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Mobile Robot Path Planning Utilizing Probability Recursive Function

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***Abstract*—**In this work a software simulation model has been proposed for two driven wheels mobile robot path planning; that can navigate in dynamic environment with static distributed obstacles. The work involves utilizing Bezier curve method in a proposed N order matrix form; for engineering the mobile robot path. The Bezier curve drawbacks in this field have been diagnosed. Two directions: Up and Right function has been proposed; Probability Recursive Function (PRF) to overcome those drawbacks.

PRF functionality has been developed through a proposed; obstacle detection function, optimization function which has the capability of prediction the optimum path without comparison between all feasible paths, and N order Bezier curve function that ensures the drawing of the obtained path.

The simulation results that have been taken showed; the mobile robot travels successfully from starting point and reaching its goal point. All obstacles that are located in its way have been avoided. This navigation is being done successfully using the proposed PRF techniques.

***Keywords*—**Mobile robot, path planning, Bezier curve.

I. INTRODUCTION

MOBILE robot applications often require repeated traversal in a changing environment between predefined start and goal points. For example, a mobile robot could be used to transport details and sub-assemblies between a store and production lines; this task implies repeated traversal between the store and the production cells. A mobile robot can also be used for surveillance; this task implies visiting certain checkpoints on a closed territory in a predefined order [1].

Real environments where these kind of mobile robots have to operate are dynamic by nature. From the point of view of mobile robot navigation, it means that unexpected obstacles can appear and disappear. The nature and density of the obstacles is usually unknown or too difficult to model. Thus ahead camera is used to detect obstacles density and locations [2].

At the same time, a mobile robot in a dynamic environment has to fulfill its assignment as fast and safely as possible. This means choosing paths between target points that are most likely unblocked and where the robot does not spend too much time maneuvering between obstacles. A straightforward approach in robotic applications has been to choose the shortest paths to the goal [1].

It is often of interest to consider the minimum length path. Alternative formulations of a minimum cost path are also sometimes important [4]. In particular, the minimum time path is not necessarily the same thing as the minimum length path. A common case in which these two can differ occurs when the maximum vehicle velocity is a function of the curvature of the path, as is the case with many synchronous-derive vehicles. As a consequence a minimum-length path may involve lower vehicle velocities than a longer but lower curvature (and hence faster) alternative path [3].

Bezier curve had been considered in representing polynomial that is useful for computational purposes. Its properties had been discussed [5].

Bezier curves provide a good approximation of trajectories, and the results indicate that the method provides a basis for the diagnosis of navigational patterns. The method offers three indicators: goodness of fit, average curvature and number of inflexion points which study results indicate that carry important information about user performance, specifically spatial knowledge acquisition [6].

In Section II, Bezier curve's main concepts and their usage drawbacks in path planning field. Probability Recursive Function Important Aspects are described in Section III. In Section IV PRF extended function and general flow chart are proposed. In Section V, conclusions, future work are discussed.

II. BEZIER CURVE'S MAIN CONCEPTS

A Bezier curve is a polynomial defined by a set of control points. The first and the last point are called endpoints. This type of curve was developed by ***Pierre Bezier*** in the 1970's for improving the shape design of Renault cars. Nowadays Bezier curves have a large applicability in computer graphics [7].

The distinction between endpoints and control points is related to different roles played by these points, in terms of location and shape of the curve. The endpoints anchor the ends of the curve, while the control points determine the actual shape of the curve, acting like magnets that pull the curve. Therefore, given these two types of points, a Bezier curve passes through the endpoints and is tangent to the first and last sides of the open polygon defined by its points[7].

The parametric equation of N order Bezier curve is given by:

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$$B(t)=\begin{bmatrix}x(t)\\y(t)\end{bmatrix}=\sum_{k=0}^{n}C_n^k.(1-t)^{n-k}.t^k.P_k \qquad (1)$$

Where t is the parameter of the curve ($0 \le t \le 1$) and P_k are constant vectors associated with control points (including the endpoints).

C_n^k Are binomial coefficients:

$$C_n^k=\binom{n}{k}=\frac{n!}{k!(n-k)!},0\le k\le n \qquad (2)$$

A. Bernstein Polynomials

The Bernstein polynomials of degree N are defined by:

$$B_{k,n}(t)=C_n^k t^k(1-t)^{k-n} \qquad (3)$$

For $k = 0, 1... n$

There are n + 1 ***n***th-degree Bernstein polynomials. For mathematical convenience, we usually set $B_{k,n} = 0$, if $k < 0$ or $k > n$.

These polynomials are quite easy to write down the coefficients of n, k can be obtained from Pascal's triangle; the exponents on the ***t*** term increase by one as k increases; and the exponents on the $(1 - t)$ term decrease by one as k increases. In the simple cases *see* [8], [9], we obtain the Bernstein polynomials of degree 3 are:

$B_{3,0}(t) = (1-t)^3$
$B_{3,1}(t) = 3t\,(1-t)^2$
$B_{3,2}(t) = 3\,t^2\,(1-t)$
$B_{3,3}(t) = t^3$

and can be plotted for $0 \le t \le 1$ as shown below:

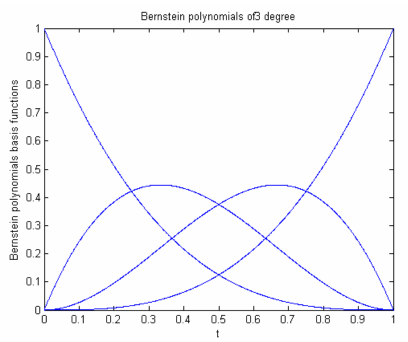


Fig. 1 Cubic Bernstein Polynomial

The number of points determining a Bezier curve defines the degree of the associated polynomial.

B. Properties of Bezier Curves

The properties of Bezier curves and the context of this work particularly support the usage of Bezier curves to approximate and analyses trajectory paths. Some of the benefits of this methodology are outlined below [6]:

1. Smoothing the trajectory path. Visual inspection suggested that good trajectories are characterized by a higher degree of smoothness, for example less or smaller zigzags than those present in poor trajectories. Therefore we needed a trajectory representation which can measure the degree of smoothness, or in other words, a differentiable curve equation.

