International Journal of Information, Control and Computer Sciences
ISSN: 2517-9942
Vol:8, No:5, 2014

Maximum Induced Subgraph of an Augmented Cube

Meng-Jou Chien, Jheng-Cheng Chen, Chang-Hsiung Tsai

***Abstract*—**Let $max_{\xi_G}(m)$ denote the maximum number of edges in a subgraph of graph G induced by m nodes. The n -dimensional augmented cube, denoted as AQ_n, a variation of the hypercube, possesses some properties superior to those of the hypercube. We study the cases when G is the augmented cube AQ_n. In this paper, we show that $max_{\xi_{AQ_n}}(m)=\sum_{i=0}^{r}(p_i+2i-\frac{1}{2})2^{p_i}$, where $p_0>p_1>\cdots>p_r$ are nonnegative integers defined by $m=\sum_{i=0}^{r}2^{p_i}$ and $m\geq 2$. We then apply this formula to find the bisection width of AQ_n.

***Keywords*—**Interconnection network, Augmented cube, Induced subgraph, Bisection width.

I. Introduction

THEtopology of an interconnection network is conveniently represented by an undirected simple graph $G=(V,E)$, where $V(G)$ and $E(G)$ is the vertex set and the edge set of G, respectively. For graph terminology and notation not defined here we refer the reader to [8]. There are a lot of interconnection network topologies proposed in literature [4]. Among these topologies, the n -dimensional hypercube, denoted by Q_n, is a popular one. Many variants of the hypercube have been proposed. The augmented cube, proposed by Choudum and Sunitha [3], is one of such variations. An n -dimensional augmented cube AQ_n can be formed as an extension of Q_n by adding some links. For any positive integer n, AQ_n is a vertex transitive, $(2n-1)$ -regular, and $(2n-1)$ -connected graph with 2^n vertices. AQ_n retains all favorable properties of Q_n since $Q_n\subset AQ_n$. Moreover, AQ_n possesses some embedding properties that Q_n does not. Previous works relating to the augmented cube can be found in [1], [2], [5], [6], [7], [9].

Let $max_{\xi_G}(m)$ denote the maximum number of edges in a subgraph of graph G induced by m nodes. Determining $max_{\xi_G}(m)$ for typical graph G not only is interesting in its own right, but the result has applications in the evaluation of bandwidth and fault tolerant of G [11]. Abdel-Ghaffar [10] solved this problem for hypercube and Yang et al. [12] solved it for recursive circulant graph $G(2^n,4)$ which is one of various of hypercubes. In this paper, we show that $max_{\xi_{AQ_n}}(m)=\sum_{i=0}^{r}(p_i+2i-\frac{1}{2})2^{p_i}$, where $p_0>p_1>\cdots>p_r$ are nonnegative integers defined by $m=\sum_{i=0}^{r}2^{p_i}$ and $m\geq 2$. We then apply this formula to find the bisection width of AQ_n.

The rest of this paper is organized as follows: In Section II, provides formal definition of AQ_n. A useful function is given and study its properties in Section III. By exploiting these properties, we show $max_{\xi_{AQ_n}}(m)=\sum_{i=0}^{r}(p_i+2i-\frac{1}{2})2^{p_i}$ in Section IV. Finally, the formula is applied to determine the bisection width of AQ_n in Section V.

Meng-Jou Chienand Jheng-Cheng Chen are with the Computer Science and Information Engineering Department, National Dong HwaUniversity,Shoufeng, Hualien 97401, Taiwan, R.O.C. (phone: 886-3863-4002; fax: 886-3863-4002; e-mail: 610121003@ems.ndhu.edu.tw,p_p971@hotmail.com).

Chang-Hsiung Tsaiis with the Computer Science and Information Engineering Department, National Dong HwaUniversity,Shoufeng, Hualien 97401, Taiwan, R.O.C. (phone: 886-3863-4001; fax: 886-3863-4001; e-mail: chtsai@mail.ndhu.edu.tw).

