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MIMO System Order Reduction Using Real-Coded Genetic Algorithm

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Abstract**—**In this paper, real-coded genetic algorithm (RCGA) optimization technique has been applied for large-scale linear dynamic multi-input-multi-output (MIMO) system. The method is based on error minimization technique where the integral square error between the transient responses of original and reduced order models has been minimized by RCGA. The reduction procedure is simple computer oriented and the approach is comparable in quality with the other well-known reduction techniques. Also, the proposed method guarantees stability of the reduced model if the original high-order MIMO system is stable. The proposed approach of MIMO system order reduction is illustrated with the help of an example and the results are compared with the recently published other well-known reduction techniques to show its superiority.

Keywords**—**Multi-input-multi-output (MIMO) system. Model order reduction. Integral squared error (ISE). Real-coded genetic algorithm.

I. Introduction

In system theory the approximation of higher order system by lower order models is one of the important challenges. Because by using the reduced lower order model, the implementation of analysis, simulation and various system designs become easier. Various order-reduction methods are available for linear continuous time domain system as well as systems in frequency domain. Further, the extension of single-input single-output (SISO) methods to reduce multi-input multi-output (MIMO) systems has also been carried out in [1]-[3].

Various order reduction methods are available in [4]-[8] based on the minimization of the integral square error (ISE) criterion. However, a familiar aspect in the methods explained in [4]-[7] is that the denominator coefficients values of the low order system (LOS) are selected arbitrarily by some stability preserving methods such as dominant pole, Routh approximation methods, etc. and then the numerator coefficients of the LOS are determined by minimization of the ISE. In [8], Howitt and Luss recommended a procedure, in which both the numerator and denominator coefficients are considered to be free parameters and are chosen to minimize the ISE in impulse or step responses.

These days, Genetic algorithm (GA) is becoming popular to solve the optimization problems in different fields of application mainly because of their robustness in finding an optimal solution and ability to provide a near optimal solution close to a global minimum. Unlike strict mathematical methods, the GA does not require the condition that the variables in the optimization problem be continuous and different; it only requires that the problem to be solved can be computed. The present attempt is towards evolving a new algorithm for order reduction, where all the numerator and denominator parameters are considered to be free parameters and the error minimization by RCGA is employed to optimize these parameters. The proposed algorithm consists of searching all the parameters by minimizing the integral square error between the transient responses of original and LOS using RCGA. The proposed approach is illustrated with the help of an example and the results are compared with the recently published techniques.

II. Real-Coded Generic Algorithm

Genetic algorithm (GA) has been used to solve difficult engineering problems that are complex and difficult to solve by conventional optimization methods. GA maintains and manipulates a population of solutions and implements a survival of the fittest strategy in their search for better solutions. The fittest individuals of any population tend to reproduce and survive to the next generation thus improving successive generations. The inferior individuals can also survive and reproduce. Implementation of GA requires the determination of six fundamental issues: chromosome representation, selection function, the genetic operators, initialization, termination and evaluation function. Brief descriptions about these issues are provided in the following sections.

A. Chromosome representation

Chromosome representation scheme determines how the problem is structured in the GA and also determines the genetic operators that are used. Each individual or chromosome is made up of a sequence of genes. Various types of representations of an individual or chromosome are: binary

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digits, floating point numbers, integers, real values, matrices, etc. Generally natural representations are more efficient and produce better solutions. Real-coded representation is more efficient in terms of CPU time and offers higher precision with more consistent results.

B. Selection Function

To produce successive generations, selection of individuals plays a very significant role in a genetic algorithm. The selection function determines which of the individuals will survive and move on to the next generation. A probabilistic selection is performed based upon the individual's fitness such that the superior individuals have more chance of being selected. There are several schemes for the selection process: roulette wheel selection and its extensions, scaling techniques, tournament, normal geometric, elitist models and ranking methods.

The selection approach assigns a probability of selection Pj to each individuals based on its fitness value. In the present study, normalized geometric selection function has been used.

In normalized geometric ranking, the probability of selecting an individual Pi is defined as:

$$P_i = q^{'}(1-q)^{r-1} \tag{1}$$

$$q^{'} = \frac{q}{1-(1-q)^{p}} \tag{2}$$

where,
q = probability of selecting the best individual
r = rank of the individual (with best equals 1)
P = population size

