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Microwave shielding of magnetized hydrogen plasma in carbon nanotubes

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Abstract—We derive simple sets of equations to describe the microwave response of a thin film of magnetized hydrogen plasma in the presence of carbon nanotubes, which were grown by iron-catalyzed high-pressure disproportionation (HiPco). By considering the interference effects due to multiple reflections between thin plasma film interfaces, we present the effects of the continuously changing external magnetic field and plasma parameters on the reflected power, absorbed power, and transmitted power in the system. The simulation results show that the interference effects play an important role in the reflectance, transmittance and absorptance of microwave radiation at the magnetized plasma slab. As a consequence, the interference effects lead to a sinusoidal variation of the reflected intensity and can greatly reduce the amount of reflection power, but the absorption power increases.

I. INTRODUCTION

THE production of iron-catalyzed high-pressure disproportionation (HiPco)-grown single-walled carbon nanotubes (SWCNTs) [1,2] studied and optimized with respect to a number of process parameters including temperature, carbon monoxide pressure, and Fe(CO)$_5$ catalyst concentration. Carbon nanotubes (CNTs) produced via HiPco process have been extensively explored for potential applications in electromagnetic shielding or absorbing materials [3-9]. Imholt and co-workers [3] and Wadhawan and et al. [4] reported that CNTs under vacuum (less than 10-5 Torr) irradiated with a microwave field of 300 W produced a visibly bright plasma accompanied by rapid degassing of the CNTs. Naab and co-workers [5] repeated the experiments of Refs. 3 and 4 under the same microwave field conditions, using a particle-induced x-ray emission (PIXE) experiment, showed that metallic impurities do not play a predominant role for the absorption of microwave energy in CNTs. Peng et al. [6,7], by using the fluid theory and Lambert-Beer law, investigated the microwave loss mechanisms of hydrogen plasma in HiPco CNTs, theoretically. The experimental phenomenon of strong microwave absorption (around 2.45 GHz) by HiPco CNTs was well explained by Peng's model.

In the preceding paper [9], by calculating the multiple reflection effects, we described the behavior of the microwave propagation in the magnetized uniform hydrogen plasma slab in the presence of CNTs. We obtained the reflection, absorption, and transmission coefficients of the system, when interference effects due to multiple reflections between plasma film interfaces are small. We studied the effects of the continuous changing of the plasma parameters and the magnetic field strength on the absorbed, reflected, and transmitted powers.

However, the previous investigation [9] based on the involved light intensities, is correct only, if the hydrogen plasma slab in the presence of CNTs is thicker than the coherence length of the light. If the slab thickness becomes much smaller than the coherence length, we cannot simply add the intensities of the individual light beams to get the total intensity, but we have to add the amplitudes of the electric field strength observing the actual phase. This leads to the appearance of interference effects [10-12] in the reflected and transmitted signals. The present work focus on the effects of interference on microwave response of the system. We will further on call this treatment the coherent description of the multiple reflections as opposed to the incoherent description in Eqs. (3) to (11) in our previous work [9].

II. THEORETICAL MODEL

We now revisit the problem of the analysis of the microwave response of the isotropic and homogeneous magnetized hydrogen plasma slab in the presence of CNTs, by considering the interference effects. Referring to Fig. 1, we investigate the reflectance, transmittance and absorptance of microwave radiation at the magnetized plasma slab in the presence of CNTs between two semi-infinite media. The whole structure is described by (see Fig. 1)

$$n(x) = \begin{cases} n_1, & \mathrm{x} < 0, \\ n_2, & 0 < \mathrm{x} < \mathrm{d}, \\ n_3, & \mathrm{d} < \mathrm{x}. \end{cases} \tag{1}$$

where n_1, n_2, and n_3 are the refractive indices. Optical and dielectric properties of the magnetized plasma slab in the presence of CNTs can be characterized by a complex refractive index

$$n_2 = n - i\kappa, \tag{2}$$

where n is the real and κ the imaginary part of the refractive index. κ is also often denoted as extinction coefficient. The plasma slab is further characterized by its thickness d. The background magnetic field is uniform and perpendicular to the surface of the slab.

