heat rejected from the system and work done on the system as negative. In present system, since work is to be done on the compressor and heat is rejected to the surrounding or the ambient at the exit of the turbine, hence these are negative while work output of the turbine and heat input on account of fuel burning in the combustor are considered to have positive values. In order to develop the process subgraphs of the system, each element within the system is replaced by its terminal graph

International Journal of Mechanical, Industrial and Aerospace Sciences
ISSN: 2517-9950
Vol:4, No:3, 2010

### C. Terminal Elements Modeling Concepts

In the present system, we are having three main elements along with surrounding. Each of the three main elements i.e. compressor, combustor and the turbine may be replaced by their terminal graphs related to mass flow rates and energy flow rates. Each element may have two or more than two terminals. Related to energy flow, each element has three terminals while that for mass flow each element has two terminals. The ambient or the surrounding act as a heat sink for the system as heat rejected at the exhaust of the turbine does not affect the input ambient conditions for the compressor as well as does not increase its own temperature i.e. ambient temperature significantly irrespective of the level of heat rejected at the turbine or heat exchange with other subsystems in the system itself. The various elements as well as energy sources and sinks in terms of their terminal graphs related to mass flow rates as well as energy flow rates are given in Table I.

### D. System Process Graph Model

The terminal graph of the elements of the system related to energy transformation processes involved in them are generally termed as process subgraphs of these elements. The process graph model (PGM) of the system is obtained by interconnecting element terminal graphs at the appropriate terminals to form the (n+1) separate graphs called process subgraphs, where 'n' of these graphs correspond to work effects and the (n+1)th graph gives all energy transformations.

Thus in order to construct the process graph model of the system, significant work effects are to be identified along with associated flows along with number of interfaces. Each interface is represented by a vertex and edges drawn to obtain the (n+1) separate terminal graphs. The terminal equations, one for each edge of the terminal graph describe the behaviour of the element, for a given process, by relating the variables of the terminal graph with the characteristics of the elements. The terminal equations related to the work effects (WEMs) and the energy effects (EFMs) as derived using first law of thermodynamics for steady flow processes for the subsystems of the simple open cycle gas turbine will be explained in further sections in this paper.

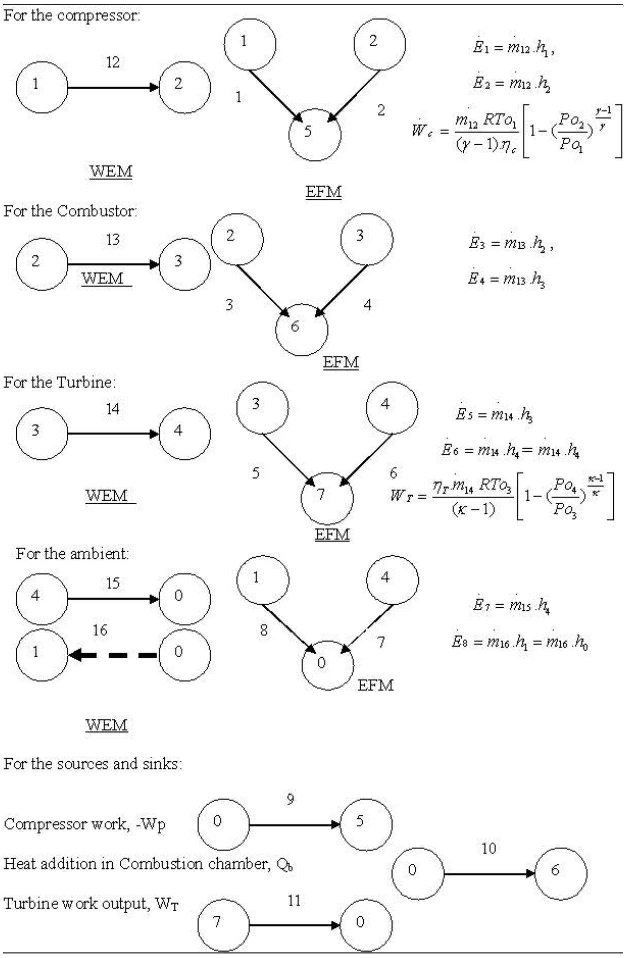
### E. Terminal Equations

The terminal equations, one for each edge of the terminal graph, specifying the behaviour of the element for a given process, by relating the variables represented by the terminal graph with the characteristics of the elements are obtained. The terminal equations for the work effects and the energy flows for the compressor, combustor and the turbine along with ambient are also shown in Table I.