2. Continuous versus Discrete representation. The present shape of trajectories are rather coarse, since points have been recorded for each movement greater than half a virtual meter, and for each turn greater than 30◦ . By approximating a Bezier curve to each trajectory we aim also at overcoming the coarse data gathered through an event-based process (rather than time-based). The higher granularity, employed by curve approximation, offers a suitable model of the trajectory, preserving in the same time its shape. In addition, it allows computing some properties on each point of the curve.

3. Quantitative measurements. Fitting a Bezier curve to each trajectory offers additional benefits in terms of quantitative measures, such as goodness of fit, curvature and number of inflexion points.

C. N Order Bezier Curve Matrix Form Properties

The N order Bezier curve has the following properties:

1. Matrix formulation of N order is of $[N+1 \times N+1]$.
2. The matrix formulation elements are distributed under the major diagonal, so:
a. Sum of rows' elements is zero, except the first row is one.
b. Sum of columns' elements is zero, except the last column is one.
3. The last row is symmetric with the first column according to minor diagonal.
4. The matrix formulation of N order is reflexive:
a. If the order is ***even*** then we must determine the binomial from zero till $N/2$ and carried out reflexivity as shown below:

Each two elements that the addition of there indices are equal to N are reflected as follows:

$$B(N-j)=B(j) \qquad (5)$$

Where: **j** the indices of original elements corresponding to reflection, i.e. $j = 0, 1\dots N/2$.

b. If the order is ***odd*** then we must determine the Binomial from zero till the nearest integer number of $N/2$ and carried out reflexivity as shown below:

Each two elements that are the addition of there indices are equal to N are reflected as follows:

$$B(N-j)=-B(j) \qquad (6)$$

5. The sum of the absolute values of the first column elements is equal to 2^N.

In order to generate the matrix formulation for N order the following steps must be directed:

1. Find the Binomial vector elements according to property No.4.
2. Decrease N by 1 and repeat the procedure in step (1) (N+1) times to determine (N+1) binomial vectors.
3. Each binomial vector is multiplied by the absolute value of the element in the N binomial vector that carried the index of the same value of the subdivision binomial order, as shown in the applications below.

D. Some Applications of This Algorithm

Some applications of this algorithm are as shown below:

N=2 No. of elements in the matrix is [3 x 3].
Sum of the absolute values of the first column is $2^2 = 4$.

$(1)^2 = ((1-t) + t)^2 = (1-t)^2\, p_0 + 2\, t\, (1-t)\, p_1 + t^2\, p_2$

$(1-t)^2 = 1 - 2.t + t^2$

$2.t.(1-t) = 2.t - 2.\, t^2$

$$M_{[2]} = \begin{bmatrix} 1 & t & t^2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -2 & 2 & 0 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} p_0 \\ p_1 \\ p_2 \end{bmatrix} \qquad \boxed{2\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}}$$

N =5 No. of elements in the matrix is [6 x 6].
Sum of the absolute values of the first column is $2^5 = 32$.

$((1-t) + t)^5 = (1-t)^5\, p_0 + 5.t.\, (1-t)^4\, p_1 + 10.\, t^2.(1-t)^3\, p_2 + 10.t^3.\, (1-t)^2.p_3 + 5.\, t^4.\, (1-t)\, p_4 + t^5.\, p_5$

$(1-t)^5 = 1 - 5.t + 10.t^2 - 10.t^3 + 5.\, t^4 - t^5$

$5.\, t.\, (1-t)^4 = 5.t.(\, 1 - 4.t + 6.t^2 - 4.t^3 + t^4) = 5.t - 20.t^2 + 30.t^3 - 20.\, t^4 + 5.\, t^5$

$10.\, t^2.(1-t)^3 = 10.\, t^2.(\, 1 - 3.t + 3.\, t^2 - t^3\,) = 10.t^2 - 30.t^3 + 30.t^4 - 10.\, t^5$

$10.\, t^3.\, (1-t)^2 = 10.t^3.(\, 1 - 2.t + t^2\,) = 10.t^3 - 20.\, t^4 + 10.t^5$

$5.\, t^4.\, (1-t) = 5.\, t^4 - 5.\, t^5$

$$\boxed{5\begin{bmatrix} 1 \\ -4 \\ 6 \\ -4 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ -20 \\ 30 \\ -20 \\ 5 \end{bmatrix}} \qquad \boxed{5\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 5 \\ -5 \end{bmatrix}}$$

$$M_{[5]} = \begin{bmatrix} 1 & t & t^2 & t^3 & t^4 & t^5 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -5 & 5 & 0 & 0 & 0 & 0 \\ 10 & -20 & 10 & 0 & 0 & 0 \\ -10 & 30 & -30 & 10 & 0 & 0 \\ 5 & -20 & 30 & -20 & 5 & 0 \\ 1 & 5 & -10 & 10 & -5 & 1 \end{bmatrix} \begin{bmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \end{bmatrix}$$

$$\boxed{10\begin{bmatrix} 1 \\ -3 \\ 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 10 \\ -30 \\ 30 \\ -10 \end{bmatrix}} \qquad \boxed{10\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ -20 \\ 10 \end{bmatrix}}$$

The general matrix form is:

$$B(t) = \begin{bmatrix} 1 & t & t^2 & \ldots & t^n \end{bmatrix} \begin{bmatrix} b_{0,n}b_{0,n} & 0 & 0 & 0 & 0 \\ b_{0,n}b_{1,n} & b_{1,n}b_{0,n-1} & 0 & 0 & 0 \\ b_{0,n}b_{2,n} & b_{1,n}b_{1,n-1} & b_{2,n}b_{0,n-2} & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ b_{0,n}b_{n,n} & b_{1,n}b_{n-1,n-1} & b_{2,n}b_{n-2,n-2} & \cdots & b_{n,n}b_{0,0} \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}$$

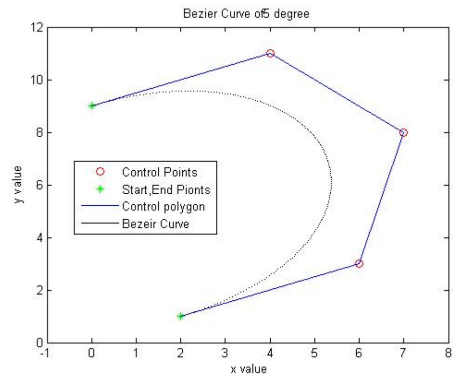
E. Bezier Curve Advantages and Disadvantages in Path Planning Field

