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Micromechanics Modeling of 3D Network Smart Orthotropic Structures

E. M. Hassan, A. L. Kalamkarov

***Abstract*—**Two micromechanical models for 3D smart composite with embedded periodic or nearly periodic network of generally orthotropic reinforcements and actuators are developed and applied to cubic structures with unidirectional orientation of constituents. Analytical formulas for the effective piezothermoelastic coefficients are derived using the Asymptotic Homogenization Method (AHM). Finite Element Analysis (FEA) is subsequently developed and used to examine the aforementioned periodic 3D network reinforced smart structures. The deformation responses from the FE simulations are used to extract effective coefficients. The results from both techniques are compared. This work considers piezoelectric materials that respond linearly to changes in electric field, electric displacement, mechanical stress and strain and thermal effects. This combination of electric fields and thermo-mechanical response in smart composite structures is characterized by piezoelectric and thermal expansion coefficients. The problem is represented by unit-cell and the models are developed using the AHM and the FEA to determine the effective piezoelectric and thermal expansion coefficients. Each unit cell contains a number of orthotropic inclusions in the form of structural reinforcements and actuators. Using matrix representation of the coupled response of the unit cell, the effective piezoelectric and thermal expansion coefficients are calculated and compared with results of the asymptotic homogenization method. A very good agreement is shown between these two approaches.

***Keywords*—**Asymptotic Homogenization Method, Effective Piezothermoelastic Coefficients, Finite Element Analysis, 3D Smart Network Composite Structures.

I. Introduction

THERE have been considerable theoretical techniques to obtain reasonable estimates on the microscopic (local) and macroscopic (global) scales within the context of mechanics of smart composites. Governing equations describing the behaviour of periodic or nearly periodic smart structures are given by a set of partial differential equations characterized by the presence of rapidly varying coefficients due to the presence of numerous embedded inclusions in close proximity to one another. One technique that permits the decoupling of the two scales is the AHM. The mathematical framework of the AHM can be found in [1]-[4]. Many micromechanics models have been established using the unit cell analyses. A wide variety of elasticity and thermo-elasticity periodic problems pertaining to composite materials and thin-walled composite structures reinforced shells and plates was examined in [5]. Expressions for the effective elastic, piezoelectric and hygrothermal expansion coefficients for general 3D periodic smart composite structures were derived in [6]-[8].

A considerable number of micromechanically oriented numerical approaches based on the finite element method have been developed and extensively used in the analysis of the mechanical properties of composites with spatial repetition of small microstructure since the work of [9]. In [10] a 2D finite element approach for a microscopic region of a unidirectional composite using a generalized plane strain formulation which includes longitudinal shear loading has been developed. Method of cells [11] and its generalization [12] have proven to be successful micromechanical analysis tools for the prediction of the overall behaviour of various types of composites with known properties and geometrical arrangement of individual constituents and give consistently accurate results for the elastic properties. A review of the work conducted using the two theories have been given by [13]. An alternative approach that retains the philosophy of Aboudi's Method of Cells has been presented and the equations of equilibrium were applied to a representative volume element and in addition a unified method of homogenization of micromechanical effects was presented in [14]. The finite element method has been extensively used to examine unit cell problems and to determine the effective properties and damage mechanisms of composites. The applications considered include unidirectional laminates [15], cross-ply laminates [16], woven and braided textile composites and piezoelectric foams [17]–[19] and many others. Pertaining to the various finite element models, the unit cells employed can be subjected to mechanical, thermal, electrical or other loading types. The introduction of the loading conditions to the unit cell is expressed, in general, in terms of macroscopic or averaged field quantities, such as stress or strain. The use of the unit cells of different shapes for the analysis and modeling of unidirectional fiber reinforced composites was studied in [20], [21] by considering symmetries in the material and deriving appropriate periodic boundary conditions for the unit cell. The loads on the unit cell and its response in terms of macroscopic stresses or strains have been addressed in such a way that the effective properties of the material can be obtained from the micromechanical analysis of the unit cell in a standard manner. In [22] appropriate constraints on a representative volume element under various loadings have been determined from symmetry and periodicity conditions. A comprehensive unit cell model has been developed for studying composites with periodic hexagonal or square