## VI. SYSTEM GRAPH THEORETIC REPRESENTATION

For the simple open cycle gas turbine, the process subgraphs of the subsystems or the elements representing the constraints placed on the system across variables as well as system through variables with interconnection of the elements are constructed. When terminal graphs related to mass flow rate for each subsystem are associated along with terminal associativity constraints then it is called as process subgraph of the system related to mass flow or the system through variables. Similarly, when the terminal graphs related to energy flow rates for each subsystem are associated along with terminal associativity constraints then it is termed as process subgraph

TABLE I
WORK EFFECTS (WEMs) AND ENERGY EFFECT (EFMs) OF THE SUBSYSTEMS



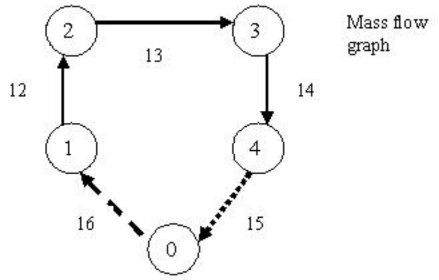




Fig. 5. Mass flow graph of a simple cycle gas turbine

of the system. In the present paper, the process subgraphs of the simple open cycle gas turbine related to mass flow and the energy flow are constructed using terminal flow graphs of the subsystems given in Table I. The process flow graphs so constructed for the simple open cycle gas turbine related to mass flow and the energy flow rate are represented in Figure 5 and Figure 6 respectively.

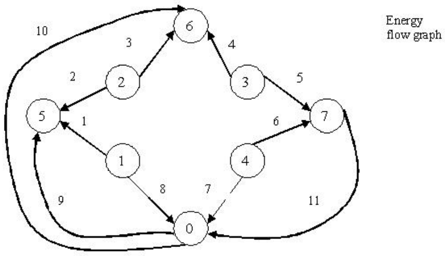



Fig. 6. Energy flow graph of a simple cycle gas turbine

## VII. SOLUTION

In reference to the number of equations required to solve all the system across variables and system through variables, the terminal equations associated with each element together with the equation of the system as per the vertex Postulate and the circuit Postulate are to be solved simultaneously. In order to have a unique solution, the total number of independent equations is to be equal to total number of variables in the system. The stepwise solution of the simple open cycle gas turbine through the establishment of relationship amongst system theory, vertex postulate and the circuit postulates as below:

### A. Developing The System Equations

In the case of Simple open cycle as turbine, it is worth to mention that the emissions of the turbine are directly discharges to the ambient atmosphere and the fresh air and fuel are admitted to the Compressor and the Combustor respectively. Since, the heat capacity of the ambient is generally assumed to be very high as compared to other subsystems in the cycle, the discharge of gas turbine to the ambient effect the local temperature marginally and the intake of the compressor takes place at the normal temperature of the ambient irrespective of discharge temperature of the gas turbine. Since, the cycle operates for a specified intake and discharge volumes or masses of air and fuel, net mass transfer to the system through its subsystems from the ambient and vice-versa is taken as zero and the cycle is generally analyzed for Steady Flow process assumptions. In the present case, from the mass flow process graph, the corresponding Incident matrix [A] is given by equation (9)as:

$$[A] = \begin{array}{cc} & \begin{array}{ccccc} 12 & 13 & 14 & 15 & 16 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 0 \end{array} & \begin{pmatrix} 1 & 0 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{pmatrix} \end{array} \tag{9}$$

The corresponding reduced matrix $[A_r]$ is obtained by eliminating one of the row from the complete incident matrix [A]. Generally, the row for the ambient atmosphere is eliminated to get the reduced incident matrix as the standard systems through variables and across variables is known. Therefore, the reduced Incident matrix $[A_r]$ of the simple open cycle gas turbine is given by equation (10)as:

$$[A_r] = \begin{array}{cc} & \begin{array}{ccccc} 12 & 13 & 14 & 15 & 16 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{pmatrix} 1 & 0 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \end{pmatrix} \end{array} \tag{10}$$

by multiplying the above reduced incident matrix $[A_r]$ with a vector of system through variable i.e. Mass flow rates through the subsystems , we obtain the mathematical statement as represented by equation (11) for the vertex postulates as:

$$\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{pmatrix} 1 & 0 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \end{pmatrix} \begin{pmatrix} \dot{m_{12}} \\ \dot{m_{13}} \\ \dot{m_{14}} \\ \dot{m_{15}} \\ \dot{m_{16}} \end{pmatrix} = 0 \tag{11}$$

