

# Measuring the Efficiency of Medical Equipment

Panagiotis H. Tsarouhas

**Abstract**—the reliability analysis of the medical equipments can help to increase the availability and the efficiency of the systems. In this manuscript we present a simple method of decomposition that could be easily applied on the complex medical systems. Using this method we can easily calculate the effect of the subsystems or components on the reliability of the overall system. Furthermore, to investigate the effect of subsystems or components on system performance, we perform a numerical study varying every time the worst reliability of subsystem or component with another which has higher reliability. It can also be useful to engineers and designers of medical equipment, who wishes to optimize the complex systems.

**Keywords**—Reliability, Availability, Series-parallel System, medical equipment.

## I. INTRODUCTION

**M**EDICAL engineering is needed for the healthcare industry, that has a turnover approaching \$145 billion annually and is currently expanding approximately at a rate of 7% annually. The reliability of technological systems is very important for medical engineering. Failure of medical systems can result in negative effects that would be injury or death of a patient and can have serious legal implications. Thus, the problem arises of reliable medical devices or equipment that may have a high level of reliability. Failure intensity increases with age of medical equipment; therefore the equipment requires technological repair and monitoring. Toporkov [1] reported up to 80% of medical equipment presently used in public health organizations is worn-out or obsolete, which makes it difficult to guarantee not only reliability and efficiency but also safety of medical equipment. Lucian and Leape [2] point out that evidence from various sources indicates that a number of hospitalized patients suffer treatment-caused injuries most of which are from system's failures.

The literature on the field of medical systems as a 'human error' that may cause injury or death among hospitalized patients is vast [3]-[6]. On the other hand the literature on medical systems that require the proper equipment and the operation is limited. Baker [7] analysed a database of failures of many types of medical equipment, to study the dependence of failure rate on equipment age and on time since repair. Toporkov [8] presented several criteria and methods for assessing reliability of medical equipment. Tavakoli et al [9] showed a case study to discuss when it is necessary to renew medical equipment, and repairing it with inappropriate parts causes severe iatrogenic problems. High technology of medical equipment makes reference to the most expensive

diagnostic and treatment tools such as magnetic imaging scanners, lithotripsy scanners etc. Polley and Shanklin [10] discussed findings and managerial implications of two sequential studies concerning the process of organizational buying for first time acquisition of technologically advanced and reliable medical equipment by hospitals. Balakrishnan [11] reported that companies which sell medical equipment gained about 15% of total revenues from annual maintenance contracts.

In order to be more competitive in the market, many designers and manufacturers are dedicated to improving the reliability of medical equipment or product components. Typical approaches to achieve higher system reliability are: (1) increasing the reliability of system components, and (2) using redundant components in various subsystems in the system [12], [13]. These approaches refer to series and parallel configurations that increase the reliability of the system together with raising the cost of purchase and installations and require particular maintenance. The second approach, in addition requires available space for the installation and additional capital.

There are numerous examples of parallel systems in which the backup system operates in the event of a failure in the primary system, e.g. critical control systems in aeroplanes, auxiliary power supply systems in hospitals. Another example is an electrical utility company using dual metering systems operating in a parallel mode, where if one metering system fails, even momentarily, the other metering system automatically starts operating, thus preventing monthly the loss of several thousand dollars of billable electricity watt-hours [14]. In a manufacturing process, two or more machines may operate in a parallel mode in an effort either to increase output or to improve production reliability where one machine serves in a redundant or backup capacity under conditions in which the other machine, either intentionally or unintentionally, ceases to operate [15], [16]. Goel and Singh [17] presented reliability analysis of a standby complex system having imperfect switchover device and availability analysis of manufacturing system in a dairy plant. Tsarouhas [18] examines the quality approach of reliability and presents a simple framework to identify it. Tsarouhas and Nazlis [19] presented a simple expression of reliability for industrial systems that affect the maintenance activity.

In this manuscript, we propose a simple method of decomposition of the original system, in two final subsystems in serial or in parallel configurations. Every final subsystem represents one or more subsystems of the original system. This simple method is more comprehensive, in order to investigate the performance of the overall system. The same method may be implementing two final components in series or in parallel configurations of the subsystem. Thus, we can separate the worst operation of subsystem or component from

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the rest of system, and we determine the combination of components with deferent reliabilities in the worst subsystem, in order to achieve maximum overall system reliability.