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arrangements of continuous fibers by means of the finite element method [23]. The work involved piezoelectric composites and the developed model was employed to account for local fluctuations of the fields. A full set of effective moduli were determined. The concept of 'macroscopic degrees of freedom' and the implementation of periodicity conditions for composites with periodic microstructure composed of linear or nonlinear constituents were discussed in [24]. The authors in [25] applied a numerical procedure for the computation of the overall macroscopic elasticity moduli of linear composite materials with periodic micro-structure. The underlying key approach is a finite element discretization of the boundary value problem for the fluctuation field on the micro-structure of the composite. A number of possible unit cell models can be developed according to the material microstructure. The authors of [26], [27] have developed a 3D unit cell model for both unidirectional and cross ply laminates. The proposed unified boundary conditions satisfy not only the boundary displacement periodicity but also the boundary traction periodicity of the unit cell. The study of [28] has integrated the asymptotic homogenization method into a finite element simulation to derive overall material properties for metal matrix composites reinforced with spherical ceramic particles. A technique to evaluate effective material properties related to unidirectional fiber reinforced composites having rhombic periodic microstructures and isotropic and transversely isotropic behaviour has been presented in [29].

II. Micromechanical Models of 3D Network Reinforced Smart Composite Structures

A. Piezothermoelastic Smart Composite Structures

Piezoelectric materials (e.g., Lead Zirconate Titanate [PZT] and Barium Titanate), have the property of converting electrical energy into mechanical energy and vice versa. This work considers piezoelectric material (PZT-5A) that embedded in a dielectric matrix material. The piezoelectric actuators respond linearly to changes in electric field, electric displacement, mechanical stress and strain and thermal effects. The coupling of electric fields and thermo-mechanical response in a smart composite is characterized by the coefficients of piezoelectric and thermal expansion.

The problem is represented by a unit-cell shown in Fig. 1 and the models are developed using the AHM and FEA to determine the effective piezoelectric and thermal expansion coefficients. Each unit cell contains a number of 3D network orthotropic inclusions in the form of structural reinforcements and actuators. Following this introduction, the rest of the paper is organized as follows: The basic problem formulation and the general asymptotic homogenization model for 3D smart network reinforced composite structures have been derived and followed by the development of the finite element model. A network of 3D smart structures is used to formulate and compare results of both, analytical and numerical approaches to illustrate the domain of applicability of the derived models.

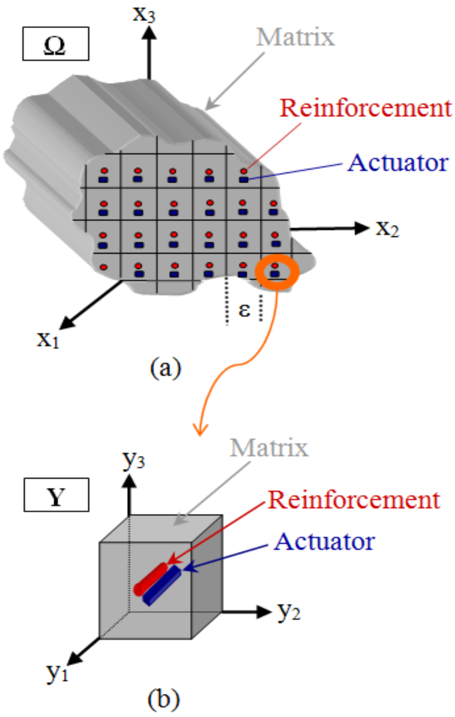

Fig. 1 (a) 3D Smart Composite Structure Ω, (b) Unit Cell *Y*

B. Asymptotic Homogenization Method, Governing Equations and the Unit Cell Problems

The electromechanical deformation of the structure shown in Fig. 1 is expressed using the balance of linear momentum and Gauss's law. In the absence of body forces and currents, the smart composite given in Fig. 1 should satisfy a boundary value problem and its piezothermoelastic behaviour is described by the following constitutive equations:

$$\sigma_{ij}^{\varepsilon} = C_{ijkl} e_{kl}^{\varepsilon} - P_{ijk} E_k - \Theta_{ij} T \tag{1}$$

$$D_i = P_{ijk} e_{kl} + \kappa_{ij} E_j \tag{2}$$

with a linearized strain-displacement relation assuming small displacement gradients,

