

# Mathematical Approach for Large Deformation Analysis of the Stiffened Coupled Shear Walls

M. J. Fadaee, H. Saffari, and H. Khosravi

**Abstract**—Shear walls are used in most of the tall buildings for carrying the lateral load. When openings for doors or windows are necessary to be existed in the shear walls, a special type of the shear walls is used called "coupled shear walls" which in some cases is stiffened by specific beams and so, called "stiffened coupled shear walls".

In this paper, a mathematical method for geometrically nonlinear analysis of the stiffened coupled shear walls has been presented. Then, a suitable formulation for determining the critical load of the stiffened coupled shear walls under gravity force has been proposed. The governing differential equations for equilibrium and deformation of the stiffened coupled shear walls have been obtained by setting up the equilibrium equations and the moment-curvature relationships for each wall. Because of the complexity of the differential equation, the energy method has been adopted for approximate solution of the equations.

**Keywords**—Buckling load, differential equation, energy method, geometrically nonlinear analysis, mathematical method, Stiffened coupled shear walls.

## I. INTRODUCTION

IN high-rise buildings, providing enough resistance and stiffness to withstand lateral forces caused by wind and earthquake is of special importance. Using shear walls is one of the methods of providing stiffness. Creating openings in a vertical row breaks up the shear wall into two or more parallel walls. Such shear walls are called coupled shear walls. The existence of the connecting beams increases the lateral stiffness and decreases the stresses in the wall. As the stiffening beams are usually arranged regularly in the height of the building, considering the regular geometry and the number of the stories, continuous medium method has been used for the analysis of such walls. By this method modeling the behavior of the structure with linear differential equations will be possible which will lead to a closed form solution. Hence, the analysis of the coupled shear walls with constant specifications throughout the height, leads to the solution of a linear differential equation with constant coefficients and critical load results have been presented for a limited range of stiffness parameters for the coupled shear walls. Reference [1]

deals with the effects of the position and stiffness of the stiffening beam on the behavior of the stiffened coupled shear walls which are placed on rigid and flexible supports. In this reference, it is shown that the position of the stiffening beam has an important effect on the behavior of the structure. In references [2] and [3] it is shown that the structure performance improves noticeably due to the presence of the stiffening beam. Height growth and efficiency improvement of high-rise buildings have contributed to more studies about their stiffness and stability. At the moment, controlling the effects of the stability decrease is one of the most important issues in designing process. In reference [4] coupled shear walls has first been divided into two separate shear walls and then the stiffness matrix of the whole system has been developed according to the boundary conditions. Hence, the analysis of the coupled shear walls with constant specifications throughout the height, leads to the solution of a linear differential equation with constant coefficients and critical load results have been presented for a limited range of stiffness parameters for the coupled shear walls. In reference [5] upper bound of critical loads for a wide range of the governing parameters has been computed.

In this paper a method has been introduced for the geometrically nonlinear analysis of the stiffened coupled shear walls. The governing equation for equilibrium and deformation of the stiffened coupled shear walls has been obtained by setting up the equilibrium equations and the moment-curvature relationships for each wall with eliminating the laminar shear from the relationships. In the governing equation, the effects of the axial force and the stiffening beam have been accounted for. The exact solution of the governing equation is very difficult so, energy method has been adopted for approximate solution. In the energy method, a shape function compatible with boundary conditions has been chosen and the total potential energy of the system has been calculated and by minimizing this function in terms of the unknown coefficients, the deformation equation of the stiffened coupled shear walls has been obtained. The critical load of the stiffened coupled shear walls has been obtained equating the determinant of the coefficients of the resultant equations to zero.

## II. THE EQUILIBRIUM EQUATIONS IN THE DIFFERENTIAL FORM

A typical stiffened coupled shear walls is shown in Fig. 1. In this figure, the stiffening beam has been placed at the height of  $h_s$  and the other specifications and parameters of

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the wall have also been shown. The connecting beams are replaced with a broad continuous medium by using continuous connection medium shown in Fig. 2. Assuming that the inflection points of the connecting beams are in the mid-span and as the vertical displacement of the connecting beam in the middle is zero, the governing differential equations and the axial force in the walls are developed in reference [6]. The moment-curvature relationships for the stiffened coupled shear walls are as follows, [6].

$$EI \frac{d^2 y_a}{dx^2} = M_e - L \int_x^H q_a dz + 2 \int_x^H P_a (\eta_a - y_a) dz \quad hs \leq x \leq H \quad (1)$$

$$EI \frac{d^2 y_b}{dx^2} = M_e - L \left( \int_{hs}^H q_a dz + \int_x^{hs} q_b dz + Q_s \right) + \quad 0 \leq x \leq hs \quad (2)$$

$$2 \left[ \int_{hs}^H P_a (\eta_a - y_b) dz + \int_x^{hs} P_b (\eta_b - y_b) dz \right]$$