The rest of this paper is organized as follows. In Section II, we present the failure distribution that contains reliability, availability, maintainability functions, and Bathtub hazard rate. In Section III, we analysis the reliability network that consists of series and parallel configuration. In Section IV, we compare the series-parallel systems and finally, we draw conclusions in Section V.

II. THE FAILURE DISTRIBUTION

The reliability engineering systems has become an important issue during their design due to the demands of continuous operation without failure that can be very harmful i.e. medical systems. Usually the required reliability of engineering systems is specified during the design phase.

A. The reliability, availability, and maintainability functions

Reliability of a system is the probability that the item will perform its intended function throughout a specified time period when operated in a normal environment [20]. The reliability of a system at operational time  $t$  can be expressed as

$$R(t) = \Pr(T_f \geq t) \tag{1}$$

where the continuous random variable  $T_f$  is the time to failure. Also  $Reliability + Unreliability = 1$  is valid.

The hazard rate or failure rate function is often used in reliability, and is defined as

$$\lambda(t) = \frac{f(t)}{R(t)} \tag{2}$$

where the continuous random variable  $f(t)$  is the probability density function that describes the shape of the failure distribution.

The parameters of reliability are mean time to failure/repair, failure/repair rate and maximum number of failures in a specific time interval. Some of the reasons for failure occurrence may be i.e. undetectable defects, abuse, low safety factor etc.

The mean time to failure (MTTF) is defined by

$$MTTF = \int_0^{\infty} R(t) dt \tag{3}$$

The maintainability quantifies the repair time of the failed system and is defined as the probability that the failed system will be restored to its satisfactory operational state when maintenance is performed. Maintainability is related to the duration of outages.

$$M(t) = \Pr(T_r \leq t) \tag{4}$$

where the continuous random variable  $T_r$  is the time to repair. Also  $Maintainability + Unmaintainability = 1$  is valid.

The most important measure of maintainability is the mean time to repair (MTTR) that focuses on downtime, and is defined by

$$MTTR = \int_0^{\infty} (1 - M(t)) dt \tag{5}$$

Availability is the probability that a system is available for use when required. The availability depends on both reliability and maintainability because first of all failure and repair distribution must be defined. The average availability over the interval  $[0, T]$  is defined as follows [21]:

$$A(T) = \left(\frac{1}{T}\right) \int_0^T A(t) dt$$

The steady state or long-run equilibrium availability is defined

$$A = \lim_{T \rightarrow \infty} A(T) = \frac{MTTF}{MTTR + MTTF} \tag{6}$$

and  $Availability + Unavailability = 1$

High availability means high reliability with suitable maintainability which characterises the efficiency of the entire system. Therefore system availability can never be less than system reliability.

B. Bathtub hazard rate

The hazard or failure rate function shown in Figure 1 represents the failure behaviour of various engineering components because the failure rate of such components is a function of time. Because of its shape it is known as the 'bathtub curve', and is divided into three regions i.e. I, II, III.

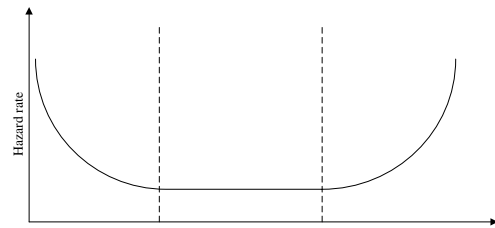


Fig. 1 Bathtub hazard rate curve

Region I is known as *burn-in* that has decreasing failure rate because failures are caused by manufacturing defects, poor workmanship and poor quality control. Region II is characterized as the *useful life*, during which the component failure rate remains constant due to random loads, human error, natural failures and abuse. Region III is called *wear-out* that has increasing failure rate because failures are caused by fatigue, aging, corrosion and friction.

Usually, many systems exhibit constant failure rate (Region II) and the failure distribution following the exponential probability distribution, thus the eqs (1)-(6) become:

$$R(t) = \exp(-\lambda t)$$

$$\lambda(t) = \lambda$$

$$MTTF = \frac{1}{\lambda}$$

$$M(t) = 1 - \exp(-rt)$$

$$MTTR = \frac{1}{r}$$

$$A(t) = \frac{r}{r + \lambda}$$

where  $\lambda$  is the constant failure rate of the component, and  $r$  is the constant repair rate of the component, and  $t$  is the operational time.

III.RELIABILITY NETWORK

A system may have various configurations or networks in performing reliability analysis. Components within a system may be related in two primary ways: series or parallel network.