$$e_{ij}^{\varepsilon} = \tfrac{1}{2}\left(u_{i,jx}^{\varepsilon} + u_{j,ix}^{\varepsilon}\right) \tag{3}$$

where C_{ijkl} is a tensor of elastic coefficients, e_{kl} is a strain tensor which is a function of the displacement field u_i, P_{ijk} is a tensor of piezoelectric coefficients describing the effect of E_k is the electric field vector on the mechanical stress field σ_{ij}, Θ_{ij} is a thermal expansion tensor, T represents change in temperature with respect to a reference state, D_i is the electric displacement and κ_{ij} is the dielectric permittivity. In this Section, only a brief overview of the steps involved in the development of the analytical model are given in so far as it represents the starting point of the current work. The basic asymptotic expansions in terms of ε for the displacement and electric fields as well as the derived asymptotic expansions for the stress and the electric displacement have been considered. The development of asymptotic homogenization model for the 3D network smart composite structures can be found in [30]. Effective piezoelectric and thermal expansion coefficients are determined by solving the unit cell problem given in (4) and (5) followed by applying (6) and (7). The problem formulation for the smart composite structure shown in Figs. 1 and 2 begins with the introduction of the piezoelectric and thermal expansion local functions.

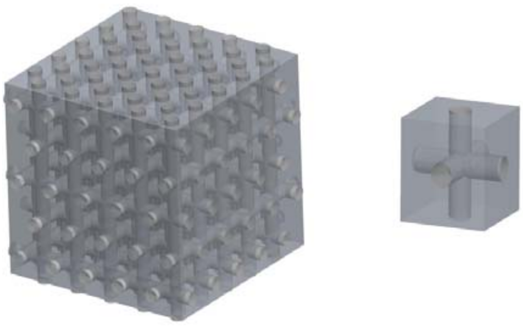

Fig. 2 Smart 3D Composite and Unit Cell

It is assumed that perfect bonding conditions at the interface between the reinforcement and actuator and the matrix.

$$\left[C_{ijmn}(y)N^k_{m,ny}(y)\right]_{,jy} = +P_{ijk,jy}(y) \tag{4}$$

$$\left[C_{ijmn}(y)N_{m,ny}(y)\right]_{,jy} = +\Theta_{ij,jy}(y) \tag{5}$$

$$\tilde{P}_{ijk} = \frac{1}{|Y|}\int_Y \left(P_{ijk}(\boldsymbol{y}) - C_{ijmn}(\boldsymbol{y})N^k_{m,ny}(\boldsymbol{y})\right) dv \tag{6}$$

$$\tilde{\Theta}_{ij} = \frac{1}{|Y|}\int_Y \left(\Theta_{ij}(\boldsymbol{y}) - C_{ijmn}(\boldsymbol{y})N_{m,ny}(\boldsymbol{y})\right) dv \tag{7}$$

The local functions b^k_{ij} can be expanded

$$b^k_{ii} = P_{iik} - \begin{bmatrix} \Gamma^k_1\left\{C_{ii11}q_{21} + C_{ii12}q_{22} + C_{ii13}q_{23}\right\} \\ +\Gamma^k_2\left\{C_{ii11}q_{31} + C_{ii12}q_{32} + C_{ii13}q_{33}\right\} \\ +\Gamma^k_3\left\{C_{ii12}q_{21} + C_{ii22}q_{22} + C_{ii23}q_{23}\right\} \\ +\Gamma^k_4\left\{C_{ii12}q_{31} + C_{ii22}q_{32} + C_{ii23}q_{33}\right\} \\ +\Gamma^k_5\left\{C_{ii13}q_{21} + C_{ii23}q_{22} + C_{ii33}q_{23}\right\} \\ +\Gamma^k_6\left\{C_{ii13}q_{31} + C_{ii23}q_{32} + C_{ii33}q_{33}\right\} \end{bmatrix} \tag{8}$$