C. Genetic Operator

The basic search mechanism of the GA is provided by the genetic operators. There are two basic types of operators: crossover and mutation. These operators are used to produce new solutions based on existing solutions in the population. Crossover takes two individuals to be parents and produces two new individuals while mutation alters one individual to produce a single new solution. The following genetic operators are usually employed: simple crossover, arithmetic crossover and heuristic crossover as crossover operator and uniform mutation, non-uniform mutation, multi-non-uniform mutation, boundary mutation as mutation operator. Arithmetic crossover and non-uniform mutation are employed in the present study as genetic operators. Crossover generates a random number r from a uniform distribution from 1 to m and creates two new individuals by using equations:

$$x_i^{'} = \begin{cases} x_i, & if \;\; i < r \\ y_i, & otherwise \end{cases} \tag{3}$$

$$y_i^{'} = \begin{cases} y_i, & if \;\; i < r \\ x_i, & otherwise \end{cases} \tag{4}$$

Arithmetic crossover produces two complimentary linear combinations of the parents, where r = U (0, 1):

$$\begin{aligned} \overline{X}' &= r\overline{X} + (1-r)\overline{Y} \\ \overline{Y}' &= r\overline{Y} + (1-r)\overline{X} \end{aligned} \tag{5}$$

Non-uniform mutation randomly selects one variable j and sets it equal to an non-uniform random number

$$\begin{aligned} x_i^{'} &= x_i + (b_i - x_i)\, f(G) \quad if\; r_1 < 0.5 \\ x_i^{'} &= x_i + (x_i + a_i)\, f(G) \quad if\; r_1 \geq 0.5 \\ x_i & \qquad\qquad\qquad\qquad otherwise \end{aligned} \tag{6}$$

where

$$f(G) = \left(r_2 \left(1 - \frac{G}{G_{\max}} \right) \right)^{b} \tag{7}$$

r_1, r_2 = uniform random numbers between 0 and 1.
G = current generation.
$G_{\max}$ = maximum number of generations.
b = shape parameter

D. Initialization, evaluation function and stopping criteria

An initial population is needed to start the genetic algorithm procedure. The initial population can be randomly generated or can be taken from other methods. Evaluation functions or objective functions of many forms can be used in a GA so that the function can map the population into a partially ordered set. The GA moves from generation to generation until a stopping criterion is met. The stopping criterion could be maximum number of generations, population convergence criteria, lack of improvement in the best solution over a specified number of generations or target value for the objective function.

III. Description of the Proposed Algorithm.

Let the transfer function matrix of the HOS of order 'n' having 'p' inputs and 'm' outputs are:

$$[G(s)] = \frac{1}{D_n(s)} \begin{bmatrix} a_{11}(s) & a_{12}(s) & \cdots & a_{1p}(s) \\ a_{21}(s) & a_{22}(s) & \cdots & a_{2p}(s) \\ \vdots & \vdots & \cdots & \vdots \\ a_{m1}(s) & a_{m2}(s) & \cdots & a_{mp}(s) \end{bmatrix} \tag{8}$$

or, $[G(s)] = [g_{ij}(s)]$ i=1, 2… m and j=1, 2… p is a m x p matrix.
The general form of $g_{ij}(s)$ is taken as:

$$g_{ij}(s) = \frac{a_{ij}(s)}{D_n(s)} = \frac{a_0 + a_1 s + a_2 s^2 + ... + a_{n-1} s^{n-1}}{b_0 + b_1 s + b_2 s^2 + ... + b_{n-1} s^{n-1} + s^n} \tag{9}$$

D(s) is the nth order denominator polynomial of the transmission matrix for the HOS.

Let, the transfer function matrix of the HOS of order '*n*' having '*p*' inputs and '*m*' outputs to be synthesized is:

$$[R(s)] = \frac{1}{D_r(s)} \begin{bmatrix} b_{11}(s) & b_{12}(s) & \cdots & b_{1p}(s) \\ b_{21}(s) & b_{22}(s) & \cdots & b_{2p}(s) \\ \vdots & \vdots & \cdots & \vdots \\ b_{m1}(s) & b_{m2}(s) & \cdots & b_{mp}(s) \end{bmatrix} \quad (10)$$

or, $[R(s)] = [r_{ij}(s)]$ i=1, 2… m and j=1, 2… p is a m x p matrix.
The general form of $r_{ij}(s)$ is taken as:

$$r_{ij}(s) = \frac{b_{ij}(s)}{D_r(s)} = \frac{\mu_0 + \mu_1 s + \mu_2 s^2 + ... + \mu_{r-1} s^{r-1}}{b_0 + b_1 s + b_2 s^2 + ... + b_{r-1} s^{r-1} + s^r} \quad (11)$$

$D_r(s)$ is the r^th order denominator polynomial of the transmission matrix for the reduced LOS.
The reduction process from an n^th order HOS to a reduced r^th order LOS consists of the following two steps :

Step-1

The coefficients of the denominator of the reduced LOS are found out by employing GA. The integral square error (ISE) is found out between the transient responses between the HOS and LOS and is given as

$$E = \int_0^\infty \left[y(t) - y_r(t) \right]^2 dt \quad (12)$$

where $y(t)$ and $y_r(t)$ are the unit step responses of original G(s) and reduced R(s) order systems.