A. Series Network

We consider the simple case of serial configuration that is the most commonly encountered reliability block diagram in engineering practice. In a serial configuration, all the consisting components of the system should be operating to maintain the required operation of the system. Thus, failure of any one component of the system will cause failure of the whole system. The series  $n$ -components are represented by the reliability block diagram of Figure 2. Let  $R_i(t)$  denote the reliability function of the component  $i$ , and we suppose that the system consists of  $n$  components. Then the reliability of the system for  $t$  hours of operation is given by [22]:

$$R_s(t) = R_1(t) * R_2(t) * \dots * R_n(t) = \prod_{i=1}^n R_i(t), i = 1, 2, 3, \dots, n \quad (7)$$

In relation (7) we assume that all the  $n$  components are independent, in other words the failure or no failure of one component does not change the reliability of the other component. Therefore the system operates if all the  $n$  mutually independent components in series operate, or:

$$R_s(t) = R_1(t) * R_2(t) * \dots * R_n(t) \leq \min\{R_1(t), R_2(t), \dots, R_n(t)\} \quad (8)$$



Fig. 2 Block diagram of an  $n$ -components series system

The reliability of the system  $R_s(t)$  can be no greater than the smallest component reliability.

In case each component has a constant failure rate  $\lambda_i$ , then the system reliability for eq. (7) is given by

$$R(t) = \exp\left(-\sum_{i=1}^n \lambda_i t\right) = \exp(-\lambda_s t) \quad (9)$$

where  $\lambda_s = \sum_{i=1}^n \lambda_i$

In Figure 3 we present the system's reliability for serial configuration as a function of reliability of each component. We use the equation (9) to compare the system reliability for different value of  $N$  in serial configuration. Let  $N$  denote the number of components that is contained in the system, and  $N=1, 10, 50, 100, 200, 300$  or  $400$ .

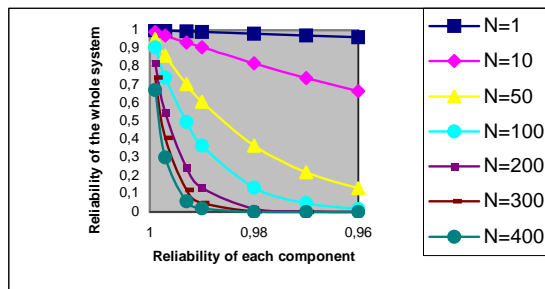


Fig. 3 Reliability of the whole System for serial configuration in terms of reliability of each component

We observed that for a system which consists of  $N=10$  components with 99% reliability each, the whole system's reliability turns out to be 90.43%, whilst a system which consists of  $N=100$  components with 99% reliability each, a whole system's reliability of 36.6% is presented. Consequently, complex medical equipment which consists of tens or even hundreds of individual components must have the highest reliability, especially if the system contains a large number of components.

B. Parallel Network

We consider the case of parallel configuration when two or more components are in parallel. The system operates if one or more components operate, and the system fails if all components fail. The parallel  $n$ -components are represented by the reliability block diagram of Figure 4. The reliability of the system for  $n$  parallel and independent components is the probability that at least one component does not fail, or [23] :

$$R_s(t) = 1 - \prod_{i=1}^n [1 - R_i(t)] \quad (10)$$

The  $R_s(t)$  can be greater than the most reliable component:

$$R_s(t) \geq \max\{R_1(t), R_2(t), \dots, R_n(t)\} \quad (11)$$

In case of all components have a constant failure rate then the system reliability for eq. (10) is given by

$$R_s(t) = 1 - \prod_{i=1}^n [1 - \exp(-\lambda_i t)] \quad (12)$$

where  $\lambda_i$  is the failure rate of  $i$ th component.

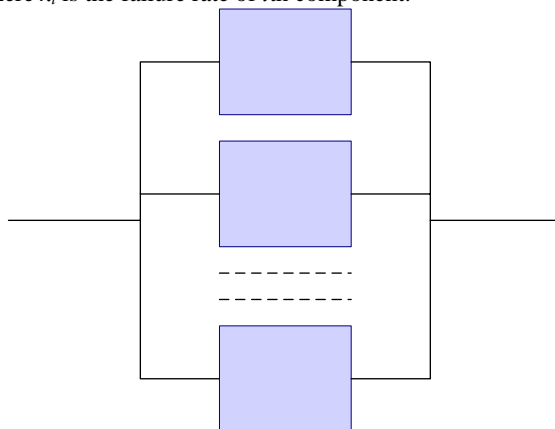


Fig. 4 Block diagram of a  $n$ -components parallel system

Therefore if we suppose a system with two-components identical and independent in parallel that have the same constant failure rate  $\lambda$  each. The reliability of this system from eq. (12) becomes:

$$R_s(t) = 1 - [1 - \exp(-\lambda t)][1 - \exp(-\lambda t)] = 2 \exp(-\lambda t) - \exp(-2\lambda t) \tag{13}$$