II. Preliminaries

Let $G=(V,E)$ be a graph, and $V(G)$ and $E(G)$ denote vertex set and edge set of graph G, respectively. For $U\subseteq V(G)$, the subgraph of G induced by U, denoted by $G[U]$, is a graph with vertex set U and all the edges of G with both vertices in U. An*m-induced subgraph* of a graph is one that is induced by m vertices. A *maximum m-inducedsubgraph* of a graph is one that has the maximum number of edges. Let $max_{\xi_G}(m)$ denote the maximum number of edges in an m -induced subgraph of graph G. Let $\xi(U)$ denote the number of edges of $G[U]$. For a pair of disjoint vertex subsets U_1 and U_2 of graph G, let $\xi(U_1,U_2)$ denote the number of edges joining U_1 and U_2.

Let $n\geq 1$ be an integer. The graph of the n -dimensional augmented cube [3], denoted by AQ_n has 2^n vertices, each labeled by an n -bit binary string $V(AQ_n)=\{u_1u_2\dots u_n \mid u_i\in\{0,1\}\}$. AQ_1 is the graph K_2 with vertex set $\{0,1\}$. For $n\geq 2$, AQ_n can be recursively constructed by two copies of AQ_{n-1}, denoted by AQ_{n-1}^0 and AQ_{n-1}^1 and by adding 2^n edge between AQ_{n-1}^0 and AQ_{n-1}^1 as follows:

Let $V(AQ_{n-1}^0)=\{(0u_2u_3\dots u_n)\mid u_i=0 \text{ or } 1 \text{ for } 2\leq i\leq n\}$ and $V(AQ_{n-1}^1)=\{(1v_2v_3\dots v_n)\mid u_i=0$ or 1 for $2\leq i\leq n\}$. A vertex $u=(0u_2u_3\dots u_n)$ of AQ_{n-1}^0 is joined to a vertex

$v = (1v_2v_3 \dots v_n)$ of AQ^1_{n-1} if and only if either (i) $u_i = v_i$ for $2 \le i \le n$; in this case, (u,v) is called a hypercube edge, or (ii) $u_i = \bar{v}_i$ for $2 \le i \le n$; in this case, (u,v) is called a complement edge.

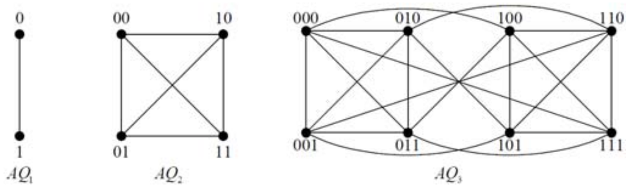

Fig.1 The augmented cubes: AQ_1, AQ_2, and AQ_3

The augmented cubes AQ_1, AQ_2, and AQ_3 are illustrated in Fig. 1. It is proved in [3] that AQ_n is a vertex transitive, $(2n-1)$ -regular, and $(2n-1)$ -connected graph with 2^n vertices for any positive integer n.

Any positive integer m can be uniquely represented by $m = \sum_{i=0}^{r} 2^{p_i}$, where $p_0 > p_1 > \dots > p_r \ge 0$. We define a useful function

$$f(m) = \begin{cases} 0 & : m \le 1 \\ \sum_{i=0}^{r} (p_i + 2i - \frac{1}{2}) 2^{p_i} & : m \ge 2 \end{cases}$$

As an example, for $m = 148 = 2^7 + 2^4 + 2^2$, we have $f(148) = (7 + 0 - \frac{1}{2})2^7 + (4 + 2 - \frac{1}{2})2^4 + (2 + 4 - \frac{1}{2})2^2 = 942$

Theorem 1 *For any* $n \ge 1$ *and* $0 < m \le 2^n$, *we have* $max_{\xi_{AQ_n}}(m) = f(m)$.

We drive several properties of the function $f(m)$ which are used to prove Theorem 1 in following sections and also give an explicit set U of vertices such that $\xi(U) = g(m)$.