The following points are summarized the main advantages and disadvantages of Bezier curve in the field of path planning:

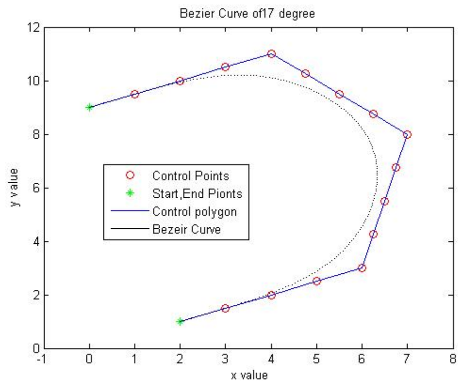
1. The Bezier curve has useful representation in its matrix form which ensures quick matrices multiplication in order to plot the mobile robot path, utilizing Bezier curve aspects.
2. Utilizing Bezier curve in mobile robot path planning has three limitations:

a. Bezier curve; with low degree gives too little flexibility. Thus, the designed mobile robot path is difficult to be followed by the robot because, mobile robot needs infinite acceleration.

b. The Bezier control points have an effect on the shape of the Bezier curve like a magnet effect. This effect causes unwanted offset distance between control polygon and Bezier curve as shown in Fig.2.a. To overcome this disadvantage, either by utilizing additional points; as shown in Fig.2.b, or utilizing a specific points' arrangement, where the control points are too far from the avoided obstacle. As a result, these two solutions are time consuming.



(a)



(b)

Fig. 2 Control points effect on the shape of Bezier curve

c. The Bezier curve control points must have a specific arrangement; undesired path shape will be obtained as shown in Fig. 3.

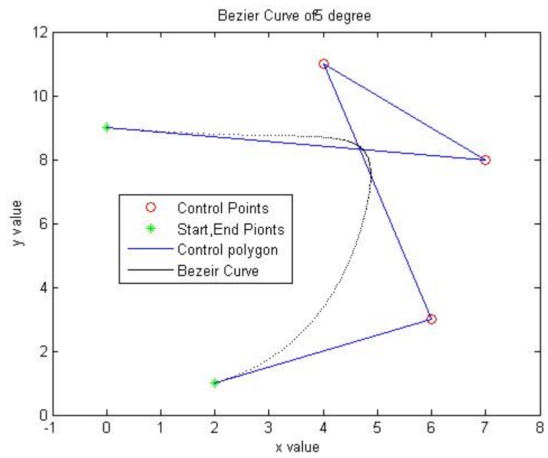


Fig. 3 Bezier Curve Control Points Arrangement Effect on the Path Shape

Therefore the Probability Recursive Function has been proposed to overcome the Bezier curve drawbacks in the field of path planning.

III. Probability Recursive Function Important Aspects

A. Binomial Two Sides

The famous two sides of binomials are success and failure that can be represented through the coin head (H) and coin tail (T) [10], or by up (U) and right (R) in calling mobile robot path planning strategies. It can be extended to involve all applicable directions (up and left), (down and right), and (down and left).

B. Probabilities Map

It is a map consists of two major sides: right side and left side refer to success and failure respectively for different occurrence rates and their effect on success/failure levels, as shown in Fig. 4.

C. Success Level versus Occurrence Rate

Consider five coins to represent (success/failure) level. For event A left side, as shown in Fig. 4, all coins on side (H) that represents success. The success level is 100% because the occurrence rate of the opposite side (T) is equal to zero.

D. Probability Index Complementary Versus Turning over Point/Axis

It is a point/axis in which all success levels have a complement failure levels side. It can be called major turning over point. So, both success and failure sides contain events like A, B, and C as shown in Fig. 1. Generally, all events except A; have a minor turning over point/axis.
For event B, as shown in Fig. 1.

P (r, N, i) *where:* *P* **is** the probability event, r is rate of T occurrence in over all trials, N is a number of coins, and i **is** an index of occurrence T in the over all trials.
For N= 5, r = 1, *P* = 5, and i = 1,2,3,4,5 respectively.
The central element is (i=3), its index equals to the near integer number to N/2 which represents (minor turning over point).So:
P (1, 5, 1), its mirror event is P (1, 5, 5), Where 1 + 5 = 6 (two probabilities (2P)).
P (1, 5, 4), its mirror event is *P* (1, 5, 2), Where 4 + 2 = 6 (two probabilities (2P)).
P (1, 5, 3) minor turning over point Where 3 = 6 / 2 (one probability (1P)).
For event C, (N= 5, r = 2, P = 4+3+2+1=10), and indices values of i1, i2 values and their complementary are illustrated in the Table I:

TABLE I
Complementary, Turning over Point Description for N=5, r=2

No.	i1	i2	Minor Turning over point axis (index = 3)	i1' i1 complement	i2' i2 complement	No. of *P*
1	1	2		5	4	2P
2	1	3		5	3	2P
3	1	4		5	2	2P
4	1	5		5	1	1P
5	2	3		4	3	2P
6	2	4		4	2	1P

- ❖ If i1+i2 ≠ 6, the addition of i1 and its complement is equal to 6, and so on for i2; that produces 2P.
- ❖ If i1+i2 = 6, as in table.1, 4 and 6 cases the addition of i1 and its complement is equal to 6 , and so on for i2. This produces 1P.

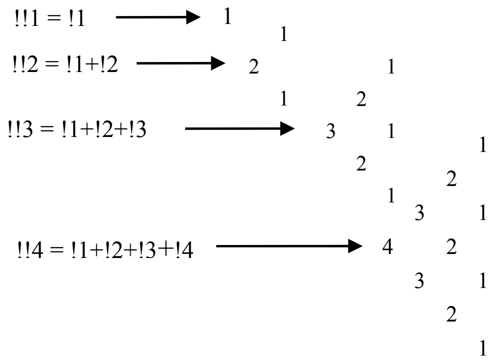
E. PRF Description Degree

For a given $(T+H)^N$, the PRF is equal to the near lowest integer number to real number (N/2). Probabilities coefficients take following shapes:
1. The first coefficient is equal to 1 that can be represented as a Point.
2. The second coefficient is equal to N that can be represented as a set of points (Line).
3. The third coefficient is the addition factorial formulation for N-1 that can be represented as set of lines struck a shape (Triangle).