The Mean Time to Failure (*MTTF*) of a particular part of equipment (of the system) is defined as the mean time that elapses from the moment the equipment goes up and starts operating after a failure, until the moment it goes down again and stops operating due to a new failure. The *MTTF* of the system is:

$$MTTF = \int_0^{\infty} R_s(t) dt = \int_0^{\infty} [2 \exp(-\lambda t) - \exp(-2\lambda t)] dt = \tag{14}$$

$$\frac{2}{\lambda} - \frac{1}{2\lambda} = 1.5 \frac{1}{\lambda}$$

From eq. (14) it is clear that, the mean time until the next failure for two components in parallel configuration may be 50% more than from one component with the same failure rate  $\lambda$ .

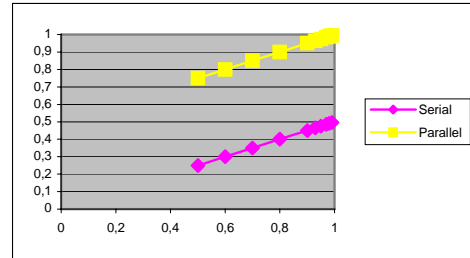
IV.COMPARISON SERIES-PARALLEL SYSTEMS

The Complex Systems consist of many subsystems and components in series-parallel configuration. The overall reliability of the complex system of components can be calculated by decomposing the system into a series of subsystems. Each subsystem should consist of a fairly simple system of components in series or in parallel. The reliabilities of each of the subsystems can then be calculated, and the overall system reliability is found by combining the subsystem reliabilities in the appropriate manner [24].

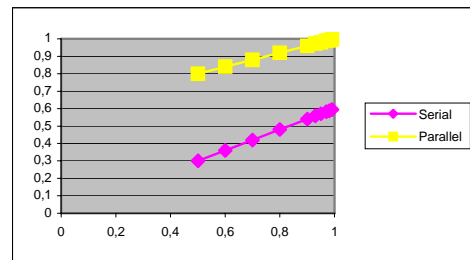
In this section, we use the eqs (7) and (10) to compare the system's reliability for serial and parallel configuration. The complex system after the decomposition in case of serial configuration becomes a serial system in two-subsystems or in case of parallel configuration, becomes a parallel system in two-subsystems. Thus, we separate the worst operation of subsystem from the rest of the system and we further investigate the performance on the overall system. The same assumption can be made for the subsystems in the complex system (i.e. medical system) that consists of many components. Let  $R_1$  and  $R_2$  denote the reliabilities that represent one or more components (or subsystems). To investigate the effect of  $R_1$  and  $R_2$  on the system reliability, we perform a numerical study varying  $R_1$  and  $R_2$  from 0.5 to 0.99. In Figure 5, we estimate the interaction of  $R_1$  and  $R_2$  on the affect of the system's reliability for both cases series-parallel network.

Also from Figure 5 we can make the following observations: (a) for high reliabilities of  $R_1$  and  $R_2 (>0.97)$ , we obtained higher reliability of the system for both serial-parallel configurations. The situation is better when it refers to parallel configuration where  $R_s$ , has a high value for shorter reliabilities (see Figure 5: f-h). Furthermore, engineers and designers must study all the parameters (cost, available space, etc.) to decide which configuration to choose. (b) The effect

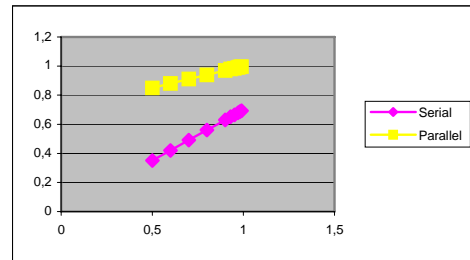
of  $R_2$  on the system reliability is very important. For reliability of  $R_2 > 0.8-0.99$ , we obtain rapid response in serial configuration, rather than in parallel configuration (see Figure 5: d-h). (c) For reliability of  $R_2 > 0.5-0.7$ , the reliability of the system has the same treatment for both configurations (see Figure 5: a-c).