no summation in i

$$b^k_{ij} = P_{ijk} - \begin{bmatrix} \Gamma^k_1\left\{C_{ij11}q_{21} + C_{ij12}q_{22} + C_{ij13}q_{23}\right\} \\ +\Gamma^k_2\left\{C_{ij11}q_{31} + C_{ij12}q_{32} + C_{ij13}q_{33}\right\} \\ +\Gamma^k_3\left\{C_{ij12}q_{21} + C_{ij22}q_{22} + C_{ij23}q_{23}\right\} \\ +\Gamma^k_4\left\{C_{ij12}q_{31} + C_{ij22}q_{32} + C_{ij23}q_{33}\right\} \\ +\Gamma^k_5\left\{C_{ij13}q_{21} + C_{ij23}q_{22} + C_{ij33}q_{23}\right\} \\ +\Gamma^k_6\left\{C_{ij13}q_{31} + C_{ij23}q_{32} + C_{ij33}q_{33}\right\} \end{bmatrix} \tag{9}$$

with i ≠ j

As for effective thermal expansion coefficients, the local functions b_{ij} can be expanded as follows:

$$b_{\mathrm{ii}} = \Theta_{ii} - \begin{bmatrix} \chi_1\left\{C_{ii11}q_{21} + C_{ii12}q_{22} + C_{ii13}q_{23}\right\} \\ +\chi_2\left\{C_{ii11}q_{31} + C_{ii12}q_{32} + C_{ii13}q_{33}\right\} \\ +\chi_3\left\{C_{ii12}q_{21} + C_{ii22}q_{22} + C_{ii23}q_{23}\right\} \\ +\chi_4\left\{C_{ii12}q_{31} + C_{ii22}q_{32} + C_{ii23}q_{33}\right\} \\ +\chi_5\left\{C_{ii13}q_{21} + C_{ii23}q_{22} + C_{ii33}q_{23}\right\} \\ +\chi_6\left\{C_{ii13}q_{31} + C_{ii23}q_{32} + C_{ii33}q_{33}\right\} \end{bmatrix} \tag{10}$$

no summation in i

$$b_{\mathrm{ij}} = \Theta_{ij} - \begin{bmatrix} \chi_1\left\{C_{ij11}q_{21} + C_{ij12}q_{22} + C_{ij13}q_{23}\right\} \\ +\chi_2\left\{C_{ij11}q_{31} + C_{ij12}q_{32} + C_{ij13}q_{33}\right\} \\ +\chi_3\left\{C_{ij12}q_{21} + C_{ij22}q_{22} + C_{ij23}q_{23}\right\} \\ +\chi_4\left\{C_{ij12}q_{31} + C_{ij22}q_{32} + C_{ij23}q_{33}\right\} \\ +\chi_5\left\{C_{ij13}q_{21} + C_{ij23}q_{22} + C_{ij33}q_{23}\right\} \\ +\chi_6\left\{C_{ij13}q_{31} + C_{ij23}q_{32} + C_{ij33}q_{33}\right\} \end{bmatrix} \tag{11}$$

with i ≠ j

where Γ^k_i and χ_i are constants to be determined from the boundary conditions. The elastic, piezoelectric and thermal expansion coefficients in (8) to (11) are referenced with respect to the local coordinates. Relationships between these coefficients and their counterparts associated with the principal material coordinate system of the inclusion are expressed by means of tensor transformation laws. In turn, these piezoelectric and thermal expansion local functions are used to calculate the homogenized expansion coefficients of the smart 3D network of orthotropic composite structure shown in Fig. 2 as follows:

$$\tilde{P}_{ijk} = \frac{1}{|Y|}\int_Y b^k_{ij}dv \tag{12}$$

$$\tilde{\Theta}_{ij} = \frac{1}{|Y|} b_{ij}\, dv \tag{13}$$

The resulting analytical expressions to be used in (8) to (11) are too lengthy to be reproduced in this study, details can be found in [31]. However, typical homogenized piezoelectric and thermal expansion coefficients will be computed and plotted in the following Sections.

III. Numerical Micromechanical Modeling Using Finite Element Analysis (FEA)

A microscopic unit cell model is used that represents the heterogeneous microstructure with periodicity condition. The 3D smart grid-reinforced structure considered is periodic which permits the isolation of discrete unit cells for the analysis. The developed finite element model takes into account geometric and material parameters, microscopic aspect such as volume fraction and the linear electro-thermo-mechanical response of a perfectly bonded cylindrical reinforcement and actuator which are aligned and poled along the principle directions and embedded in an isotropic non-piezoelectric elastic matrix as given in Fig. 3. Finite element software package ANSYS® is used for the analysis and the 3D coupled-field twenty-node Solid 226 linear piezoelectric brick element has been used to mesh the unit cell. The element has five degrees of freedom per node (3 translational, thermal and electric potential). This step has been automated using ANSYS Parametric Design Language APDL to generate all required constraint equations [31].