!1=1 $\longrightarrow$ 1
!2=1+2=3 $\longrightarrow$ 1 2 $\longrightarrow$ 1 1
!3=1+2+3=6 $\longrightarrow$ 1 2 3 1
!10=1+…+10=55 $\longrightarrow$ 1 2 3 4 10
!N-1=1+…+N-1= ((N-1)/2)*N $\longrightarrow$ 1 2 3 4 ..... $N-1$
!N=1+…+N= (N/2)*(N+1) $\longrightarrow$ 1 2 3 4 ... N

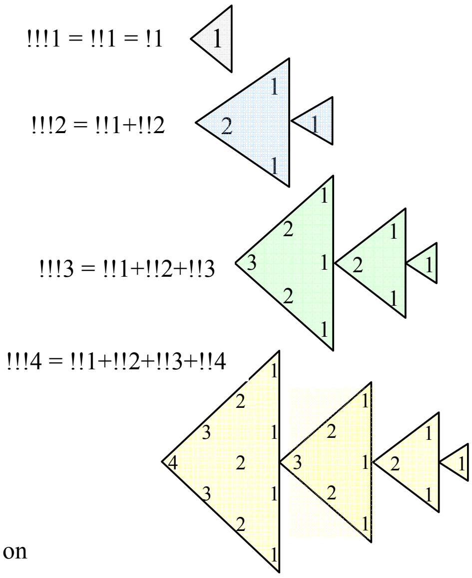
Assumption. !N expresses the addition of numbers from 1 to N, in their examples and it will be used in this paper.

4. The fourth coefficient is the addition of !'s for the element from 0 to N-2 that can be represented as a set of triangles (Pyramid), as shown below:



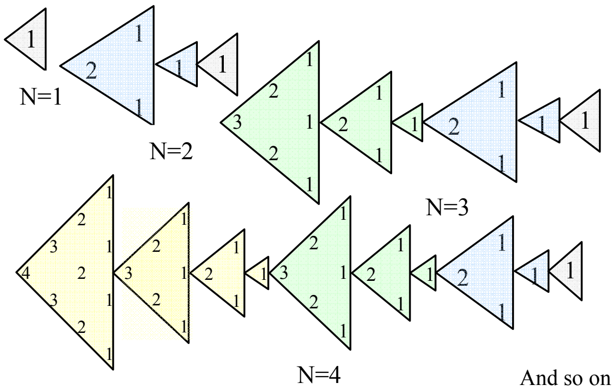
And so on

5. The fifth coefficient is the addition of !!'s elements from 0 to N-3 that can be represented as set of pyramids as shown below:



And so on

6. The sixth coefficient is the addition of !!!'s for the element from 0 to N- 4 that can be represented as group of (set of pyramids) and as shown below:



And so on

F. Recursion

Recursion, in mathematics and computer science, is a method of defining functions in which the function being defined is applied within its own definition. The term is also used more generally to describe a process of repeating objects in a self-similar way. For instance, when the surfaces of two mirrors are almost parallel with each other the nested images that occur are a form of recursion [11], as shown in Fig. 5.

0
1

N=1

0	0
0	1
1	0
1	1

N=2

0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

N=3

0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1
1	0	0	0
1	0	0	1
1	0	1	0
1	0	1	1
1	1	0	0
1	1	0	1
1	1	1	0
1	1	1	1

N=4

- ❖ Fast computation is focused through recursion criteria where arrange is not important. PRF uses recursion with arrangement as in Fig.1 and PRF engine that will discussed later.

Fig. 5 Illustration of Recursion Criteria

G. Mobile Robot Path Planning Utilizing PRF

By utilizing a regular grid map representation for a given mobile robot environment [12], from the left down corner to the up right one, which ensures, there is a finite number of paths as shown in Fig. 6.

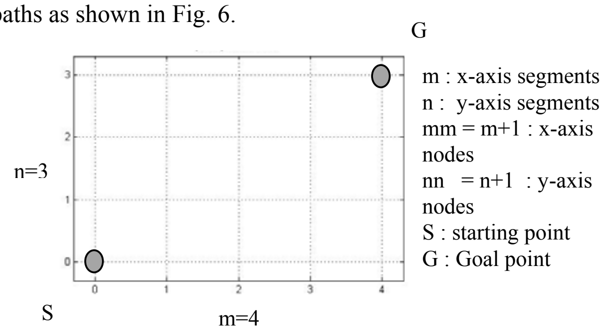


Fig. 6 [4x3] Grid map Free Environment

Where: $C_7^3 = \frac{(3+4)!}{3!*4!} = 35$ paths

So, in order to plot any possible path, two vectors are required: one for x-axis and the other for y-axis to point path. For the path shown in Fig. 7 the two vectors are as follows:

X1= [0 0 0 0 1 2 3 4]
Y1= [0 1 2 3 3 3 3 3] X1, Y1 vectors length = m+n+1

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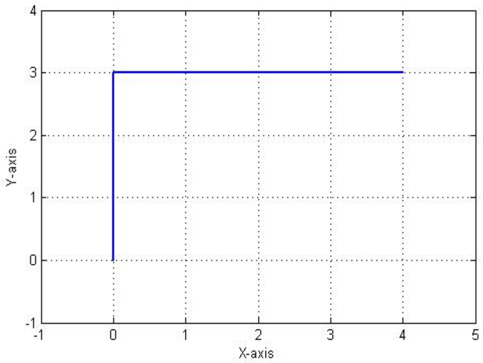


Fig. 7 The first possible path

H. PRF Engine

It is the coding of destination vector (d1) which can be coded in denotable of PRF degree. There are two major types of engines:

1. External engine (Parent)
2. Internal engines (Offspring of each other recursively), their number is equal to the minimum of m, n subtracted by one.

❖ All the PRF engines have the same boundaries.

For example, d1 = [0 0 0 1 1 1 1]

- The first three zeros move as single package according to the PRF external engine i3; i3 = 1 to mm, so the values of i3 goes from 1 to 5.
- The last two zeros in the previous package (of three zeros) move as single package according to the PRF internal engine i2, which represents offspring to i3, and parent for i1; i2 = 1 to mm, so the values of i2 from 1 to 5 initializing i3 values as reference.
- The last single zero in the previous package (of two zeros) move as a package according to the PRF internal engine i1, which represents offspring to i2; i1 = 1 to mm, so the values of i3 goes from 1 to 5 initializing i2 values as reference.