Since, for the mass flow process graph, the pressure, $Po_i$, is the system across variable measured at the vertices of the system relative to the reference i.e. the ambient, then using the fundamental property of the reduced incident matrix as per Vertex Postulate 2, the vector containing the pressure drops in the subsystems is given by the equation (12) as:

$$\begin{pmatrix} \Delta Po_{12} \\ \Delta Po_{13} \\ \Delta Po_{14} \\ \Delta Po_{15} \\ \Delta Po_{16} \end{pmatrix} = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} Po_1 \\ Po_2 \\ Po_3 \\ Po_4 \end{pmatrix} \tag{12}$$

Similarly, for the energy flow process graph, the incident matrix , the reduced incident matrix and the corresponding mathematical statement of the energy flow process graph as per vertex postulate are given by equation (13), (14) and (15) respectively.

$$[A] = \begin{array}{cc} & \begin{array}{ccccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 0 \end{array} & \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 1 & 1 & -1 \end{pmatrix} \end{array} \tag{13}$$

$$[A_r] = \begin{array}{c} \\ 1\\2\\3\\4\\5\\6\\7 \end{array} \begin{array}{c} \begin{array}{ccccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \end{array} \tag{14}$$

$$\begin{array}{c} \\ 1\\2\\3\\4\\5\\6\\7 \end{array} \begin{array}{c} \begin{array}{ccccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \end{array} \begin{pmatrix} \dot{E_1} \\ \dot{E_2} \\ \dot{E_3} \\ \dot{E_4} \\ \dot{E_5} \\ \dot{E_6} \\ \dot{E_7} \\ \dot{E_8} \\ \dot{E_9} \\ \dot{E_{10}} \\ \dot{E_{11}} \end{pmatrix} = 0 \tag{15}$$

as already done in case of pressure variable, the temperature drops $\Delta To_i$ ( i = 1,2,3,4,...11) associated with the edges of the graph and may be expressed as a function of vertex across variables $T_j$, (j= 1,2,3,4,.....7) measured at each vertex relative to the reference i.e. the ambient atmosphere (point O). Therefore,the resultant equation (16) is represented as:

$$\begin{pmatrix} \Delta To_1 \\ \Delta To_2 \\ \Delta To_3 \\ \Delta To_4 \\ \Delta To_5 \\ \Delta To_6 \\ \Delta To_7 \\ \Delta To_8 \\ \Delta To_9 \\ \Delta To_{10} \\ \Delta To_{11} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} To_1 \\ To_2 \\ To_3 \\ To_4 \\ To_5 \\ To_6 \\ To_7 \end{pmatrix} \tag{16}$$

### B. Application Of First Law Of Thermodynamics

The analysis through the first law of thermodynamics is an energy accounting procedure in which the energy transfer to and from the systems and the subsystems are taken into account. In case of the simple open cycle gas turbine, the flow of the air, air- fuel mixture or the hot gases as a result of combustion are represented as steady flow processes. Therefore, according to first law of thermodynamics applied to to the steady flow energy states of each element in the system, the equation (17) becomes

$$\dot{E_1} + Q_{input} = \dot{E_2} + W_{output} \tag{17}$$

where notations have their usual meaning with standard sign conventions. i.e. the work output and the heat input rate as positive and the work done on the system as well as the heat rejection by the system as negative over the elements or the subsystems of the system.