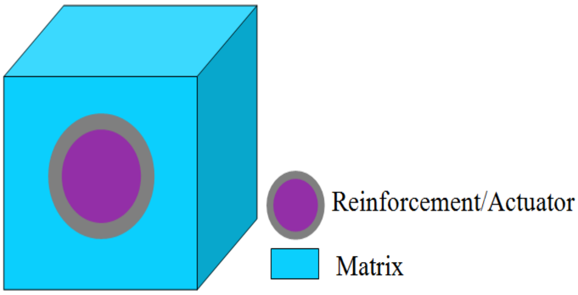

Fig. 3 A Two-phase Composite

A. Periodic Boundary Conditions and Unit Cell Model

In invoking the unit cell approach for characterizing the electromechanical behavior of piezoelectric and thermo 3D network of smart structures, it is essential to ensure that the deformation characteristics of the local or microscopic unit cells are representative of the deformation of the global or macroscopic smart structures. Hence, particular attention has been taken to ensure that the deformation under designed boundary conditions across the boundaries of the unit cell is compatible with the deformation of the neighboring unit cells. The developed finite element analysis consists of three basic steps.

1) The analysis begins with the determination and prescription of periodic boundary conditions to a unit cell. Hence, the unit cell shown in Fig. 4 is subjected to controlled electrical field and thermal loadings by constraining opposite surfaces of the unit cell to have equivalent electro-thermo-mechanical deformations and to ensure that there is no separation or overlap between the neighboring unit cells. Analogies between the prescribed boundary conditions and displacement field for a periodically arranged structure are expressed in [9].
   The equations describing the displacement fields on different boundary surfaces $(\phi^+/\phi^-,\varphi^+/\varphi^-,\omega^+/\omega^-)$ of the unit cell with single reinforcement and actuator shown in Fig. 4 are summarized in (13). It should be noted that the periodicity function is identical at the two opposite boundaries. The periodicity boundary conditions (14) can be applied in the finite element model in form of constraint equations written in APDL [31].

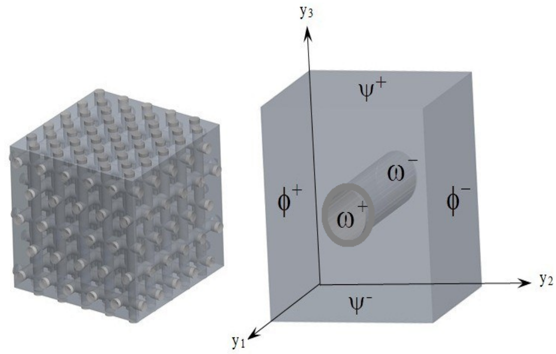

Fig. 4 3D Unit cell for smart composite and notation of the Unit Cell Boundary Surfaces

$$
\begin{aligned}
u_i^{\phi+} &= \tilde{\varepsilon}_{ij} y_j^{\phi+} + u_i^* \; ; \; u_i^{\phi-} = \tilde{\varepsilon}_{ij} y_j^{\phi-} + u_i^* \\
u_i^{\psi+} &= \tilde{\varepsilon}_{ij} y_j^{\psi+} + u_i^* \; ; \; u_i^{\psi-} = \tilde{\varepsilon}_{ij} y_j^{\psi-} + u_i^* \\
u_i^{\omega+} &= \tilde{\varepsilon}_{ij} y_j^{\omega+} + u_i^* \; ; \; u_i^{\omega-} = \tilde{\varepsilon}_{ij} y_j^{\omega-} + u_i^*
\end{aligned}
\tag{14}
$$

The impose periodicity of the boundary conditions (14) on the surfaces of the unit cell are enforced as multifreedom constraints and are valid not only for displacements but also for other types of degree of freedom (e.g. electric potential and temperature). Rigid body motion can be prevented by fixing the displacements and rotations of at least one arbitrary point of the unit cell. Fig. 5 is considered to identify the nodes for which the periodicity condition is applied. This will ensure that all edges of the unit cell and surfaces of the constituents remain parallel for the prescribed loading conditions.