Since the whole medium is homogeneous in the z direction (i.e., $\partial n/\partial z = 0$), the electric field vector of a general plane-

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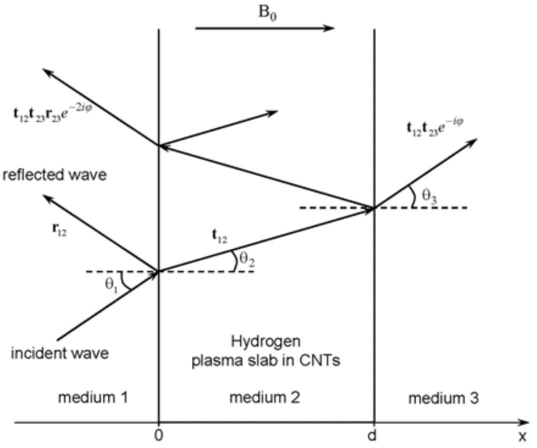

Fig. 1. Schematic representation of the hydrogen plasma slab in the presence of CNTs characterized by its thickness *d*.

wave solution of the wave equation can be of the form

$$\mathbf{E} = \mathbf{E}(x)\exp[i(\omega t - \beta z)], \tag{3}$$

where the parameter β is the z component of the propagation wave vector and ω is the angular frequency. In Eq. (3), we assume that the microwave is propagating in the xz plane and we further assume that the electric field is either an *s* wave (with $\mathbf{E}\|\mathbf{y}$) or a *p* wave (with $\mathbf{H}\|\mathbf{y}$). The amplitude reflection and transmission coefficients r_p, r_s and t_p, t_s are given by Fresnel's equations [10-14]. For *s* polarized (TE wave), we have

$$r_{\ell m} = \frac{k_{\ell x} - k_{mx}}{k_{\ell x} + k_{mx}}, \tag{4}$$

$$t_{\ell m} = \frac{2k_{\ell x}}{k_{\ell x} + k_{mx}}, \tag{5}$$

and for *p* polarized (TM wave), one obtain

$$r_{\ell m} = \frac{n_\ell^2 k_{mx} - n_m^2 k_{\ell x}}{n_\ell^2 k_{mx} + n_m^2 k_{\ell x}}, \tag{6}$$

$$t_{\ell m} = \frac{2n_\ell^2 k_{mx}}{n_\ell^2 k_{mx} + n_m^2 k_{\ell x}}, \tag{7}$$

where $\ell = 1, 2$ and $m = \ell + 1$ and

$$k_{ix} = \left[\left(\frac{n_i\omega}{c}\right)^2 - \beta^2\right]^{1/2} = \frac{\omega}{c} n_i \cos\theta_i, \qquad i = \ell, m \tag{8}$$

are the x components of the wave vectors. The relation between θ_1, θ_2 and θ_3 is given by Snell's law:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 = n_3 \sin\theta_3, \tag{9}$$

where θ_i, with $i = \ell, m$ is the ray angle measured from the x axis, as indicated in Fig. 1.

The derivation of the formulas for the reflectance and transmittance is very similar to the derivation of Eqs. (3) and (4) in Ref. 9. Such derivation was first carried out by G.B. Airy [10,12]. Let consider a beam incident from the left, as depicted in Fig. 1. The incident beam is partially reflected and partially transmitted at the first interface. The transmitted part is subsequently reflected back and forth between the two interfaces as shown. However, in the presence of the interference effects, instead of the intensity coefficients $R_{\ell m}$ and $T_{\ell m}$ (see Eqs. (3) and (4) in Ref. 9), we use the amplitude coefficients $r_{\ell m}$ and $t_{\ell m}$, and we have to include the phase factor -2ϕ that accounts for the geometric path difference between any two successive reflected or transmitted rays. Let the amplitude of the incident wave be 1. Taking the phase difference into account as a factor $\exp(-2i\phi)$ and adding the amplitudes of the reflected rays, we obtain

$$\begin{aligned} r &= r_{12} + t_{12}r_{23}t_{21}e^{-2i\phi} \\ &+ t_{12}r_{23}r_{21}r_{23}t_{21}e^{-4i\phi} + t_{12}r_{23}\left(r_{21}r_{23}\right)^2 t_{21}e^{-6i\phi} + \dots \\ &= r_{12} + t_{12}r_{23}t_{21}e^{-2i\phi}\sum_{j=1}^{\infty}\left(r_{21}r_{23}e^{-2i\phi}\right)^{j-1} \\ &= r_{12} + \frac{t_{12}r_{23}t_{21}e^{-2i\phi}}{1 - r_{21}r_{23}e^{-2i\phi}}. \end{aligned} \tag{10}$$