For [4x3] environment, where the path engine [1 3 2], the path is designed as below

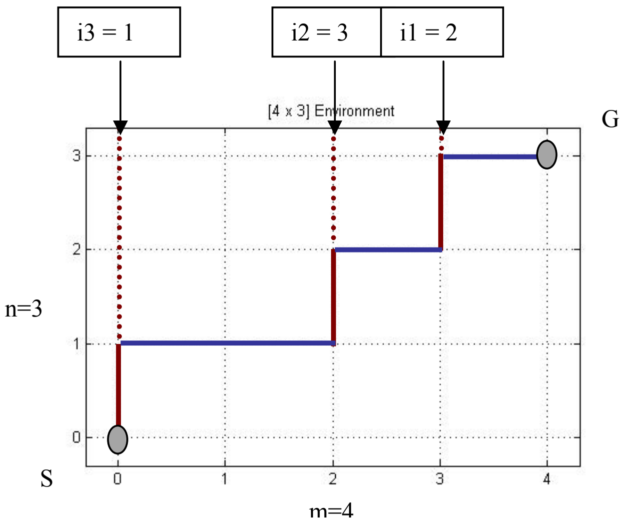


Fig. 8 [1 3 2] PRF engine in [4x3]environment

But, for [2 2 2] engine, the path design is as shown in Fig. 6.

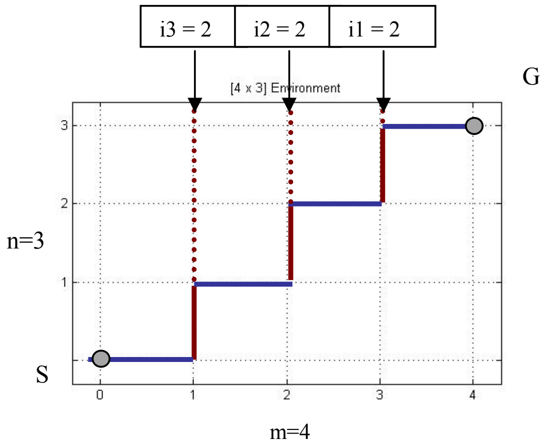


Fig. 9 Another [4x3] PRF engine

Generally, PRF eliminates the negative magnetic effect of the Bezier curve control points on the shape of Bezier curve; since it ensures an efficient arrangement of the Bezier curve control points.

IV. PRF Extended Functions

To construct a Probability Recursive Function (PRF) supported algorithm for the mobile robot path planning in obstacle environment problem. First, feasible paths that satisfy safety; collision-free with spread obstacles in the environment are detected through the usage of PRF for the detection of the static obstacles in: Obstacle Probability Recursive Function (OB-PRF). Second, prediction the sequence of optimum paths according to their length concerning the proposed vertices criteria for each path, by utilizing PRF in the field of optimization; through Vertices Probability Recursive Function (V-PRF). Third, geometric modeling in engineering the mobile robot optimum path by compromising both PRF and Bezier curve aspects in: N-order Bezier curve Probability Recursive Function (BC-PRF).

In the next item, the design concept of PRF that satisfies the above three functions are discussed, and they proposed in an algorithm.

A. Obstacle Probability Recursive Function (OB-PRF)

For the given environment, discretization is being done utilizing regular grid maps representations [12]. Scanning is being done for the environment by utilizing ahead camera as shown in Fig. 10, in order to localize all the spread obstacles.

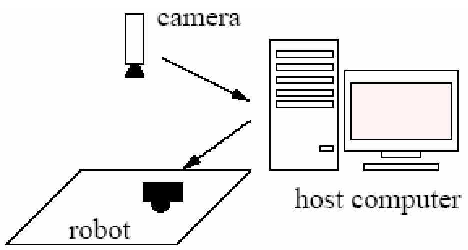
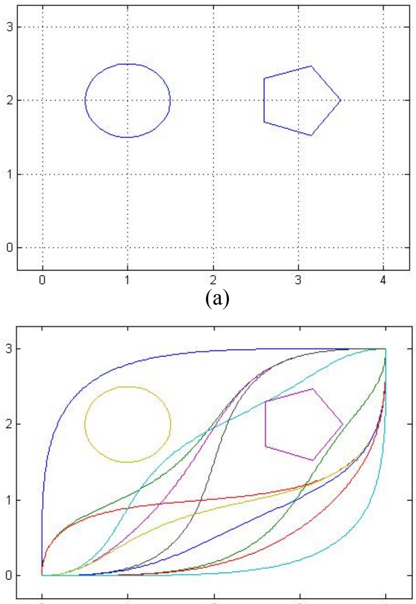


Fig. 10 Localization system

Determine (specify) all the environment nodes that are the obstacles occupied, and assign them as fault nodes. Find the effect of these fault nodes on the PRF engine, that it is used for generating all mobile robot feasible paths in free obstacle environment. Replace each infeasible path by term (X) that declare that this is fault path with in all PRF possible paths as shown in Case.1.

Case.1*:* For [4x3] environment that has two obstacles as shown in Fig.11.a. After applying OB-PRF, all feasible paths that ensure collision-free with obstacles can be obtained as shown in Fig.11.b.



(a)

(b)

Fig. 11 OB-PRF, a) The obstacle environment, b) All feasible paths.