III. Properties of $f(m)$

For a positive integer m, we define $l(m) = \lfloor log_2 m \rfloor$ and $m' = m - 2^{l(m)}$.Obviously, $2^{l(m)} \le m < 2^{l(m)+1}$ and $0 \le m' < \frac{m}{2}$.

Proposition 1 *Let* m *be a positive. Then,* $f(m) = f(2^{l(m)}) + f(m') + 2m'$

Proof. We may write $m = 2^{p_0} + 2^{p_1} + \dots + 2^{p_r}$ for some integer $r \ge 0$ and $p_0 > p_1 > \dots > p_r \ge 0$. Clearly, $l(m) = p_0$. From the definition of $f(m)$, $f(m) = (2l(m)-1)2^{l(m)-1} + \sum_{i=1}^{r}(p_i + 2i - \frac{1}{2})2^{p_i}$.

Since $m' = 2^{p_1} + 2^{p_2} + \dots + 2^{p_r}$, we also have $f(m') = \sum_{i=1}^{r}[p_i + 2(i-1) - \frac{1}{2}]2^{p_i}$.

We conclude from the above that $f(m) = (2l(m)-1)2^{l(m)-1} + f(m') + \sum_{i=1}^{r} 2 \times 2^{p_i} = f(2^{l(m)}) + f(m') + 2m'$ because $f(2^{l(m)}) = (2l(m)-1)2^{l(m)-1}$.

Proposition 2 *For any positive integers* m_1 *and* m_2, *we have* $f(m_1 + m_2) \ge f(m_1) + f(m_2) + 2min\{m_1, m_2\}$.

Proof. Clearly equality holds for $m_1 = 1$ or $m_2 = 1$. The proof is by induction on $m_1 + m_2$. Without loss of generality, we may assume that $m_1 \ge m_2 \ge 2$. In particular, we want to prove that $f(m_1 + m_2) \ge f(m_1) + f(m_2) + 2m_2$, where the induction hypothesis implies that

$$f(m_1' + m_2) \ge f(m_1') + f(m_2) + 2min\{m_1', m_2\} \quad (1)$$

$$f(m_1' + m_2') \ge f(m_1') + f(m_2') + 2min\{m_1', m_2'\} \quad (2)$$

Notice that $2^{l(m_1)} \le m_1 \le m_1 + m_2 \le 2m_1 < 2^{l(m_1)+2}$ and, in particular, $l(m_1 + m_2)$ equals either $l(m_1)$ or $l(m_1)+1$. We consider all possible cases:

Case 1: $l(m_1 + m_2) = l(m_1)$

In this case, $(m_1 + m_2)' = m_1 + m_2 - 2^{l(m_1+m_2)} = m_1 + m_2 - 2^{l(m_1)} = m_1' + m_2$. Proposition 1 gives $f(m_1) = (2l(m_1)-1)2^{l(m_1)-1} + f(m_1') + 2m_1'$ and

$$f(m_1 + m_2) = (2l(m_1 + m_2) - 1)2^{l(m_1+m_2)-1} + f((m_1 + m_2)') + 2(m_1 + m_2)'$$
$$= (2l(m_1) - 1)2^{l(m_1)-1} + f(m_1' + m_2) + 2(m_1' + m_2)$$

Hence,

$$f(m_1 + m_2) = f(m_1) - f(m_1') + f(m_1' + m_2) + 2m_2$$
$$\ge f(m_1) + f(m_2) + 2min\{m_1', m_2\} + 2m_2$$
$$\ge f(m_1) + f(m_2) + 2m_2$$

, where the first inequality follows from (1).

