International Journal of Engineering, Mathematical and Physical Sciences
ISSN: 2517-9934
Vol:5, No:9, 2011

Mathematical modeling of an avalanche release and estimation of flow parameters by numerical method

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Abstract—Avalanche release of snow has been modeled in the present studies. Snow is assumed to be represented by semi-solid and the governing equations have been studied from the concept of continuum approach. The dynamical equations have been solved for two different zones [starting zone and track zone] by using appropriate initial and boundary conditions. Effect of density (ρ), Eddy viscosity (η), Slope angle (θ), Slab depth (R) on the flow parameters have been observed in the present studies. Numerical methods have been employed for computing the non linear differential equations. One of the most interesting and fundamental innovation in the present studies is getting initial condition for the computation of velocity by numerical approach. This information of the velocity has obtained through the concept of fracture mechanics applicable to snow. The results on the flow parameters have found to be in qualitative agreement with the published results.

Keywords—Snow Avalanche, Fracture mechanics, Avalanche Velocity, Avalanche Zones.

I. Introduction

SNOW avalanche is a sudden downward movement of snow from top hill resulting into tons and tons of snow arriving down the hill. The end effect is quite devastating and destructive to the property and sometimes even loss to human lives. The exact cause of an avalanche is really not known however, it is believed due to instability in the snow pack [1]. Also it is believed that there will be initiation of crack at the beginning and this crack will grow and result into an avalanche [2]. Temperature metamorphism within snow pack will result into instability due to growth of Depth hoar crystals on the ground and also due to an ice layer in the snow pack [3]. The studies on avalanche will certainly help in taking precaution against getting buried under an avalanche. It also helps toward protecting the structures when meant to protect either building (tunnels, roads, etc) or personnel. In view of the importance to human lives, structures and environment (forest, vegetations), studies on snow avalanche has been taken up in the present investigation.

II. Analysis

Avalanche release can be modeled in accounting the snow movement down the hill in three different zones: starting zone, track zone and run-out zone (Fig.1)

The basic governing equation, for the motion of the snow, down the hill is:

$$\rho\ u\ \frac{\partial u}{\partial x} = -\frac{\partial^2 u}{\partial x^2} + F \qquad (1)$$

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Manuscript revised January 5, 2011.

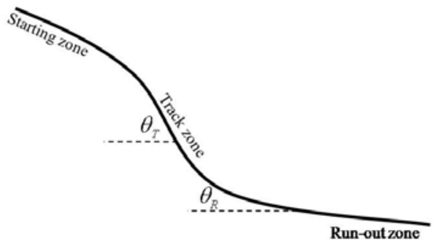

Fig. 1. Three stages of an avalanche release

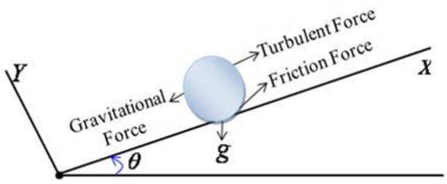

Fig. 2. Forces acting on a slab

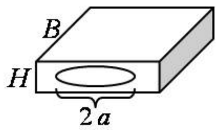

Fig. 3. Snow slab dimension

Where ρ is density of snow, μ is viscosity, u is velocity, F is body force, x is the distance down the hill. The negative sign in the first term on the right hand side of Equation (1) indicates the frictional forces acts in the opposite direction of the movement of an avalanche. Equation (1) can be further simplified by using (Fig.2):

$$\rho\ u\ \frac{\partial u}{\partial x} = -\frac{\partial^2 u}{\partial x^2} + \rho g\ (\sin\theta - \mu\cos\theta) - \frac{\rho g}{R\eta}\ u^2 \qquad (2)$$

Where η is Eddy viscosity, θ is slope angle and R is slab depth, $R = \frac{BH}{2H+B}$ since $H << B$ then R is approximated to H ($R \approx H$) (Fig. 3).

Equation (2) will be required to solve for two different zones: (I). Starting zone (II). Track zone.

In order to solve Equation (2), we need two initial or boundary conditions as following conditions:

1. $u|_{x=x_s} = 0.0029$
2. $\frac{\partial u}{\partial x}|_{x=x_s} = \lim_{x\to 0} \frac{u(x)-u(0)}{x-0} = \frac{u_\epsilon}{\epsilon_x}$

Where u_ϵ is fracture velocity, ϵ_x Strain in direction of x and x_s is starting zone.

$$u_\epsilon = \sqrt{\frac{2\pi E}{\rho k}\left(1 - \frac{a_0}{a}\right)}$$

And $k = \frac{0.38^2}{2\pi}$, $a = a_0 + \epsilon_x$ then fracture velocity is estimated to be:

$$u_\epsilon = 24.7 \quad m/s$$

The data required for the computation is taken from Refs. ([4], [5], [6], [7] and [8]) and are tabulated in Table 1.