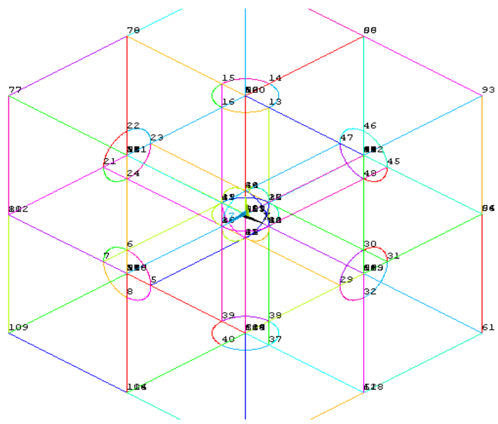

Fig. 5 Unit cell with its boundary node sets utilized

2) The non-homogenous electric fields and electrical displacements obtained from the unit cell analysis are reduced to homogenized or averaged quantities through an averaging procedure. Once the local (microscopic) stress, strain, electric potential and electric displacements fields in the unit cell are determined under the applied loading conditions, the coupled average stresses and strains and the equivalent average electric fields and

electric displacements can be captured and expressed as given in (15)-(18).

$$\tilde{\sigma}_{ij} = \tfrac{1}{V}\int_V \sigma_{ij}\, dV \tag{15}$$

$$\tilde{\varepsilon}_{ij} = \tfrac{1}{V}\int_V \varepsilon_{ij}\, dV \tag{16}$$

$$\tilde{E}_{i} = \tfrac{1}{V}\int_V E_{i}\, dV \tag{17}$$

$$\tilde{D}_{i} = \tfrac{1}{V}\int_V D_{i}\, dV \tag{18}$$

where V is the volume of the periodic unit cell

3) The corresponding effective piezoelectric coefficients of the pertinent smart composite structure can be predicted using the calculated average non-zero value in the strain and electric field vector and the calculated average values in the stress and electrical displacement vector using the constitutive equation (19).

$$\begin{bmatrix}\tilde{\sigma}_{11}\\ \tilde{\sigma}_{22}\\ \tilde{\sigma}_{33}\\ \tilde{\sigma}_{23}\\ \tilde{\sigma}_{13}\\ \tilde{\sigma}_{12}\\ D_1\\ D_2\\ D_3\end{bmatrix} = \begin{bmatrix}\tilde{C}_{11} & \tilde{C}_{12} & \tilde{C}_{13} & 0 & 0 & 0 & 0 & 0 & -P_{31}\\ \tilde{C}_{21} & \tilde{C}_{22} & \tilde{C}_{23} & 0 & 0 & 0 & 0 & 0 & -P_{32}\\ \tilde{C}_{31} & \tilde{C}_{32} & \tilde{C}_{33} & 0 & 0 & 0 & 0 & 0 & -P_{33}\\ 0 & 0 & 0 & \tilde{C}_{44} & 0 & 0 & 0 & -P_{24} & 0\\ 0 & 0 & 0 & 0 & \tilde{C}_{55} & 0 & -P_{15} & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & \tilde{C}_{66} & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & P_{15} & 0 & \kappa_{11} & 0 & 0\\ 0 & 0 & 0 & P_{24} & 0 & 0 & 0 & \kappa_{22} & 0\\ P_{31} & P_{32} & P_{33} & 0 & 0 & 0 & 0 & 0 & \kappa_{33}\end{bmatrix}\begin{bmatrix}\tilde{\varepsilon}_{11}\\ \tilde{\varepsilon}_{22}\\ \tilde{\varepsilon}_{33}\\ \tilde{\varepsilon}_{23}\\ \tilde{\varepsilon}_{13}\\ \tilde{\varepsilon}_{12}\\ E_1\\ E_2\\ E_3\end{bmatrix} \tag{19}$$

As for the effective coefficient of thermal expansion, an increase in temperature is applied to the unit cell to provide thermal loadings and due to the temperature difference stresses are developed. Appropriate average procedure is invoked to capture the homogeneous coupled response of the unit cell. The constitutive relations in (20) are then solved to determine the variation of the effective coefficients of thermal expansion.