Real Coded Generic Algorithm is employed to minimize the integral square error (ISE) in order to find out the denominator coefficients of the LOS.

Step-2

Once the denominator coefficients of the reduced LOS are found out, GA is used to find out the numerator coefficients of LOS. Here also the Real Coded Generic Algorithm is implemented in such a way to minimize the ISE by keeping the values of the denominator coefficients, which are found out in step 1, of the reduced LOS constant.

IV. Numeric Example

To demonstrate the proposed method, one numeric example of a higher order system (6^th order) is taken from literature[9]-[10] and the proposed algorithm is employed to obtain a second-order reduced model.
Consider a system having two inputs and two outputs with transfer matrix of

$$[G(s)] = \begin{bmatrix} \dfrac{2(s+5)}{(s+1)(s+10)} & \dfrac{(s+4)}{(s+2)(s+5)} \\ \dfrac{(s+10)}{(s+1)(s+20)} & \dfrac{(s+6)}{(s+2)(s+3)} \end{bmatrix} = \frac{1}{D_6(s)} \begin{bmatrix} a_{11}(s) & a_{12}(s) \\ a_{21}(s) & a_{22}(s) \end{bmatrix} \quad (13)$$

where $D_6(s)$ is the common denominator polynomial. As can be verified the MIMO system is a 6^th order system and is given as

$$D_6(s) = (s+1)(s+2)(s+3)(s+5)(s+10)(s+20)$$
$$= 6000 + 13100s + 10060s^2 + 3491s^3 + 571s^4 + 41s^5 + s^6$$

and

$$a_{11}(s) = 6000 + 7700s + 3610s^2 + 762s^3 + 70s^4 + 2s^5$$
$$a_{12}(s) = 2400 + 4100s + 2182s^2 + 459s^3 + 38s^4 + s^5$$
$$a_{21}(s) = 3000 + 3700s + 1650s^2 + 331s^3 + 30s^4 + s^5$$
$$a_{22}(s) = 6000 + 9100s + 3660s^2 + 601s^3 + 41s^4 + s^5$$

The proposed algorithm is applied to minimize the function E (ISE). The ISE is calculated for each element ($r_{ij}(s)$) of the transfer function matrix of the LOS and it is given by Eqn. (14)

$$E = \int_0^\infty \left[g_{ij}(t) - r_{ij}(t) \right]^2 dt \quad (14)$$

where i=1,2 ; j=1,2; and $g_{ij}(t)$ and $r_{ij}(t)$ are the unit step response of the original and reduced order model, respectively.
The general form of the transfer function matrix of the reduced second order LOS is taken as

$$[R(s)] = \frac{1}{D_r(s)} \begin{bmatrix} b_{11}(s) & b_{12}(s) \\ b_{21}(s) & b_{22}(s) \end{bmatrix} \quad (15)$$

Where $D_r(s)$ is the common denominator polynomial of the reduced model. For the implementation of RCGA normal geometric selection is employed which is a ranking selection function based on the normalized geometric distribution. Arithmetic crossover takes two parents and performs an interpolation along the line formed by the two parents. Non uniform mutation changes one of the parameters of the parent based on a non-uniform probability distribution. This Gaussian distribution starts wide, and narrows to a point distribution as the current generation approaches the maximum generation. The follow chart of proposed approach is shown in Fig. 1.

The reduced 2^nd order common denominator polynomial is found by implementing our method as:

$$D_r(s) = 1.0459s^2 + 4.1138s + 2.9920$$

and

$$b_{11}(s) = 1.3935s + 2.9918$$
$$b_{12}(s) = 1.0773s + 1.1985$$
$$b_{21}(s) = 0.6247s + 1.4963$$
$$b_{22}(s) = 1.8935s + 3.0069$$

The step responses of original and reduced order models are compared in Figs. 2(a-d). A comparison of proposed method with the other well-known order-reduction techniques available in literature is given in the Table 1. The comparison is made by calculating the ISE given by Eqn. (14) pertaining to a step input. The ISE is calculated for each element of the transfer function matrix of the LOS with respect to the corresponding original system.