From the associated terminal graphs related to the energy i.e. the Energy flow subgraph of the simple open cycle gas turbine, reduced incident matrix is obtained. By applying the vertex postulate to the energy flow process subgraph and substituting the various rows representing energy rates on the vertices along the mass flow through the subsystems of the system into the equations containing energy transfer rates with no mass transfers i.e. the energy transfer rates for the Compressor input work, heat addition due to fuel burning in the combustor and the net turbine output, we get the equation (18) as

$$\begin{aligned} \dot{E_9} &= -\dot{E_2} + \dot{E_8} \\ \dot{E_{10}} &= \dot{E_2} - \dot{E_4} \\ \dot{E_{11}} &= -\dot{E_4} + \dot{E_6} \end{aligned} \tag{18}$$

Representing the set of above equations in matrix form, equation (19) can be written as

$$\begin{pmatrix} \dot{E_9} \\ \dot{E_{10}} \\ \dot{E_{11}} \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 \end{pmatrix} \begin{pmatrix} \dot{E_2} \\ \dot{E_4} \\ \dot{E_6} \\ \dot{E_8} \end{pmatrix} \tag{19}$$

Since the flow of the air as well as the exhaust gases through various components of the simple open cycle gas turbine is assumed to be Steady type flow and the mass flow rate of air is constant throughout the flow passage with no bleed i.e. $\dot{m_{12}}$ = $\dot{m_{13}}$ = $\dot{m_{14}}$ = $\dot{m_{15}}$ = $\dot{m_{16}}$ = $\dot{m}$. Therefore applying the Steady flow Energy equation (SFEE) to various sub-systems and substituting the terminal equations in the above equation, we get equation (20) as

$$\begin{pmatrix} \dot{E_9} \\ \dot{E_{10}} \\ \dot{E_{11}} \end{pmatrix} = [R]. \begin{pmatrix} \dot{m}.ho_1 - \frac{m.R.To_1}{(\gamma-1).\eta_c}[1 - (\frac{Po_2}{Po_1})^{\frac{\gamma-1}{\gamma}}] \\ \dot{m}.ho_3 \\ \dot{m}.ho_3 - (\frac{m.R.To_3).\eta_T}{\kappa-1}[1 - (\frac{Po_4}{Po_3})^{\frac{\kappa-1}{\kappa}}] \\ \dot{m}.ho_1 \end{pmatrix} \tag{20}$$

where

$$[R] = \begin{pmatrix} -1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 \end{pmatrix}$$

The above set of equations in matrix form gives the solutions of the simple cycle gas turbine for various energy transfer rates like compressor input work; heat addition to the system through is element in the combustion chamber and the net turbine work output. The application of the vertex postulate leads to the same set of results as obtained by using the conventional analysis. The similar calculations related to overall thermal efficiency of the system may be carried out as per standard procedures.

### C. Application Of Second Law Of Thermodynamics

The first and second law of thermodynamics yields an equation which specifies the upper bound of work output ,equation (21),as:

$$\dot{W}_u \leq -\sum_{j=1}^{N} m_j.\phi_j + \sum_{i=1}^{M} m_i.\phi_i - \dot{\Phi} \quad (21)$$

where M = Total number of inputs to the system
N = Total number of outputs in the system and the dissipation in the system , equation (22), is given by

$$I = -\dot{W}_u - \sum_{j=1}^{N} m_j.\phi_j + \sum_{i=1}^{M} m_i.\phi_i - \dot{\Phi} \quad (22)$$

since for a steady flow process, $\dot{\phi}$ is zero, hence we get equation (23), equation (24) as

$$I = -\dot{W}_u - \sum_{j=1}^{N} m_j.\phi_j + \sum_{i=1}^{M} m_i.\phi_i \quad (23)$$

or

$$I = -\dot{W}_u - \sum_{j=1}^{N} \dot{\Phi}_j + \sum_{i=1}^{M} \dot{\Phi}_i \quad (24)$$

from the definition of exergy factor $\chi$, the exergy $\dot{\Phi}$ is equal to $\chi.\dot{E}$. Furthermore, work output may be considered as exergy output with exergy factor $\chi$, equal to unity. Therefore, in energy flow graph of the system, sources are used to model all energy inputs and outputs. An edge representing a source is drawn from a vertex to a datum vertex. For energy inputs, this would imply that the through variable $\dot{E}_i$, i =1,2,3, .., N will have a negative value. Therefore, above equation becomes (equation (25))as

$$I = -\sum_{j=1}^{N+1} \chi_j.\dot{E}_j - \sum_{i=1}^{M} \chi_i.\dot{E}_i \quad (25)$$

since the datum state is considered as dead state for the system with exergy factor equal to zero, the across variables associated with the sources are given as