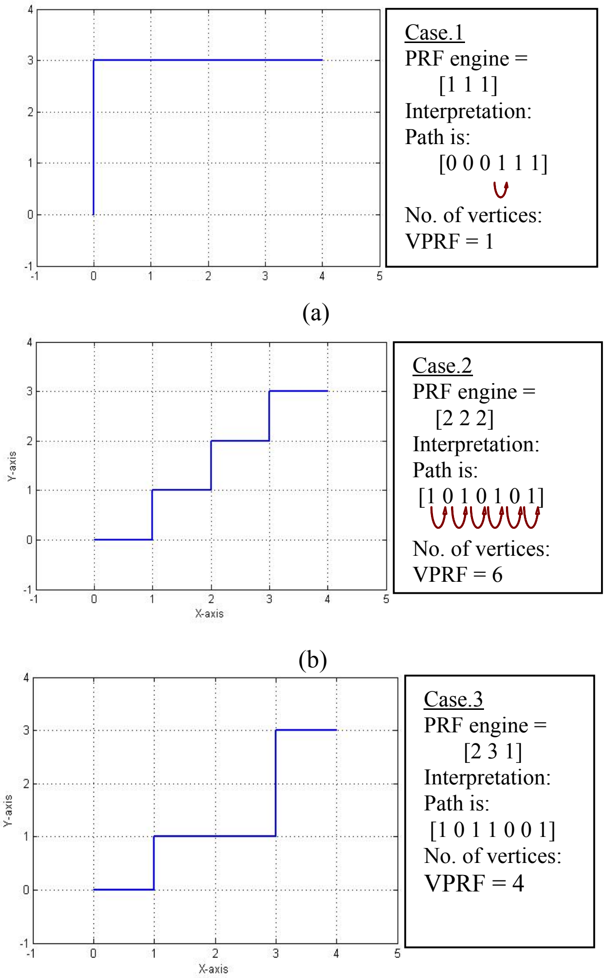
1 OB-PRF Mechanism

1. Specify the obstacles location.
2. Specify the fault grid nodes in the environment; that are occupied by obstacles, which represent the PRF engines constraints.

B. Vertices Probability Recursive Function (V-PRF)

It is often of interest to consider the minimum length path. Alternative formulations of a minimum cost path are also sometimes important. In particular, the minimum time path is not necessarily the same thing as the minimum length path. A common case in which these two can differ occurs when the maximum vehicle velocity is a function of the curvature of the path, as is the case with many synchronous-derive vehicles [3].

Through research and observation it seems by compromising PRF and N-order Bezier matrix that the path length is changed reversely with number of path vertices, for Case: in [4X3] environment there are 35 feasible paths and there vertices are shown in PRF matrix form as shown Fig. 12 and Fig. 13 appear how the path vertices are calculated.



(a)

(b)

(c)

Fig. 12 Three Cases on using VPRF

Finally, by sorting these paths incrementally, optimum paths vector is obtained and there indices. Which facilitate quick search for exact optimum path and Fig. 15 shown the application of that on Case. 1.

1. VPRF Mechanism

For [m x n] environment:
VPRF degree = m+1, VPRF engines = n
For n > 1, VPRF engines are divided into two parts:

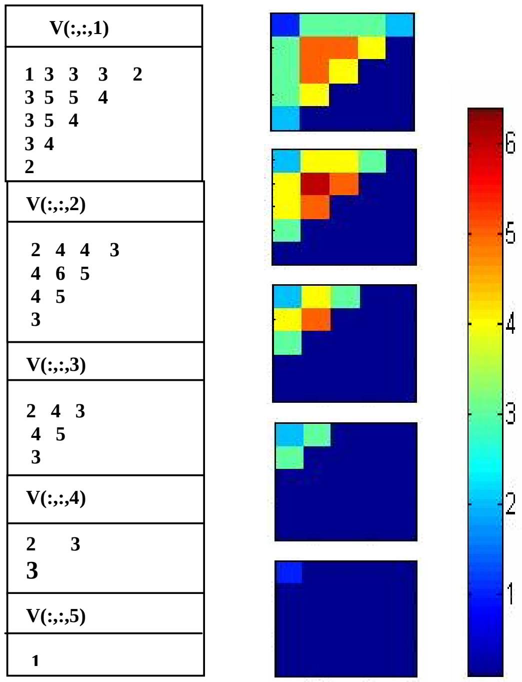
1. External engine take the following form:

1	2	2	…	2	1

2. All the Internal engines take the following form:

0	+2	+2	…	+2	+1

For [3x2] environment, VPRF can be found as follows:
There are 5*4/ 2 = 10 paths that can be represented in PRF engine and there vertices in Table II.

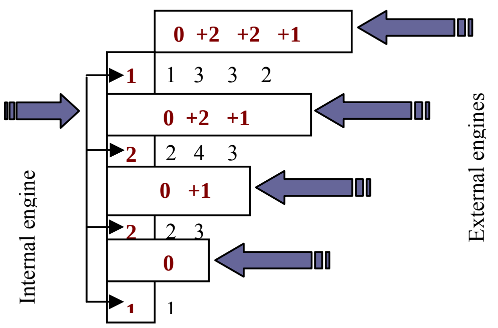


(a) (b) (c)

Fig. 13 Vertices for all Feasible Paths in Free Environment a) In VPRF Form, b) Matlab Figure, and c) The VPRF bar

TABLE II
VPRF FORM FOR [3x2] ENVIRONMENT

	All feasible paths					PRF Engine		VPRF
1	0	0	1	1	1	1	1	1
2	0	1	0	1	1	1	2	3
3	0	1	1	0	1	1	3	3
4	0	1	1	1	0	1	4	2
5	1	0	0	1	1	2	1	2
6	1	0	1	0	1	2	2	4
7	1	0	1	1	0	2	3	3
8	1	1	0	0	1	3	1	2
9	1	1	0	1	0	3	2	3
10	1	1	1	0	0	4	1	1



For [4x3] environment, see Fig. 13.

C. N-Order Bezier curve matrix and PRF (BC-PRF)

BC-PRF is generating by compromising the Probability Recursive Function for generating the mobile robot paths that is utilizing regular grid map representation, with the N-order Bezier curve matrix form algorithm.

PRF gave a very useful Bezier curve control points arrangement that reflect there magnet effect from negative to positive effect, especially path's vertices clear as an indicator of path length as show in V-PRF, in between all feasible paths obtained from OB-PRF. And the N-order Bezier curve matrix ensures quick manipulations in order to plot the optimum mobile robot path, as shown in Fig. 14.