Similarly, the transmission coefficient t is obtained as

$$\begin{aligned} t &= t_{12}t_{23}e^{-i\phi} + t_{12}r_{23}r_{21}t_{23}e^{-3i\phi} + t_{12}\left(r_{23}r_{21}\right)^2 t_{23}e^{-5i\phi} + \dots \\ &= t_{12}t_{23}e^{-i\phi}\sum_{j=1}^{\infty}\left(r_{21}r_{23}e^{-2i\phi}\right)^{j-1} = \frac{t_{12}t_{23}e^{-i\phi}}{1 - r_{21}r_{23}e^{-2i\phi}}. \end{aligned} \tag{11}$$

The phase 2ϕ follows from a simple geometrical consideration of the phase difference which is given by the path difference of two neighboring interfering light beams and is given by

$$\phi = k_{2x}d = \frac{2\pi d}{\lambda} n_2 \cos\theta_2, \tag{12}$$

and is proportional to the thickness d and the index n_2 of the layer. Using the identities $r_{21} = -r_{12}$ and $t_{12}t_{21} - r_{12}r_{21} = 1$ which follow from Fresnel's equations, we can simplify Eqs. (10) and (11) to:

$$r = \frac{r_{12} + r_{23}e^{-2i\phi}}{1 + r_{12}r_{23}e^{-2i\phi}}, \tag{13}$$

$$t = \frac{t_{12}t_{23}e^{-i\phi}}{1 + r_{12}r_{23}e^{-2i\phi}}. \tag{14}$$

The above formulas (Eqs. (13) and (14)) are valid for both polarizations.

III. NUMERICAL RESULT AND DISCUSSION

We now present the simulation results of the reflectance, transmittance and absorptance of microwave radiation by magnetized hydrogen plasma slab in the presence of CNTs. To compare the results with previous work [9], we consider now the common case of perpendicular incidence (θ_1 and $\theta_2 = 0$) and assume that we are dealing with a free standing plasma slab, where medium 1 and 3 are air. The complex dielectric constants for our system, can be obtained as follows [9]:

$$\varepsilon_r = 1 - \frac{\upsilon_p^2(\upsilon - \upsilon_c)}{\upsilon\left[(\upsilon - \upsilon_c)^2 + \upsilon_e^2\right]} - i\frac{\upsilon_p^2\upsilon_e}{\upsilon\left[(\upsilon - \upsilon_c)^2 + \upsilon_e^2\right]}, \tag{15}$$

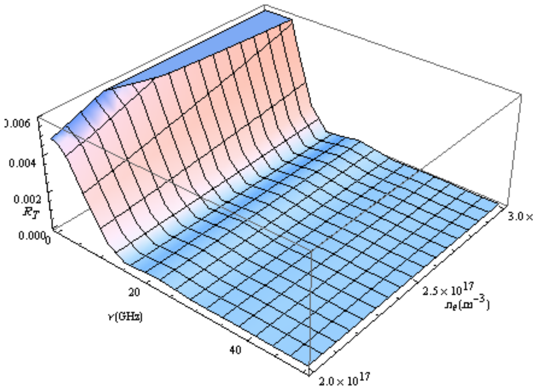

Fig. 2. Dependence of total reflected power of hydrogen plasma slab in the presence of CNTs on microwave frequency for different value of n, where $\upsilon_e = 22$ GHz and $\upsilon_c = 5$ GHz.