$\Delta\chi_i = \chi_i - \chi_0 = \chi_i$, for i= 1, 2, 3, 4,...... M;
$\Delta\chi_j = \chi_j - \chi_0 = \chi_j$, for j= 1, 2, 3, 4,...... N+1

by putting up the values of $\Delta\chi_i$ and $\Delta\chi_j$ , the dissipation is given by equation (26) as

$$I = -\sum_{j=1}^{N+1} \Delta\chi_j.\dot{E}_j - \sum_{i=1}^{M} \Delta\chi_i.\dot{E}_i \quad (26)$$

Regrouping all the edges of the energy flow graph into three categories i.e. M input edges, N+1 output edges and P process graphs, we have equation (27) and equation (28) as

$$\sum_{j=1}^{N+1} \Delta\chi_j.\dot{E}_j + \sum_{i=1}^{M} \Delta\chi_i.\dot{E}_i + \sum_{p=1}^{P} \Delta\chi_p.\dot{E}_p = 0 \quad (27)$$

or

$$\sum_{p=1}^{P} \Delta\chi_p.\dot{E}_p = -\sum_{j=1}^{N+1} \Delta\chi_j.\dot{E}_j - \sum_{i=1}^{M} \Delta\chi_i.\dot{E}_i \quad (28)$$

therefore, for a steady flow system, the dissipation at each element is represented by equation (29) as

$$I = \sum_{p=1}^{P} \Delta\chi_p.\dot{E}_p = \sum_{p=1}^{P} \Delta\phi_p = \sum_{p=1}^{P} \dot{m}_p \Delta b_p \quad (29)$$

where the flow exergy, $b = h - To.S$

This may be computed from the mass flow graph developed for the simple open cycle gas turbine. Now, using the set of above equations, the energy flow graph for the simple open cycle gas turbine based on second law of thermodynamics is shown in Figure 7.

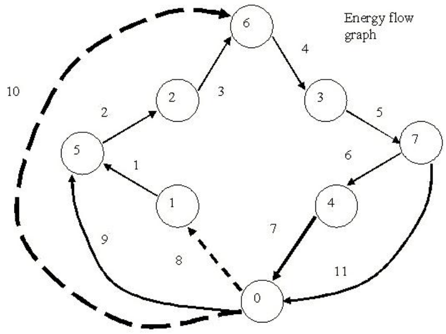



Fig. 7. Energy flow graph based on second law of thermodynamics

Following data is used for the computation and analysis of the simple open cycle gas turbine for the implementation of the proposed approach: $Po_1 = 1.0$ Mpa, $To_1 = 15^o$ C , $\eta_c = 0.86, \eta_T = 0.89, \kappa = 1.333$ for expanding gases, $\gamma = 1.4$ for air, compression ratio = 12.0, Pressure drop in combustion chamber = 6% of compressor delivery pressure, assuming that the quantity of fuel is $<<$ quantity of air in the process flow. However, the heat content of the fuel helps primarily the Turbine inlet temperature to reach $To_3 = 1077^oC$.

The analysis for the simple open cycle gas turbine for given assumptions and the boundary conditions is performed on unit mass basis over all the vertices and the edges and are tabulated in Table II and Table III respectively.

TABLE II
THERMODYNAMIC PROPERTIES AT THE VERTICES OF PROCESS GRAPH MODEL

| Sr. No. | Vertex in the Process Graph Model | P (MPa) | T ($^oC$) | H (Kj/Kg) | S (Kj/Kg.K) | b (Kj/Kg) |
|---|---|---|---|---|---|---|
| 1 | 1 | 0.100 | 15.0 | 88.1820 | 0.365442 | 15.0936 |
| 2 | 2 | 1.200 | 361.25 | 469.0358 | 1.171979 | 234.640 |
| 3 | 3 | 1.126 | 1077.0 | 1255.575 | 2.022825 | 851.010 |
| 4 | 4 | 0.100 | 531.7 | 627.1571 | 1.428665 | 341.4241 |

The exergy dissipated or increase in the fluid over each element can be calculated by summing up the product of the through and across variables over the energy flow graph. Using these obtained results, the thermal efficiency and the effectiveness of the system can be calculated in Table IV.