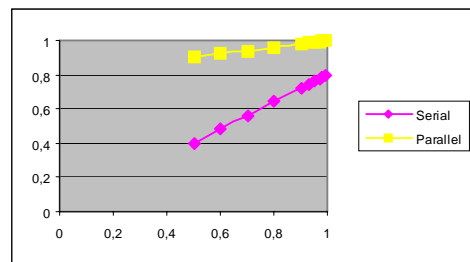
(a) System reliability for  $R_1$  increasing and  $R_2 = 0.5$



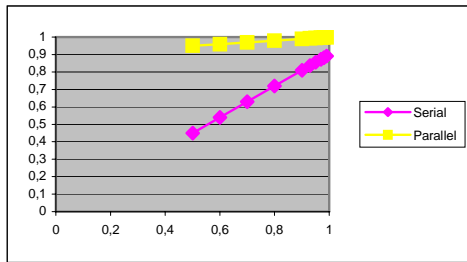
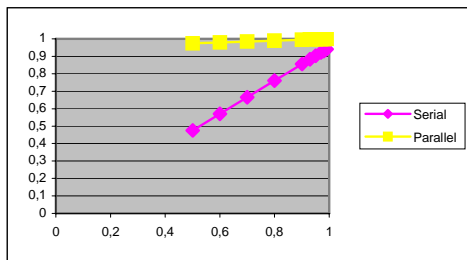
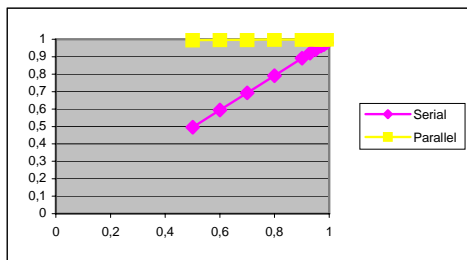
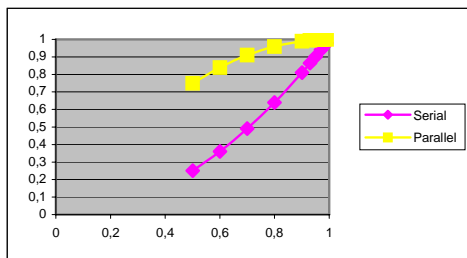
(b) System reliability for  $R_1$  increasing and  $R_2 = 0.6$



(c) System reliability for  $R_1$  increasing and  $R_2 = 0.7$



(d) System reliability for  $R_1$  increasing and  $R_2 = 0.8$

(e) System reliability for  $R_1$  increasing and  $R_2 = 0.9$ (f) System reliability for  $R_1$  increasing and  $R_2 = 0.95$ (g) System reliability for  $R_1$  increasing and  $R_2 = 0.99$ (h) System reliability for  $R_1$  increasing and  $R_2$  increasingFig. 5 Diagrams for different values of  $R_1$  and  $R_2$ 

In other words, the  $R_2$  play a significant role on the performance's system, especially in parallel configuration. Finally, we obtain the same results and diagrams if we maintain the reliability of  $R_1$  constant and change the reliabilities of  $R_2$ .

## V.CONCLUSIONS

In this manuscript we presented a simple method of decomposition of the complex medical system. This method can easily calculate the effect of the subsystems or components on the reliability of the overall system. In addition, to investigate the effect of subsystems or

components on system performance, we performed a numerical study varying every time the worst reliability of a subsystem or component with another which has higher reliability, and we observed that:

- given the reliability of medical equipment the more components the system has, the higher the reliability of each component may be
- for low reliabilities of subsystems or components the parallel configuration is preferable instead of the serial configuration
- for high reliabilities of subsystems or components we can use both parallel or serial configuration
- to maintain the same reliability of the overall system, the serial configuration could have higher reliabilities of subsystems or components, whereas the parallel configuration could have shorter reliabilities.

Before the engineer or designer makes a decision either to replace a subsystem (component) or to decide to select the parallel configuration of medical equipment, it would be better to evaluate the cost of replacement and installation in relation to the efficiency of the system and the availability of space for the installation.