Case 2: $l(m_1 + m_2) = l(m_1) + 1$ and $l(m_1) = l(m_2)$

In this case, $(m_1 + m_2)' = (m_1 + m_2) - 2^{l(m_1+m_2)} = m_1 + m_2 - 2^{l(m_1)+1}$ $= m_1 - 2^{l(m_1)} + m_2 - 2^{l(m_2)} = m_1' + m_2'$. Proposition 1 gives $f(m_1) = (2l(m_1)-1)2^{l(m_1)-1} + f(m_1') + 2m_1'$, $f(m_2) = (2l(m_2)-1)2^{l(m_2)-1} + f(m_2') + 2m_2'$ and

$$f(m_1 + m_2) = (2l(m_1 + m_2) - 1)2^{l(m_1+m_2)-1} + f((m_1 + m_2)') + 2(m_1 + m_2)'$$
$$= (2l(m_1) + 1)2^{l(m_1)} + f(m_1' + m_2') + 2m_1' + 2m_2'.$$

Since $l(m_1) = l(m_2)$ and $m_1 \ge m_2 \ge 2$ implies $m_1' \ge m_2' \ge 0$, we have

$$f(m_1 + m_2) = f(m_1) + f(m_2) + 2^{l(m_1)+1} + f(m_1' + m_2') - f(m_1') - f(m_2'),$$
$$\ge f(m_1) + f(m_2) + 2^{l(m_1)+1} + 2m_2' = f(m_1) + f(m_2) + 2m_2$$

where the inequality follows from (2).

Case 3: $l(m_1 + m_2) = l(m_1) + 1$ and $l(m_1) > l(m_2)$

In this case, $(m_1 + m_2)' = (m_1 + m_2) - 2^{l(m_1+m_2)} = m_1 + m_2 - 2^{l(m_1)+1}$ $= m_1 - 2^{l(m_1)} + m_2 - 2^{l(m_1)} = m_1' + m_2 - 2^{l(m_1)}$. Furthermore, as $2^{l(m_1)+1} = 2^{l(m_1+m_2)} \le m_1 + m_2 < 2^{l(m_1)+1} + 2^{l(m_2)+1} \le 2^{l(m_1)+1} + 2^{l(m_1)}$, we get $2^{l(m_1)} \le m_1 + m_2 - 2^{l(m_1)} < 2^{l(m_1)+1}$.

Since $m_1' + m_2 = m_1 + m_2 - 2^{l(m_1)}$, we deduce that $l(m_1' + m_2) = l(m_1)$ and $(m_1' + m_2)' = (m_1' + m_2) - 2^{l(m_1' + m_2)} = m_1' + m_2 - 2^{l(m_1)}$.

Proposition 1 gives $f(m_1) = (2l(m_1) - 1)2^{l(m_1)-1} + f(m_1') + 2m_1'$,

$$f(m_1 + m_2) = (2l(m_1 + m_2) - 1)2^{l(m_1+m_2)-1} + f((m_1 + m_2)') + 2(m_1 + m_2)'$$
$$= (2l(m_1) + 1)2^{l(m_1)} + f(m_1' + m_2 - 2^{l(m_1)}) + 2m_1' + 2m_2 - 2^{l(m_1)+1}$$

and

$$f(m_1' + m_2) = (2l(m_1' + m_2) - 1)2^{l(m_1'+m_2)-1} + f((m_1' + m_2)') + 2(m_1' + m_2)'$$
$$= (2l(m_1) - 1)2^{l(m_1)-1} + f(m_1' + m_2 - 2^{l(m_1)}) + 2m_1' + 2m_2 - 2^{l(m_1)+1}.$$

The above expressions for $f(m_1)$, $f(m_1+m_2)$, and $f(m_1'+m_2)$ yield

$$\begin{aligned} f(m_1 + m_2) &= f(m_1' + m_2) + (2l(m_1) + 3)2^{l(m_1)-1} \\ &= f(m_1) + f(m_1' + m_2) - f(m_1') - 2m_1' + 2^{l(m_1)+1}, \\ &\geq f(m_1) + f(m_2) + 2min\{m_1', m_2\} - 2m_1' + 2^{l(m_1)+1} \\ &= f(m_1) + f(m_2) + 2min\{2^{l(m_1)}, m_2 - m_1' + 2^{l(m_1)}\} \end{aligned}$$

where the inequality follows from (1). Since $m_1' < m_1/2 < 2^{l(m_1)}$ and $m_2 < 2^{l(m_2)+1} \leq 2^{l(m_1)}$, we have $min\{2^{l(m_1)}, m_2 - m_1' + 2^{l(m_1)}\} \geq min\{2^{l(m_1)}, m_2\} = m_2$.