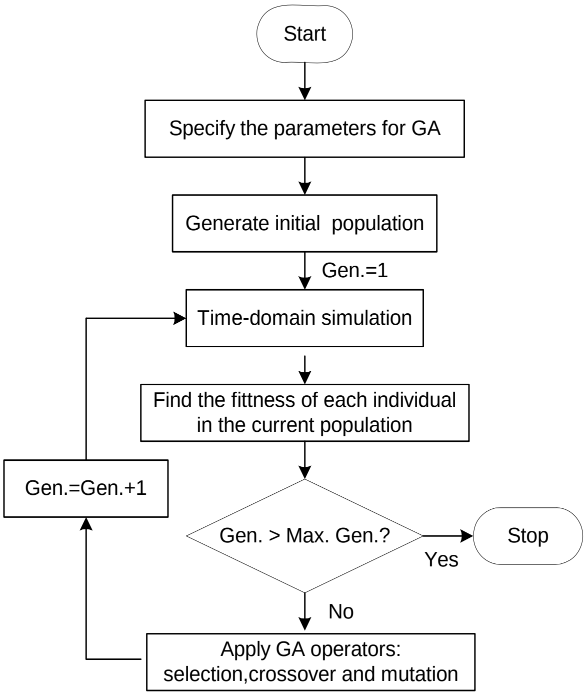
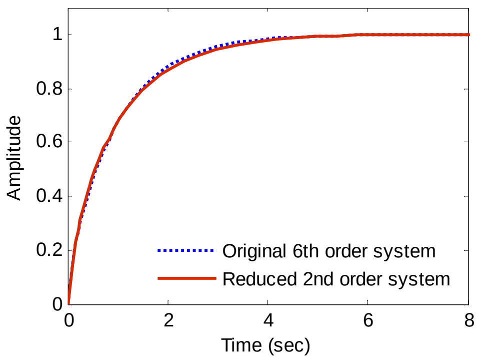
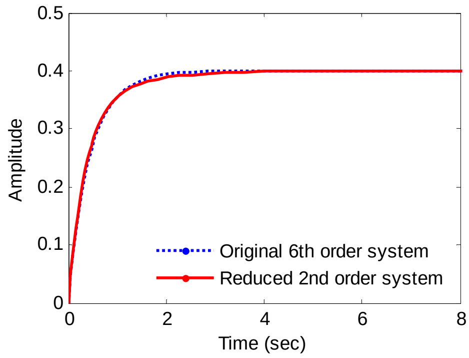


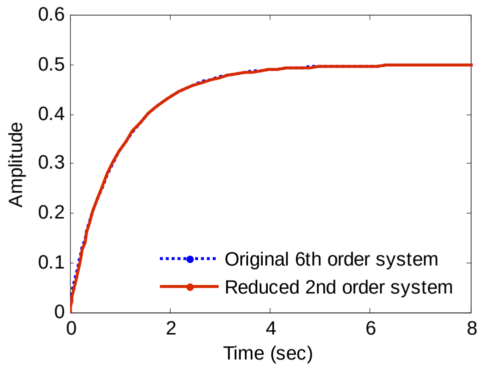
Fig. 1 Flowchart of the proposed approach



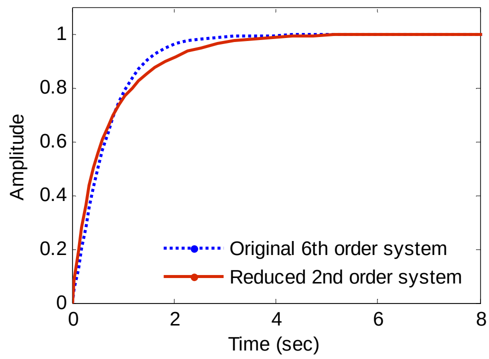
(a)



(b)



(c)



(d)

Fig. 2 Comparison of step response (a) Output 1 for input 1 (b) Output 1 for input 2 (c) Output 2 for input 1 (d) Output 2 for input 2

V. Conclusion

An approach based on the error minimization technique employing real-coded genetic algorithm has been presented, to derive stable reduced order models for linear time invariant multi-input multi-output dynamic systems. The algorithm has been implemented in MATLAB 7.0.1 on an Inetl Core 2 Duo processor and the computation time is negligible being less than 1 minute. The matching of the step response is assured reasonably well in the method. The ISE in between the transient parts of original and reduced order systems is calculated and compared in the tabular form as given in Tables 1, from which it is clear that the proposed algorithm compares well with the other existing techniques of model order reduction. The method also preserves the model stability and avoids any steady-state error between the time responses of the original and reduced systems.

TABLE I
COMPARISON OF REDUCTION METHODS

Reduction method	r_{11}	r_{12}	r_{21}	r_{22}
Proposed method	$4.0656x10^{-4}$	$7.772x10^{-5}$	$3.2448x10^{-5}$	0.0068
Vishwakarma and Prasad [9]	0.001515	$7.845x10^{-5}$	$2.9984x10^{-4}$	0.0047
Parmar et al. [10]	0.014498	0.008744	0.002538	0.015741
Prasad and Pal [11]	0.136484	0.002446	0.040291	0.067902
Safonov and Chiang [12]	0.590617	0.037129	0.007328	1.066123
Prasad et al. [13]	0.030689	0.000256	0.261963	0.021683

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