## REFERENCES

- [1] A.A. Toporkov. Assessment of Service Life of Sophisticated Medical Equipment. *Biomedical Engineering*, Vol. 41, No 3, p.p. 122-127, 2007 .
- [2] L. Lucian., M.D. Leape. A systems Analysis Approach to medical error. *Journal of Evaluation in Clinical Practice*, Vol. 3, No. 3, pp. 213-222, 1997.
- [3] S.E Bedell., D.C Deitz, D. Leeman & T.L. Delbanco. Incidence and characteristics of preventable iatrogenic cardiac arrests. *Journal of the American Medical Association*, 265, 2815-2820, 1991.
- [4] F. Rosner, J.T. Berger, P. Kark, J. Potash, and A.J. Bennett. Disclosure and prevention of medical errors. *Archives of Internal Medicine*, Vol. 160, pp. 2089-92, 2000.
- [5] J. Reason. *Human Error*. Cambridge University Press, Cambridge, 1990.
- [6] L.L. Leape, D.W. Bates, D.J. Cullen, J. Cooper, H.J. Demonaco, T. Gallivan, R. Hallisey, J. Ives, N. Laird , G. Laffel , N. Nemeskal, L.A. Petersen, K. Porter, D. Servi, B.F. Shea, S. Small, B. Sweitzer, B.T. Thompson & M. Vander Vliet . Systems analysis of adverse drug events. *Journal of the American Medical Association*, 274, 35-43, 1995.
- [7] R.D. Baker. Data-based modelling of the failure rate of repairable equipment. *Lifetime Data Analysis*, 7, 65-83, 2001.
- [8] A.A. Toporkov. Criteria and Methods for Assessing Reliability of Medical Equipment. *Biomedical Engineering*, Vol. 42, No 1, p.p. 11-16, 2008.
- [9] H. Tavakoli, M. Karami, J. Rezai, K. Esfandiari, and P. Khashayar. When Renewing Medical Equipment is necessary: a case report. *International Journal of Health care Quality Assurance*, Vol. 20, No 7, p.p. 616-619, 2007.
- [10] J.P Polley., and L.W. Shanklin. Marketing High-technology Medical equipment to Hospitals. *Journals of Business & Industrial Marketing*, Vol. 8, No. 4, pp. 32-42, 1993.
- [11] S. Balakrishnan.. Quality for Life in the Medical Equipment Industry: a case study. *Competitiveness Review*, 9 (2), pp. 36-48, 1999.
- [12] W. Kuo, V.R. Prasad. An annotated overview of system-reliability optimization. *IEEE Transactions on Reliability*, 49(2), 176-87, 2000.
- [13] Y.C. Hsieh, T.C. Chen, D.L. Bricker. Genetic algorithm for reliability design problems. *Microelectronics Reliability*, 38, 1599-605., 1998
- [14] "Redundant metering operates reliably". *Electrical World*, June, 78-82, 1992
- [15] G. Taguchi., A.E. Elsayed, and T. Hsiang. *Quality Engineering in Production Systems*, McGraw-Hill, New York, NY, pp. 135-138, 1989.
- [16] H.J. Weiss, and M.E. Gershon. *Production and Operations Management*, 2nd ed., Allyn and Bacon, Boston, MA, 1993.

- [17] P. Goel, and J. Singh. Reliability analysis of a standby complex system having imperfect switch-over device. *Microelectron Reliability*, 35, 285-8, 1995.
- [18] P. Tsarouhas. Reliability and Maintenance Management for Industrial Systems. *Asian Journal of Information Technology*, 4(5), 498-501., 2005.
- [19] P. Tsarouhas, D. Nazlis. Industrial Systems Maintenance under the Light of Reliability. *Information Technology Journal*, Vol. 5, No. 1, pp. 13-17, 2006.
- [20] I.W.R. Brischke, D.N.P. Murthy. Case study in reliability and maintenance. John Wiley & Sons, Hoboken, New Jersey, 2003.
- [21] C.E. Ebeling. *An Introduction to Reliability and Maintainability Engineering*. McGraw Hill, New York, NY, 1997.
- [22] I.Bazovsky. *Reliability Theory and Practice*. Prentice-Hall, Englewood Cliffs, NJ, 1961.
- [23] U. D. Kumar, J. Crocker, J. Knezevic, , M. El-Haram.. *Reliability, maintenance and logistic support-A life cycle approach*. Kluwer Academic Press, Boston, MA, 2000.
- [24] A. Hayter, (2002). *Probability and statistics for engineers and scientists*. 2<sup>th</sup> edition. Duxbury, CA, USA, 2002.

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