Therefore, $f(m_1 + m_2) \geq f(m_1) + f(m_2) + 2min\{m_1, m_2\}$.

IV. Proof of Theorem 1

A partition of a set S is a collection of disjoint subsets of S whose union equals S. Then the following lemma is obviously.

Lemma 1 *[12] Let U be a vertex subset of graph G. Let $\{U_0, U_1, \ldots, U_k\}$ be a partition of U. Then* $\xi(U) = \sum_{i=0}^{k} \xi(U_i) + \sum_{0 \leq i < j \leq k} \xi(U_i, U_j)$.

Let U be a set of vertices on the AQ_n, let $U^{(a)} = U \cap V(AQ_{n-1}^a)$ where $a = 0$ or 1. We have the following observation.

Lemma 2 *For a set U of vertices on AQ_n, $n > 1$, we have* $\xi(U) \leq \xi(U^{(0)}) + \xi(U^{(1)}) + 2min\{|U^{(0)}|, |U^{(1)}|\}$.

Proof. Since $\{U^{(0)}, U^{(1)}\}$ is a partition of U, by Lemma 1, $\xi(U) = \xi(U^{(0)}) + \xi(U^{(1)}) + |\xi(U^{(0)}, U^{(1)})|$. Without loss of generality, we may assume that $|U^{(0)}| \leq |U^{(1)}|$. One can observe that $U^{(0)}$ and $U^{(1)}$ are vertex subsets of AQ_{n-1}^0 and AQ_{n-1}^1 respectively. The proof is divided into two parts as follows.

Case 1: $|U^{(0)}| = 0$.

This implies $U = U^{(1)}$. It is obvious that $\xi(U^{(0)}) = 0$ and $min\{|U^{(0)}|, |U^{(1)}|\} = 0$. Thus $\xi(U) \leq \xi(U^{(0)}) + \xi(U^{(1)}) + 2min\{|U^{(0)}|, |U^{(1)}|\}$.

Case 2: $|U^{(0)}| \neq 0$.

By definition, every vertex of AQ_{n-1}^0 connects to exactly two vertices of AQ_{n-1}^1. Hence, for any vertex $u \in U^{(0)}$, at most two vertices in $U^{(1)}$ are adjacent to u. Therefore, $\xi(U^{(0)}, U^{(1)}) \leq 2|U^{(0)}|$. As a result, $\xi(U) \leq \xi(U^{(0)}) + \xi(U^{(1)}) + 2min\{|U^{(0)}|, |U^{(1)}|\}$.

Lemma 3 *For any integer $n \geq 1$ and $0 \leq m \leq 2^n$, we have* $max_{\xi_{AQ_n}}(m) \leq f(m)$.

Proof. It suffices to show that $\xi(U) \leq f(m)$ for every set $U \in V(AQ_n)$. The proof is induction on n. It is obviously true for $n = 1,2$. Suppose the claim is true for $n = k$. Let U be an arbitrary set of m vertices in AQ_n. Thus $\{U^{(0)}, U^{(1)}\}$ is a partition of U, and $U^{(0)} \subseteq V(AQ_{n-1}^0)$ and $U^{(1)} \subseteq V(AQ_{n-1}^1)$. By Lemma 2, the induction hypothesis, and Proposition 2, we have

$$\begin{aligned} \xi(U) &\leq \xi(U^{(0)}) + \xi(U^{(1)}) + 2min\{|U^{(0)}|, |U^{(1)}|\} \\ &\leq f(|U^{(0)}|) + f(|U^{(1)}|) + 2min\{|U^{(0)}|, |U^{(1)}|\} \\ &\leq f(|U^{(0)}| + |U^{(1)}|) \\ &= f(m). \end{aligned}$$

Thus the lemma is proved.