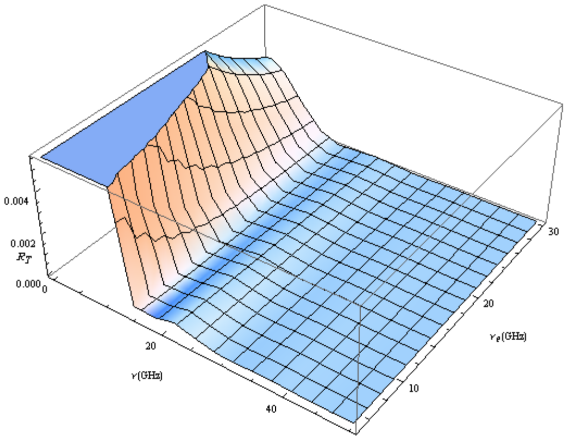

Fig. 3. Dependence of total reflected power of hydrogen plasma slab in the presence of CNTs on microwave frequency for different value of υ_e, where $n = 2.181 \times 10^{17} m^{-3}$ and $\upsilon_c = 5$ GHz.

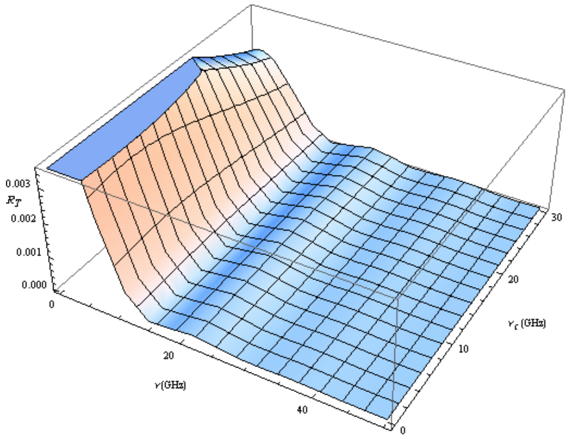

Fig. 4. Dependence of total reflected power of hydrogen plasma slab in the presence of CNTs on microwave frequency for different value of υ_c, where $n = 2.181 \times 10^{17} m^{-3}$ and $\upsilon_e = 22$ GHz.

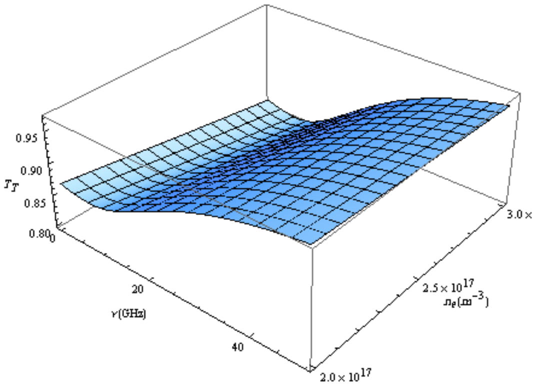

Fig. 5. Dependence of total transmitted power of hydrogen plasma slab in the presence of CNTs on microwave frequency for different value of n, where $\upsilon_e = 22$ GHz and $\upsilon_c = 5$ GHz.

where

$$\upsilon_p = \sqrt{\frac{(n_{e,plasma} + n_{e,CNTs})e^2}{\varepsilon_0 m_e}}$$

is the plasma frequency, $\upsilon_e = \upsilon_{e,plasma} + \upsilon_{e,CNTs}$ is the effective collision frequency between the electrons and neutral plasma particles and between electrons and CNTs and υ_c is the electron gyrofrequency. $n_{e,plasma}$ and $n_{e,CNTs}$ are the electron density of hydrogen plasma and electron density of CNTs, respectively. Under normal condition, the free electron density in plasma is generally $10^{17} - 10^{19}\ m^{-3}$ [9], where we take $n = n_{e,plasma} + n_{e,CNTs}$ as the order of magnitudes of $10^{17}\ m^{-3}$. The order of magnitudes of υ_e is usually $10^9 - 10^{10}$ Hz and we take υ_e as the order of magnitudes of 10^9 Hz. Also, we assume that the slab width is $1\ cm$. The reflectance is defined as the fraction of energy reflected from the system and is given by