TABLE I
DATA OF FLOW PARAMETERS

Symbol	Quantity	Range	Typical Value
θ	Slope angle	$30-45$	38^o
η	Eddy viscosity	$400-1000$	$600\ m/s^2$
ρ	Density of snow	$100-300$	$200\ kg/m^3$
R	Slab depth	$0.4-1$	$0.5\ m$
L	Slab length	$1-3$	$2\ m$
μ	Viscosity of snow	$0.1-0.3$	$0.2\ kg/m.s$
a_0	Initial crack length	$5-20$	$10mm$
ϵ_x	Strain in x-direction	$1.1-1.3$	$1.3mm$

III. RESULTS AND DISCUSSIONS

The nonlinear ordinary differential equation (2) with the associated initial conditions has been solved numerically by using the fourth-order Runge-Kutta method for the computational purpose. The results are shown in Figs. 4-8

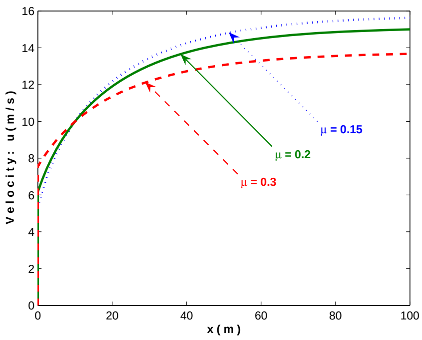

Fig. 4. Variation of velocity with x for different Snow Viscosity μ

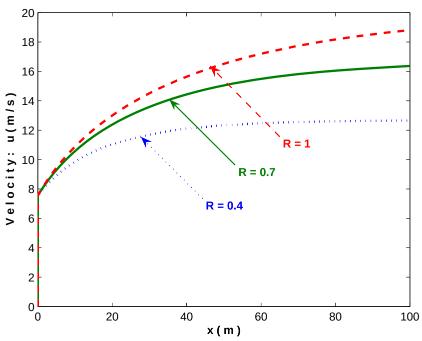

Fig. 5. Variation of velocity with x for different Slab depth R

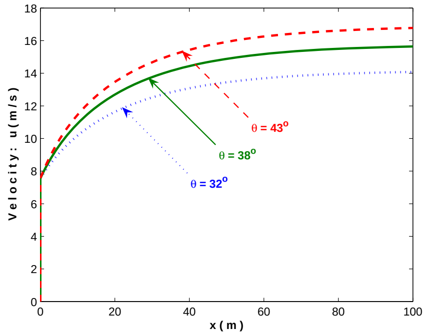

Fig. 6. Variation of velocity with x for different Slope angle θ

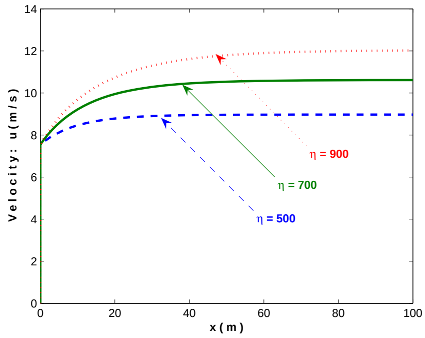

Fig. 7. Variation of velocity with x for different Eddy viscosity η

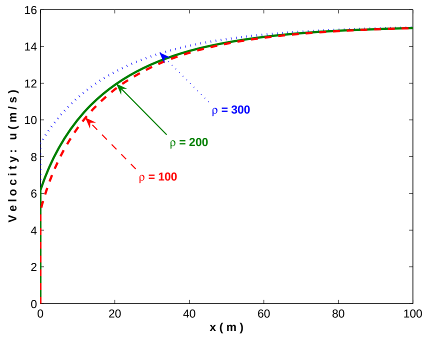

Fig. 8. Variation of velocity with x for different Snow density ρ

The results indicate that the snow avalanche velocity increases with the increasing x and for increasing eddy viscosity (η), slab depth (R) and slope angle (θ), and for decreasing viscosity (μ). The effect of density (ρ) on the avalanche velocity has observed that, higher the density of snow yield higher avalanche velocity (between start zone and track zone) which agrees with the physical observations.

IV. CONCLUSION

Avalanche Dynamics has been modeled in the present investigations. One of the most difficult parts in the modeling of avalanche dynamics is the prediction of an avalanche. Since the material so complex and changes its behavior due to adverse atmospheric conditions, hence for modeling of an avalanche has to be done by the numerical approach. All numerical based models published earlier has some assumptions that a preconceived notion of some initial velocity to start. Some of the model even used approach of open channel flow for proposing the models in starting and track zone. In the present investigations, the approach is basically uses the concepts of calculus variations and fundamental of fracture mechanics. This approach is very new and very fundamental in snow science. Overall the results are found to be in qualitative agreement with the physics of flows and deformations.

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