Next, we give for any integer $n \geq 1$ and $0 \leq m \leq 2^n$, a set, denoted by $U_{m,n}$, of m vertices on the AQ_n for which $\xi(U_{m,n}) = f(m)$. The set $U_{m,n}$ is defined by

$U_{m,n} = \{(s_1 s_2 \cdots s_n) \in V(AQ_n) \mid \sum_{i=1}^{n} s_i 2^{i-1} < m\}$, i.e., $U_{m,n}$ consists of all vectors that are binary expansions of nonnegative integers less than m.

Lemma 4 *For any integer $n \geq 1$ and $0 \leq m \leq 2^n$, we have* $\xi(U_{m,n}) = f(m)$.

Proof. The proof is induction on n. Clearly the statement holds for $n = 1$. Suppose the claim is true for $n \leq k-1$. Now we consider the following three cases when $n = k$.

Case 1: $0 \leq m \leq 2^{k-1}$

In this case, $U_{m,k}^{(0)} = U_{m,k-1}$, $m = |U_{m,k}| = |U_{m,k}^{(0)}|$, and $U_{m,k}^{(1)}$ is empty. By Lemma 2, we have $\xi(U_{m,k}) = \xi(U_{m,k}^{(0)}) = \xi(U_{m,k-1})$. By induction hypothesis, $\xi(U_{m,k-1}) = f(m)$; this implies $\xi(U_{m,k}) = f(m)$.

Case 2: $2^{k-1} < m \leq 2^k$

In this case, $U_{m,k}^{(0)} = V(AQ_{k-1}^0)$ and $|U_{m,k}^{(1)}| = m'$ where $m' = m - 2^{k-1}$. Thus for any vertex $u \in U_{m,k}^{(0)}$, there are exactly two vertices in $U_{m,k}^{(1)}$ adjacent to u. This implies $\xi(U_{m,k}^{(0)}, U_{m,k}^{(1)}) = 2|U_{m,k}^{(1)}| = 2m'$.

Since $\{U_{m,k}^{(0)}, U_{m,k}^{(1)}\}$ is a partition of $U_{m,k}$, by Lemma 1, $\xi(U_{m,k}) = \xi(U_{m,k}^{(0)}) + \xi(U_{m,k}^{(1)}) + \xi(U_{m,k}^{(0)}, U_{m,k}^{(1)})$. By the induction hypothesis, we have

$$\begin{aligned}\xi(U_{m,k}) &= \xi(U_{m,k}^{(0)}) + \xi(U_{m,k}^{(1)}) + \xi(U_{m,k}^{(0)}, U_{m,k}^{(1)}) \\ &= f(|U_{m,k}^{(0)}|) + f(|U_{m,k}^{(1)}|) + \xi(U_{m,k}^{(0)}, U_{m,k}^{(1)}) \\ &= f(2^{k-1}) + f(m') + 2m'.\end{aligned}$$

Therefore, by Proposition 1, $\xi(U_{m,k}) = f(m)$ because $l(m) = k-1$.

Case 3: $m = 2^k$

In this case, $U_{m,k}$ contain all the vertices in the AQ_k and $\xi(U_{m,k}) = (2k-1)2^{k-1}$. By definition of $f(m)$, we have $f(2^k) = (k-\frac{1}{2})2^k = (2k-1)2^{k-1}$. Hence, $\xi(U_{m,k}) = f(m)$.

From Lemma 3 and Lemma 4, we have $max_{\xi_{AQ_n}}(m) = \xi(U_{m,n}) = f(m)$. Thus Theorem 1 is proved.

V. Application to Bisection Width

The bisection width of graph G, denoted by $bisection(G)$, is the minimum cardinality of an edge cut of G that splits G into two equally-size parts. The arm of this section is to determine the bisection width of AQ_n.

Lemma 5 *For a set U of vertices of n-regular graph G, we have* $\xi(U, V(G)-U) = n \times |U| - 2\xi(U)$.