$$\begin{bmatrix}\tilde{\sigma}_{11}\\ \tilde{\sigma}_{22}\\ \tilde{\sigma}_{33}\\ \tilde{\sigma}_{23}\\ \tilde{\sigma}_{13}\\ \tilde{\sigma}_{12}\end{bmatrix} = \begin{bmatrix}\tilde{C}_{11} & \tilde{C}_{12} & \tilde{C}_{13} & 0 & 0 & 0\\ \tilde{C}_{21} & \tilde{C}_{22} & \tilde{C}_{23} & 0 & 0 & 0\\ \tilde{C}_{31} & \tilde{C}_{32} & \tilde{C}_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & \tilde{C}_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & \tilde{C}_{55} & 0\\ 0 & 0 & 0 & 0 & 0 & \tilde{C}_{66}\end{bmatrix}\begin{bmatrix}\tilde{\varepsilon}_{11} - \Theta_{11}\Delta T\\ \tilde{\varepsilon}_{22} - \Theta_{22}\Delta T\\ \tilde{\varepsilon}_{33} - \Theta_{33}\Delta T\\ \tilde{\varepsilon}_{23}\\ \tilde{\varepsilon}_{13}\\ \tilde{\varepsilon}_{12}\end{bmatrix} \tag{20}$$

IV. Examples: 3D Smart Network reinforced Composite Structures

In this Section typical effective piezoelectric and thermal expansion coefficients will be computed and plotted. The representative effective piezoelectric and thermal expansion coefficients are determined and compared with respect to the unit cell spatial arrangement and the volume fraction of the reinforcements and actuators. For illustration purposes, the actuators and reinforcements have material properties given in Tables I and II. The developed numerical model applied to a 3D periodic network of orthotropic smart composite with cubic orientations of reinforcement and actuators as shown in Fig. 6. A perfect bonding at the fiber matrix interface has been assumed. Various models of increasingly finer discretization have been developed to attain a satisfactory convergence of the mesh at the boundaries of the unit cells and around the interface, particularly for models when reinforcements and actuators have larger volume fractions. A not solved (Mesh Facet) element was added to provide greater control over element sizes and to allow for volume meshing with or without mid-side nodes and most importantly to ensure equal meshing configurations on opposite boundary surfaces of the unit cell.

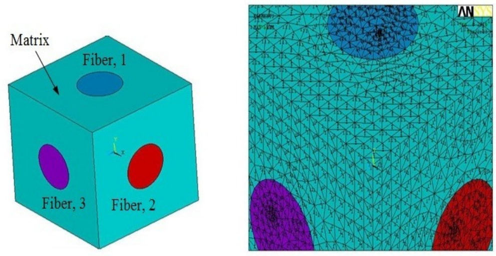

Fig. 6 Discretized Meshing of the Unit Cell

TABLE I
PROPERTIES OF THERMOPIEZOELASTIC MATERIAL PZT-5A [32]

Coefficient	Value
$C_{11}^{(p)} = C_{22}^{(p)}$ (MPa)	121000
$C_{33}^{(p)}$ (MPa)	111000
$C_{12}^{(p)}$ (MPa)	75400
$C_{13}^{(p)} = C_{23}^{(p)}$ (MPa)	75200
$C_{44}^{(p)}$ (MPa)	22600
$C_{55}^{(p)} = C_{66}^{(p)}$ (MPa)	21100
$P_{13}^{(p)} = P_{23}^{(p)}$ (C/mm^2)	-5.45E-6
$P_{33}^{(p)}$ (C/mm^2)	1.56E-5
$P_{42}^{(p)} = P_{51}^{(p)}$ (C/mm^2)	2.46E-5
$\theta_{11}^{(p)} = \theta_{22}^{(p)}$ (1/^{0}C)	-1.704E-10
$\theta_{33}^{(p)}$ (1/^{0}C)	3.732E-10

TABLE II
MATERIAL PROPERTIES OF ISOTROPIC MATRIX [33].