TABLE III
EDGE-WISE FLOW EXERGY VARIATION

| Sr. No. | Edges in the Process Graph Model | M (Kg/Sec) | $\Delta b$ (Kj/Kg) |
|---|---|---|---|
| 1 | 12 | 1.0 | 15.0936 - 234.640 = -219.546 |
| 2 | 13 | 1.0 | 234.640 - 851.010 = -616.370 |
| 3 | 14 | 1.0 | 851.010 - 341.4241 = 509.5859 |

TABLE IV
DISSIPATION AT INDIVIDUAL ELEMENTS

| Sr. No. | Subsystem | $\Delta\phi = -\sum_p \dot{m_p} \Delta b_p$ |
|---|---|---|
| 1 | Compressor | -(1.0 x -219.546) = 219.546 |
| 2 | Combustion Chamber | -(1.0 x -616.370) = 616.370 |
| 3 | Turbine | -(1.0 x 509.5859) = -509.5859 |

## VIII. CONCLUSION

In the present paper, the graph theoretic approach for the modeling and analysis of simple open cycle gas turbine has been presented. The concept of process graph models representing the interconnections of the elements as well as their behaviour have been developed for simple open cycle gas turbine unit based on graph theory and system theory. It is clear from the implementation method of the technique adopted that the modeling procedure involves sufficient number of equations in terms of unknown variables which can be used to get the unique solution of the system. The System of equations has been solved using MATLAB for the unknown governing variables. The relationships between the vertex & circuit postulates of system theory and the laws of thermodynamics have been adopted for the simple open cycle gas turbine energy flow graphs and the exergy analysis for the system. Since the results obtained by using the present approach are similar to those obtained by using the conventional techniques for such system and the approach is quite sensitive to the affect of variation in the process variable on the system behavior, the framework of such techniques can be used further for further optimization of process parameters of the simple cycle gas turbine systems.

## REFERENCES


[1] Kurzke, J., *Achieving maximum thermal efficiency with the Simple cycle Gas Turbine* 9th CEAS European Propulsion Forum: Virtual Engine - A Challenge for Integrated Computer Modelling, Roma, Italy, 15-17 October, 2003

[2] Kurzke J., *Gas Turbine cycle design Methodology: A comparison of parameter variation with Numerical Optimization* ASME Journal of Engineering for Gas Turbine and Power, Volume 121, Pages 6-11, 1999

[3] Young J. B. and Wilcox R. C., *Modeling the air cooled gas turbine: Part 1- General Thermodynamics* ASME Journal of Turbomachinery, Volume 124, Pages 207 -213, 2002

[4] Silva V. V. R., Khatib W. and Fleming P. J., *Performance optimization of Gas Turbine engine* Journal of Engineering Applications of Artificial Intelligence, Volume 18, Pages 575-583, 2005

[5] Amann C. A., *Applying Thermodynamics in search of superior engine efficiency* ASME Journal of Engineering for Gas Turbine and Power, Volume 127, Pages 670-675, 2005

[6] Guha A., *Performance and optimization of Gas Turbines with real gas effects* Proceedings of Institution of Mechanical Engineers, Volume 215 Part A, Pages 507-512, 2001

[7] Yadav J. P. and Singh O., *Thermodynamic analysis of air cooled simple gas/ steam combined cycle plant* Proceedings of the Journal of Institution of Engineers (INDIA), Volume 86, Pages 217-222, 2006

[8] Brooks F. J., *GE gas Turbine performance characteristics* GER-3567H, GE Power systems report, 2000

[9] Cohen H. , Rogers G. F. C. and Saravanamuttoo H. I. H., *Gas Turbine theory* Fourth Edition, Longman Publishers, 1996

[10] Philip P. W. and Fletcher P., *Gas Turbine performance* Second Edition, Blackwell Science, 2004

[11] Ghajar A. J., Delahoussaye R. D. and Nayak V. V., *Development and implementation of Interactive/ Visual software for Simple Cycle gas Turbine* Proceedings of American Society for Engineering Education Annual Conference and Exposition, 2005

[12] Chandrashekar M. and Wong F. C., *Thermodynamic Systems Analysis-I: A Graph Theoretic approach* Journal of Energy, Volume 7, Pages 539-566, Number 6, 1982

[13] Shai O. and Preiss K., *Graph theory representations of Engineering systems and their embedded knowledge* Journal of Artificial Intelligence in Engineering, Volume 13, Pages 273-285, 1999

[14] Shai O. ,Titus N. and Ramani K., *Combinatorial synthesis approach employing Graph networks* Journal of Advanced Engineering Informatics, Volume 22, Pages 161-171, 2008