$$R = \mid r \mid^2 . \tag{16}$$

In addition, in our model, transmittance is given by

$$T = \mid t \mid^2 . \tag{17}$$

Absorptance, which is defined as the fraction of energy dissipated, is given by

$$A = 1 - R - T. \tag{18}$$

At this stage, the dependence of the total reflected power of hydrogen plasma slab on microwave frequency for different value of n with $\upsilon_e = 22$ GHz and $\upsilon_c = 5$ GHz is shown in Fig. 2. It can be seen that the total reflected power increase with increasing hydrogen plasma density and its position shifts toward high frequencies. However, comparing Fig. 2, in this work and corresponding result in [9], it can be concluded that the interference effects can greatly reduce the amount of reflection power. The total reflected power versus the microwave frequency at different collision frequencies υ_e is shown in Fig. 3, where $n = 2.181 \times 10^{17} m^{-3}$ and $\upsilon_c = 5$ GHz. It is clear that the total reflected power rapidly decrease with increasing collision frequency in the microwave frequency $\upsilon < 15$ GHz and its position shifts toward low frequencies. It is easy to find that the interference effects lead to a sinusoidal variation of the reflected intensity. Fig. 4 shows the dependence of the total reflected power of the hydrogen plasma on the microwave

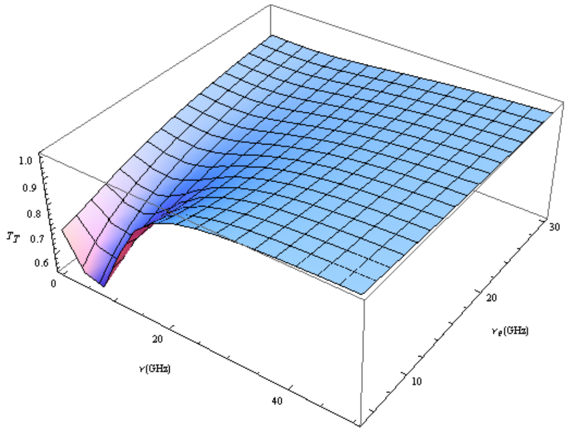

Fig. 6. Dependence of total transmitted power of hydrogen plasma slab in the presence of CNTs on microwave frequency for different value of υ_e, where $n = 2.181 \times 10^{17} m^{-3}$ and $\upsilon_c = 5$ GHz.

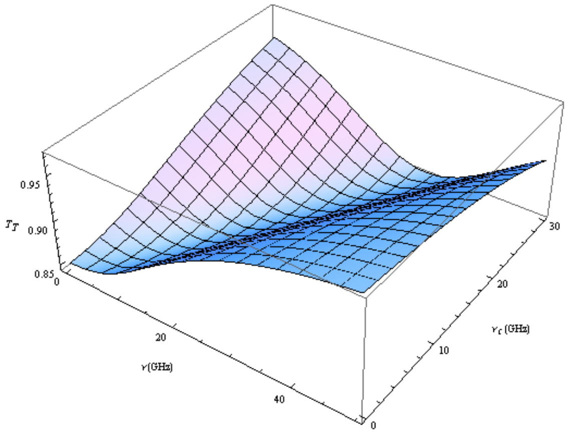

Fig. 7. Dependence of total transmitted power of hydrogen plasma slab in the presence of CNTs on microwave frequency for different value of υ_c, where $n = 2.181 \times 10^{17} m^{-3}$ and $\upsilon_e = 22$ GHz.

frequency for different value of υ_c. Accordingly, R_T decrease substantially with increasing external field strength and its position shifts toward low frequencies. Fig. 5 shows the total transmitted power of hydrogen plasma slab versus the microwave frequency and the hydrogen plasma density n with $\upsilon_e = 22$ GHz and $\upsilon_c = 5$ GHz. One can see that the total transmitted power decrease with increasing hydrogen plasma density and its position shifts toward high frequencies. However, comparing Fig. 5, in this work and corresponding result in [9], it can be concluded that the interference effects can greatly increase the amount of transmission power in low frequencies. The total transmitted power versus the microwave frequency at different collision frequencies υ_e is shown in Fig. 6, where $n = 2.181 \times 10^{17} m^{-3}$ and $\upsilon_c = 5$ GHz. It can be seen that the total transmitted power increase with increasing collision frequency and its position shifts toward low frequencies. One can see that the interference effects lead to a sinusoidal variation of the transmitted intensity. Also, Fig. 7 shows the dependence of the total transmitted power of the hydrogen plasma on the microwave frequency for different value of υ_c. The total transmitted power increase substantially with increasing external field strength for low frequencies.