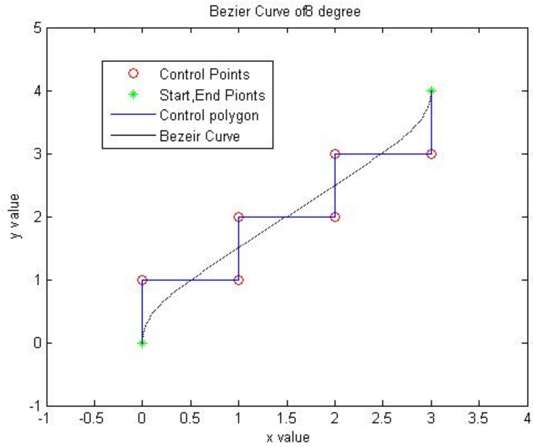
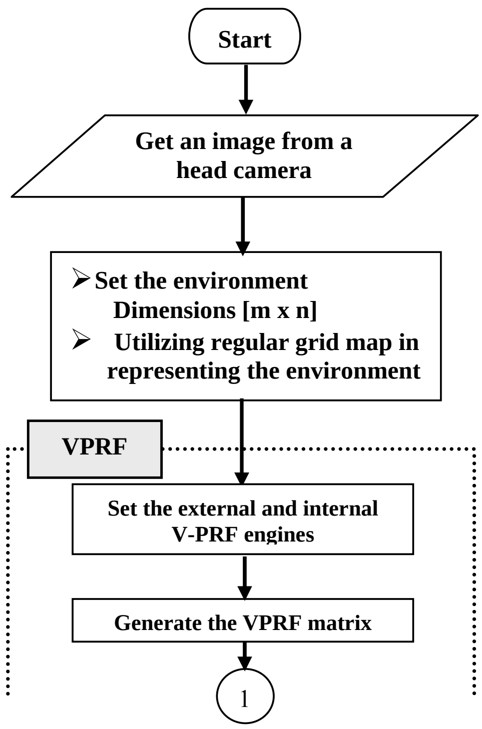
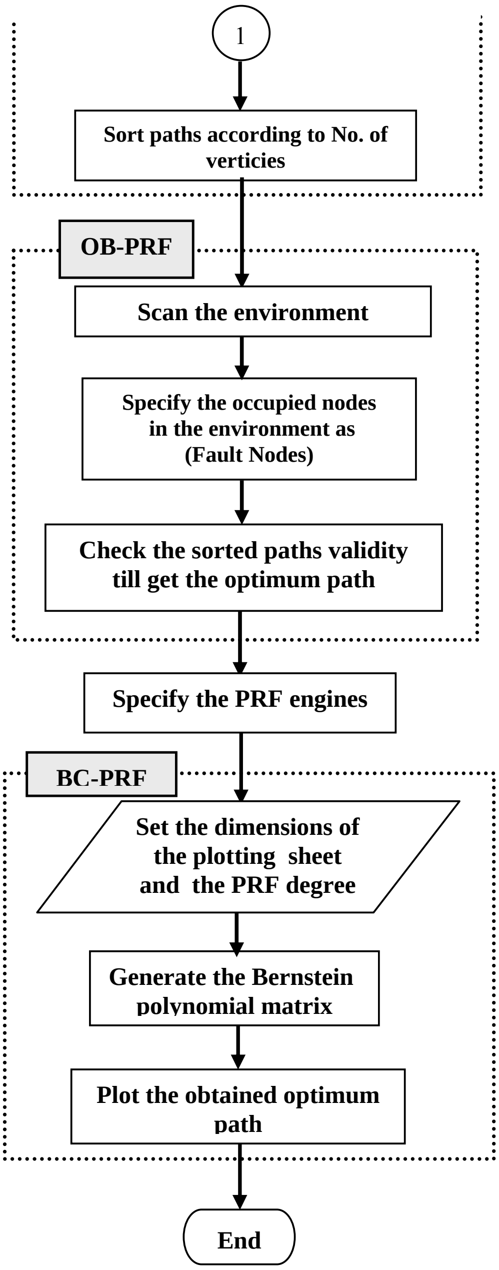


Fig. 14 Mobile Robot Path Planning Utilizing BC-PRF

D. Proposed Algorithm Flow Chart





V. Simulation Results

The following figures show the test of global- path planning method with various maps and the behavior of the mobile robot for each map is shown. In Fig. 15 the mobile robot has to go from start point to the end point in an open space. When the proposed path planning algorithm is utilized, the path of mobile robot motion will be appearing on the map as shown in Fig.15.b.

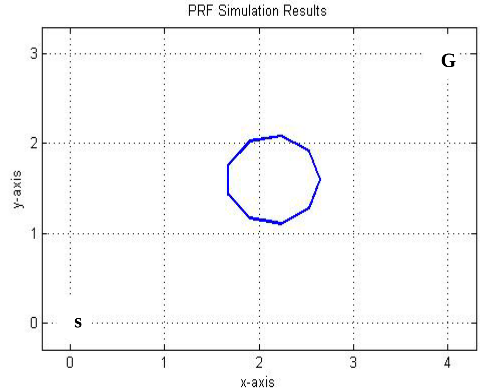


Fig. 15 (a) The First Case of Obstacle Environment

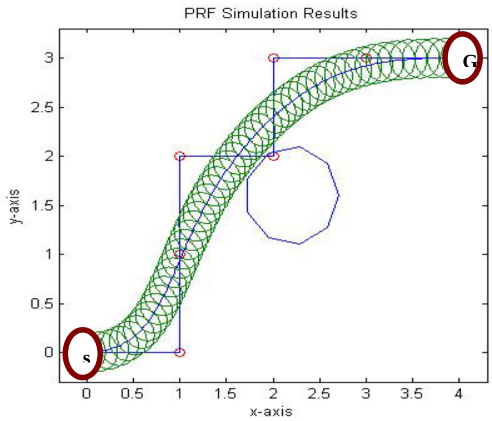


Fig. 15 (b) The First Case Mobile Robot Path

Fig. 16.a presents a map of a building with two obstacles; where there are many passages leading to the goal point. The mobile robot must go from the starting point located at (1,1) to the end point located at (4,3). The optimum path with short, smooth, and obstacles collision-free appears in Fig. 16.b.