Theorem 2 *For any integer n, we have* $bisection(AQ_n) = 2^n$

Proof. The proof is obviously true for $n = 1,2$. Suppose $n \geq 3$. For any set U of 2^{n-1} vertices of AQ_n, by Lemma 5 and Theorem 1 that

$$\begin{aligned}\xi(U, V(AQ_n)-U) &= (2n-1)\times 2^{n-1} - 2\xi(U) \\ &\geq (2n-1)\times 2^{n-1} - 2\times f(2^{n-1}) \\ &= (2n-1)\times 2^{n-1} - 2(2n-3)2^{n-2} \\ &= 2^n.\end{aligned}$$

Thus, $bisection(AQ_n) \geq 2^n$. On the other hand, let $U = V(AQ_{n-1}^0)$. Then $|U| = 2^{n-1}$ and $\xi(U, V(AQ_n)-U) = 2^n$. Therefore, we have $bisection(AQ_n) = 2^{n-1}$.

Acknowledgment

This work was supported in part by the National Science Council of the Republic of China under Contract NSC 102-2115-M-259-006-.

References

[1] M. Chen, "The distinguishing number of the augmented cube and hypercube powers," Discrete Mathematics, 308(11):2330-2336, 2008.
[2] N. W. Chang, S. Y. Hsiueh, "Conditional diagnosability of augmented cubes under the PMC model,"IEEE Transactions on Dependable and Secure Computing, 9(1):46-60, 2012.
[3] S. A. Choudum, V. Sunitha, Khaled A.S., "Augmented cubes," Networks, 40(2):71-84, 2002.
[4] T. Y. Feng, "A survey of interconnection networks,"IEEE Computer, 14:12-27, 1981.
[5] S. Y. Hsieh, J. Y. Shiu, "Cycle embedding of augmented cubes,"Applied Mathematics and Computation, 191:314-319, 2007.
[6] H. C. Hsu, L. C. Chiang, J. M. Tan, L. H. Hsu, "Fault hamiltonicity of augmented cubes,"Parallel Computing, 31:131-145, 2005.
[7] H. C. Hsu, P. L. Lai, C. H. Tsai, "Geodesic pancyclicity and balanced pancyclicity of augmented cubes,"Information Processing Letters, 101:227-232, 2007.
[8] F. T. Leighton,Introduction to Parallel Algorithms and Architectures: arrays, trees, hypercubes. San Mateo: Morgan Kaufman, 1992.
[9] M. Ma, G. Liu, J. M. Xu, "The super connectivity of augmented cubes,"Information Processing Letters, 106:59-63, 2008.
[10] K. A. S., Abdel-Ghaffar, "Maximum number of edges joining vertices on a cube,"Information processing letters, 87(2):95-99, 2003.
[11] J. Xu,Topological Structure and Analysis of Interconnection networks, Kluwer Academic Publishers, Dordrecht, 2002.
[12] X. Yang, D. J. Evans, G. M. Megson, "Maximum induced subgraph of a recursive circulant,"Information Processing Letters, :293-298, 2005.

Meng-JouChien is currently working toward the Master degree in Computer and Information Science and Engineering at the National Dong Hwa University. She received her BS degree in Computer Science and Information Engineering, National Dong Hwa University, Taiwan, in 2012. Her research interests include interconnection networks and optical communication.

Jheng-Cheng Chen received the BS degree in Information Engineering from Dahan Institute of Technology, Taiwan, in 2007 and the Master degree in Graduate Institute Of Learning Technology National Dong Hwa University, Taiwan, in 2009, respectively. He is currently working toward the Ph.D degree in Computer Science and Information Engineering at the National Dong Hwa University. His primary research interests include Graph theory and interconnection networks.

Chang-Hsiung Tsai received the BS degree in mathematics from Chung Yuan Christian University in 1989, and the MS and PhD degrees from National Chiao Tung University in 1991 and 2002, respectively. He is now with the Department of Computer Science and Information Engineering, National Dong Hwa University. His research interests include graph theory and its applications to interconnection networks, particularly fault diagnosis of network systems and graph embedding.