$P_a$  and  $P_b$  are the uniform gravitational load on the top and bottom of the stiffening beam, respectively,  $Q_s$  is the shear force in the stiffening beam,  $M_e$  is the moment caused by the external loading which calculated having uniform distributed load ( $u$ ), triangular distributed load with maximum amount ( $w$ ) at top of the structure and concentrated load ( $p$ ) at top of the structure and is obtained as follows:

$$M_e = \frac{u}{2} (H - x)^2 + \frac{w}{6H} (2H^3 - 3H^2 x + x^3) + p(H - x) \quad (3)$$

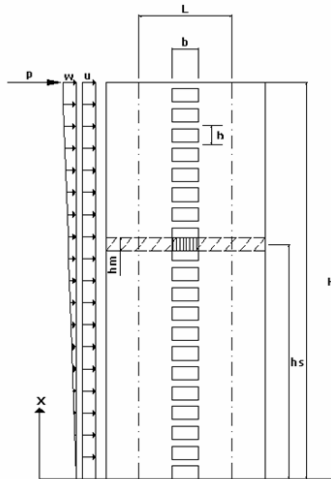


Fig. 1 Stiffened coupled shear walls

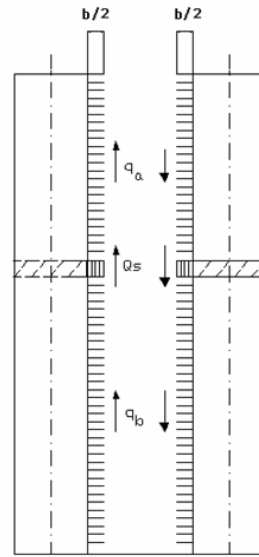


Fig. 2 Substituted structure

As it is shown in Figs. 3 and 4,  $T_{a1}$  and  $T_{a2}$  are the left and right axial forces in the walls above the stiffening beam, respectively, and  $T_{b1}$  and  $T_{b2}$  are the left and right axial forces in the walls below the stiffening beam, respectively, which can be obtained as follows, [6].

$$T_{a1} = - \int_x^H p_a dz + \frac{2}{L} \int_x^H P_a (\eta_a - y_a) dz - \frac{EI}{L} \frac{d^2 y_a}{dx^2} + \frac{M_e}{L} \quad (4)$$

$$T_{a2} = + \int_x^H p_a dz + \frac{2}{L} \int_x^H P_a (\eta_a - y_a) dz - \frac{EI}{L} \frac{d^2 y_a}{dx^2} + \frac{M_e}{L} \quad (5)$$

$$T_{b1} = - \left[ \int_{hs}^H p_a dz + \int_x^{hs} p_b dz \right] + \frac{2}{L} \left[ \int_{hs}^H P_a (\eta_a - y_a) dz + \int_x^{hs} P_b (\eta_b - y_b) dz \right] - \frac{EI}{L} \frac{d^2 y_b}{dx^2} + \frac{M_e}{L} \quad (6)$$

$$T_{b2} = \left[ \int_{hs}^H p_a dz + \int_x^{hs} p_b dz \right] + \frac{2}{L} \left[ \int_{hs}^H P_a (\eta_a - y_b) dz + \int_x^{hs} P_b (\eta_b - y_b) dz \right] - \frac{EI}{L} \frac{d^2 y_b}{dx^2} + \frac{M_e}{L} \quad (7)$$

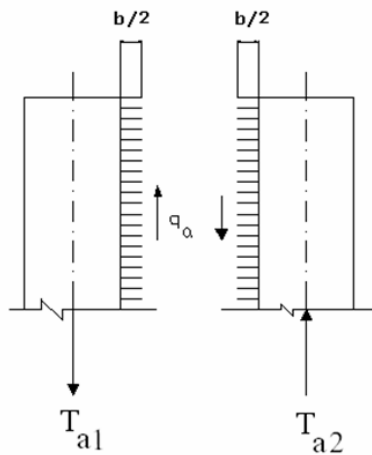


Fig. 3 The section above the stiffening beam

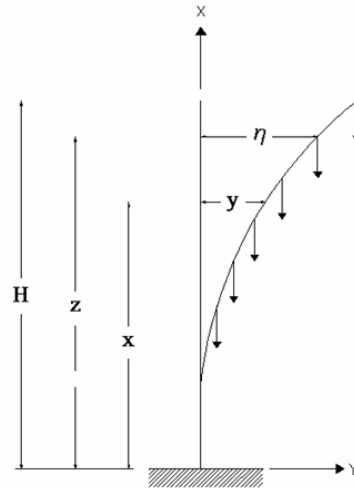


Fig. 5 The effect of the uniform gravitational load on the structure

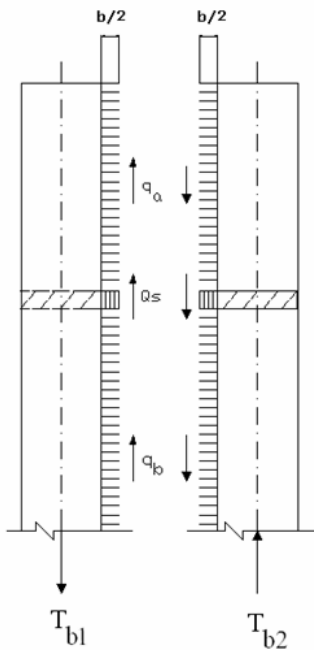


Fig. 4 The section below the stiffening beam

### III. THE DISPLACEMENT EQUATIONS IN THE DIFFERENTIAL FORM

The displacement equation in the differential form for the top of the stiffening beam for the case that the cross sectional areas of two walls are equal,  $(A_1 = A_2)$ , can be obtained as follows, [6]:

$$\frac{d^5 y_a}{dx^5} - \frac{12I_b}{hb^3 I} \left[ L^2 + \frac{2I}{A_1} - \frac{hb^3}{6EI_b} P_a (H-x) \right] \frac{d^3 y_a}{dx^3} - \frac{4P_a}{EI} \frac{d^2 y_a}{dx^2} - \frac{48P_a I_b}{hb^3 EI A_1} (H-x) \frac{dy_a}{dx} - \frac{1}{EI} \frac{d^3 M_e}{dx^3} + \frac{24I_b}{EA_1 hb^3 I} \frac{dM_e}{dx} = 0 \quad (8)$$

The displacement equation in the differential form for the bottom of the stiffening beam for the case that the cross sectional areas of two walls are equal,  $(A_1 = A_2)$ , can be obtained as follows, [6]:

$$\frac{d^5 y_b}{dx^5} - \frac{12I_b}{hb^3 I} \left[ L^2 + \frac{2I}{A_1} - \frac{hb^3}{6EI_b} P_b (h_s - x) \right] \frac{d^3 y_b}{dx^3} - \frac{4P_b}{EI} \frac{d^2 y_b}{dx^2} - \frac{48I_b}{EA_1 hb^3 I} [P_b (h_s - x)] \frac{dy_b}{dx} - \frac{1}{EI} \frac{d^3 M_e}{dx^3} + \frac{24I_b}{EA_1 hb^3 I} \frac{dM_e}{dx} = 0 \quad (9)$$

In Equations (1) to (9)  $y_a, q_a, y_b$  and  $q_b$  are the lateral displacement of the wall and laminar shear resulted from the connecting beams above and below the stiffening beam, respectively,  $L$  is the centre to centre distance between the stiffened coupled shear walls,  $b$  is the net span of the connecting beams,  $E$  is the module of elasticity,  $A_1, A_2, I_1$  and  $I_2$  are the cross sectional areas and moments of inertia of the left and right walls, respectively. The other parameters are shown in the Figs. 1 to 5.

### IV. SOLVING THE EQUATIONS

In order to find the displacements and the stresses in the

stiffened coupled shear walls through nonlinear analysis explained in the former sections, the equations (8) and (9) must be solved. Because of the complexity of these equations, the exact solution for them has not been found yet. An approximate solution of them using energy method can be found in reference [6].

#### V. CONCLUSION

In the method proposed in this paper, the geometrically nonlinear analysis of stiffened coupled shear walls has been discussed and a suitable formulation has been suggested for buckling load of the stiffened coupled shear walls under gravitational loading. The effects of axial load and stiffening beam on the behavior of the stiffened coupled shear walls, and also the effects of the axial load on the lateral displacements have been accounted for. By setting up the equilibrium equations and moment-curvature relationships for each wall, and eliminating the laminar shear from the relationships, the governing equation of the stiffened coupled shear walls displacement has been obtained. The equations are very complex and no exact solution has been found for them yet. For Approximate solution, the energy method can be used by choosing a suitable shape function which satisfies the boundary conditions. Then, the total potential energy can be calculated and by minimizing it in terms of the unknown coefficients, the stiffened coupled shear walls deformation equation can be obtained. At the end, the uniform distributed critical load for the stiffened coupled shear walls can be computed by equating the equation coefficients determinant to zero. The explanation of the approximate solution and several numerical examples can be found in reference [6].

#### REFERENCES

- [1] A. Coul, L. Bensmil, "Stiffened Coupled Shear Walls," *Journal of Structural Engineering ASCE*, vol. 117, no. 8, 1991, pp. 2205-2223.
- [2] H. C. Chan, J. S. Koung, "Elastic Design charts for Stiffened Coupled Shear Walls," *Journal of Structural Engineering ASCE*, vol. 115, no. 2, 1989, pp. 247-267.
- [3] H. C. Chan, J. S. Koung, "Stiffened Coupled Shear Walls," *Journal of Engineering Mechanics ASCE*, vol. 115, no. 4, 1989, pp. 689-703.
- [4] R. Rosman, "Stability and Dynamics of shear-wall Frame Structures," *Building Science*, vol. 9, 1974, pp. 55-63.
- [5] E. Grzelak, "The Approximate Free Vibration and Stability Analysis of Shear- Flexure Cantilevers," M.Sc. Project report, Dept. Civ. Eng., McGill Univ., Montreal.
- [6] H. Khosravi, "Geometrically Nonlinear Analysis of Stiffened Coupled Shear Walls," M.Sc. thesis, Dept. Civ. Eng., Islamic Azad Univ., Kerman-Iran, 2005.