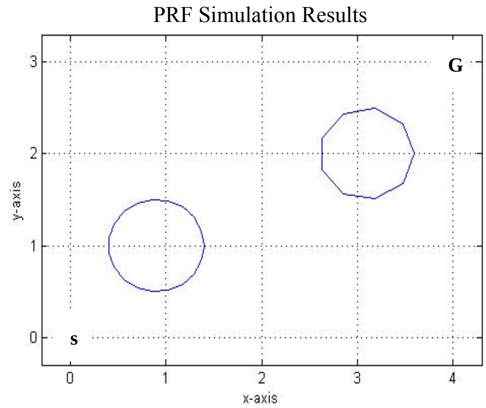


Fig. 16 (a) The Second Case of Obstacle Environment

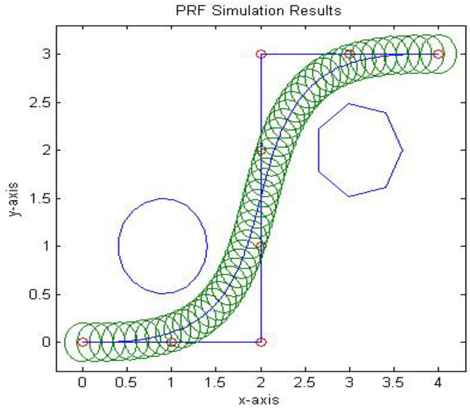


Fig. 16 (b) The Second Case Mobile Robot Path

If obstacle distribution density increases in the environment map; the PRF path planning algorithm ensures that mobile robot can reach the goal point when there is a required passage distance between at least two distributed obstacles as shown in Fig. 17.

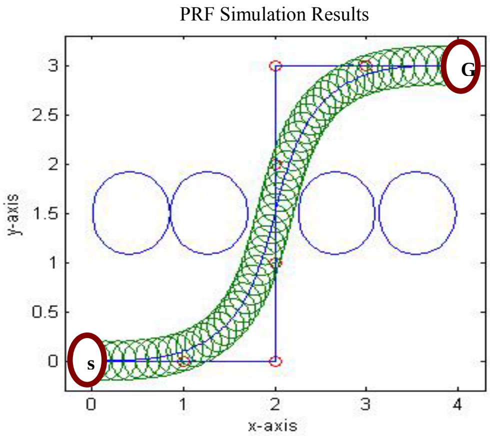


Fig. 17 (a) The Third Case Mobile Robot Path

If the distance between every two distributed obstacles is not enough to overtake obstacles, the PRF path planning algorithm dealing with the distributed obstacles as clustered obstacle and avoid it; as shown in Figs. 18, 19.

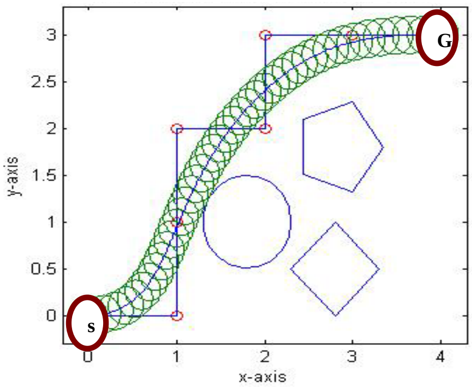


Fig. 18 The fourth case mobile robot path

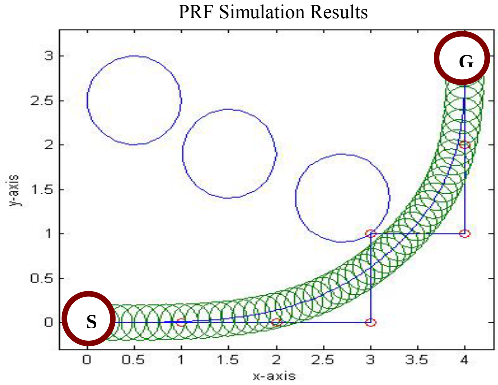


Fig. 19 The Fifth Case Mobile Robot Path

The simulation results have been taken showed; the mobile robot travels successfully from starting point and reaching its goal point. All obstacles that are located in its way have been avoided. This navigation is being done successfully using the proposed PRF techniques.

VI. Comparison with Previous Works

L.C.Kiong and at el [13] algorithm works with polygon obstacle shape environments because it depends on the reversibility vector method in connecting their vertices reaching to near optimum path, but it cannot deal with circular obstacles that are consisting of infinite number of vertices as shown in Fig. 20 (a).

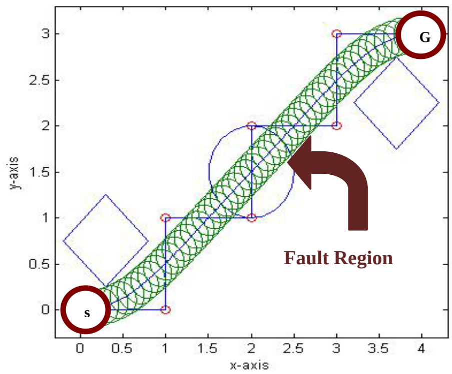


Fig. 20 (a) Wrong path for the sixth case

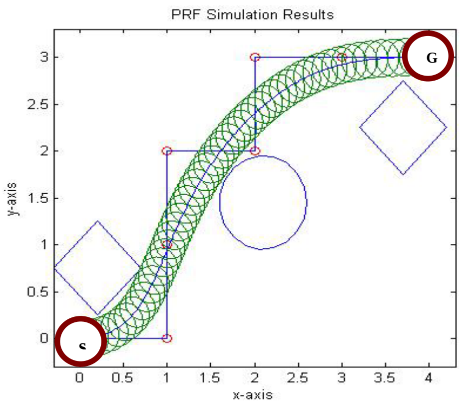


Fig. 20 (b) Right path for the seventh case

While the carried out path planning algorithm PRF can work in environments with any obstacle shape as shown in Fig. 20 (b). Similarly, PRF in comparison with Bug algorithm [14]; it ensures obstacle avoidance in continuous path that is applicable to track by mobile robot.

VII. Conclusion and Future Work

Through this research, PRF seems like a new path planning model which exhibits very attractive efficiency by extending functions: OB-PRF, V-PRF, and BC-PRF. OB-PRF eliminates the fault paths, which increases the PRF functionality efficiency in paths feasibility examination. VPRF generates a vertices matrix; faces the PRF feasibility matrix; which helps the mobile robot planner to predict the length of all feasible paths; according to the proposed number of vertices criteria independently on the length of each other, without the need to generate these paths. So, BC-PRF utilizes the PRF engine in generating the obtained index of optimum path and plots it effectively in matrix form.

Generally, PRF works to satisfy planning algorithm in: static obstacles detection, select the best path, and plot the path utilizing Bezier curve approach. PRF may be considered as mobile robot path planning library. And the future works are: Extend the PRF functionality from binomial to multinomial, Utilize Genetic Algorithm in creating the path by using PRF, and Utilize B-Spline instead of Bezier curve and examine the difference between them.

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