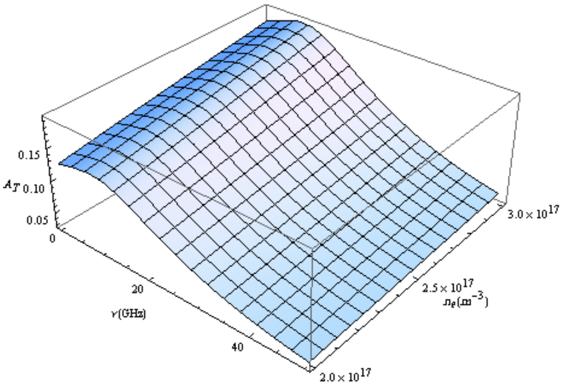

Fig. 8. Dependence of total absorbed power of hydrogen plasma slab in the presence of CNTs on microwave frequency for different value of n, where $\upsilon_e = 22$ GHz and $\upsilon_c = 5$ GHz.

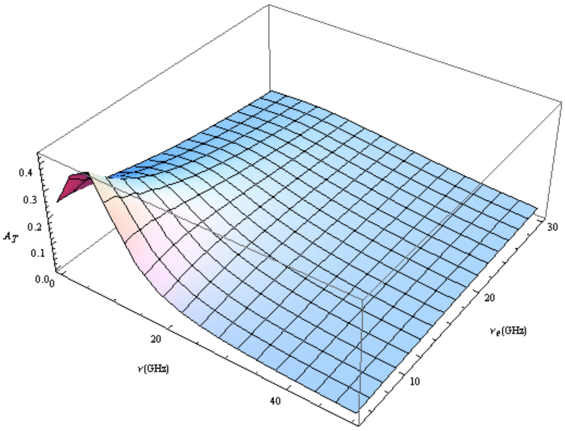

Fig. 9. Dependence of total absorbed power of hydrogen plasma slab in the presence of CNTs on microwave frequency for different value of υ_e, where $n = 2.181 \times 10^{17} m^{-3}$ and $\upsilon_c = 5$ GHz.

The dependence of the total absorbed power of hydrogen plasma slab on microwave frequency for different value of n with $\upsilon_e = 22$ GHz and $\upsilon_c = 5$ GHz, is displayed in Fig. 8. It can be seen that the total absorbed power increase with increasing hydrogen plasma density and its position shifts toward high frequencies. However, comparing Fig. 8, in this work and corresponding result in [9], it can be concluded that the interference effects can greatly increase the amount of absorbed power. The total absorbed power versus the microwave frequency and the collision frequency υ_e is exhibited in Fig. 9. One can see, by increasing collision frequency, the absorption peak decreases and its position shifts toward low frequencies. Therefore, it can be concluded that a high number density and low collision frequency of hydrogen plasma will absorb the microwaves and can greatly increase the amount of absorbtion power. We note that this increase eventually reaches a saturation value since for high number densities more collisions occur, which tends to decrease the amount of absorbed power. Finally, to see clearly the influence of the external magnetic field on the total absorbed power of hydrogen plasma slab, we plot the total absorbed power versus the microwave frequency and the electron gyrofrequency, υ_c as shown in Fig. 10. It is clear that the total absorbed power of the magnetized hydrogen plasma moves from low to high frequencies when the magnetic field strength increases.

IV. CONCLUSION

In this work, by considering the interference effects due to multiple reflections between plasma film interfaces, we extended the previous investigation [9] to describe the behavior of the microwave propagation in the magnetized uniform hydrogen plasma in the presence of CNTs. We obtained the reflectance, transmittance and absorptance of microwave radiation at the magnetized hydrogen plasma slab. We studied the effects of the continuously changing of the hydrogen parameters and the magnetic field strength on the reflectance, transmittance and absorptance in the system. It is found that the interference effects lead to a sinusoidal variation of the transmitted and reflected intensity. Also, the interference effects can greatly reduce the amount of reflection power, but the absorption power increases. These results show that in practice a thin hydrogen plasma film in HiPco CNTs could be tailored to act as a absorber of microwave radiation.

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