Matrix Material Properties	
E	ν_{12}
36000 MPa	0.35

V. Results and Discussion

The stress field and the electric displacement field components that are developed in the unit cell as a result of the applied strain, electric fields and change in temperature on the unit cell are determined. As an example of applying the matrix representation given in (19) for coupled field response in the unit cell, variation of effective piezoelectric coefficients $\tilde{P}_{333}$ and $\tilde{P}_{311}$ are given by:

$$\tilde{P}_{333} = -\frac{\tilde{\sigma}_{33}}{E_3} \tag{21}$$

$$\tilde{P}_{311} = -\frac{\tilde{\sigma}_{11}}{E_3} \tag{22}$$

Equations (21) refers to the stress response in the same direction as that of the applied electric field (y_3) and (22) represents the stress response of the structure in the (y_1) direction when an external electric field is applied in the (y_3) direction. An increase in temperature is applied to the unit cell to provide a temperature loading (ΔT) and due to the temperature difference stresses are developed. The constitutive relations in (20) are then used to determine the variation of the effective coefficients of thermal expansion in the three global directions. For instance $\tilde{\Theta}_{11}$ is expressed as follows:

$$\tilde{\Theta}_{11} = C_{1111}\Theta_{11} + C_{1122}\Theta_{22} + C_{1133}\Theta_{33} \tag{23}$$

VI. Comparison of the Analytical (AHM) and Numerical (FEA) Results

The calculated effective coefficients are compared with results from the analytical model. Typical effective piezoelectric coefficients poled in the (y_3) direction and effective thermal expansion coefficients are plotted vs. volume fractions of the reinforcements/actuators as shown in Figs. 7 and 8 respectively. FEA results in (31) have shown that the contribution of the stiffness of the matrix will not appreciably affect the homogenized coefficients. Hence, a dielectric matrix material has been considered in the current study. Notice that there is a very good agreement between numerical results calculated using the FEA and analytical solutions obtained using the AHM with small discrepancies occur for higher fiber volume fractions of reinforcements and actuators with a satisfied error less than 5%.

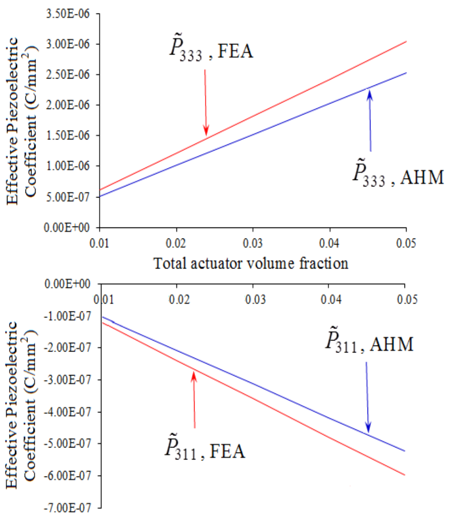

Fig. 6 Effective Piezoelectric Coefficients ($\tilde{P}_{333}$, $\tilde{P}_{311}$) vs. Actuator Volume Fraction

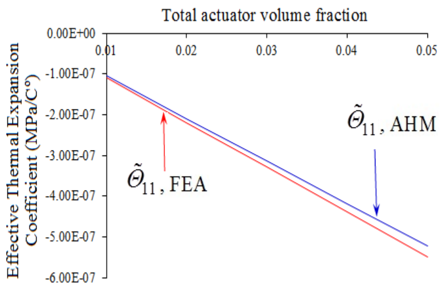

Fig. 7 Effective Thermal Expansion Coefficients ($\tilde{\Theta}_{11}$) vs. Actuator Volume Fraction

VII. Conclusion

Comprehensive micromechanical models of 3D periodic composite structures reinforced with smart network of orthotropic reinforcement and actuator have been developed. The general orthotropy of the constituent materials is very important from a practical point of view and renders the mathematical problem at hand much more complex. The AHM decouples the microscopic characteristics of the composite from its macroscopic behaviour so that each problem can be handled separately. The solution of the microscopic problem leads to the determination of the effective coefficients of piezoelectric and thermal expansion which are universal in nature and can be used to study a wide variety of boundary value problems. The FEA has been also developed and used to examine the aforementioned smart structure. The electro-thermo-mechanical deformations from the finite element simulations have been used to calculate the effective piezoelectric and thermal moduli of the structures. The results are compared and a very good agreement is shown between the two models and this indicates that the developed finite element model can be further extended to study more complex unit cell geometries.

Acknowledgment

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