Marangoni Convection in a Fluid Saturated Porous Layer with a Deformable Free Surface

Nor Fadzillah Mohd Mokhtar, Norihan Md Arifin, Roslinda Nazar, Fudziah Ismail and Mohamed Suleiman

Abstract—The stability analysis of Marangoni convection in porous media with a deformable upper free surface is investigated. The linear stability theory and the normal mode analysis are applied and the resulting eigenvalue problem is solved exactly. The Darcy law and the Brinkman model are used to describe the flow in the porous medium heated from below. The effect of the Crispation number, Bond number and the Biot number are analyzed for the stability of the system. It is found that a decrease in the Crispation number and an increase in the Bond number delay the onset of convection in porous media. In addition, the system becomes more stable when the Biot number is increases and the Da^{eff} number is decreases.

Keywords— Deformable, Marangoni, Porous, Stability.

I. INTRODUCTION

THE instability of the convection driven by buoyancy is referred to as Rayleigh-Benard instability has been extensively studied since the early analysis by Horton and Rogers [1] and Lapwood [2]. They discussed a porous medium saturated by a wetting liquid, heated from below and they concluded that the filtration Rayleigh number has a critical value equal to $4\pi^2$. The latter effect is due to the local variation of surface tension or referred to as Marangoni instability was first theoretically analysed by Pearson [3]. Sparrow, Goldstein and Jonsson [4] studied analytically the thermal instability of an internally heated fluid layer as well as a layer heated from below, with various boundary conditions. On the Marangoni instability problem, the effect of the surface deflection is later considered by Scriven and Sternling [5]. As these two kinds of instability take place at the same time, the instability mechanism is known as the Benard-Marangoni instability. Nield [6] first analyses the Benard-Marangoni instability problem. Katto and Masuoka [7] resolved some of the apparent divergence between theoretical predictions and experimental results on convective critical conditions for bottom-heated porous media by introducing the effective thermal diffusivity in the more conventional external Rayleigh number. Gupta and Joseph's [8] numerical treatment showed excellent agreement with experimental results on the heat

transport across a bottom heated porous layer. Kazimi and Gasser [9] studied the onset of convection in a porous medium with internal heat generation by employing a rigid lower surface with a free upper surface and isothermal conditions at the upper and lower surfaces. The combination of critical Rayleigh numbers presented in their paper was expected to hold true for a bed with a rigid isothermal upper boundary as well as a free isothermal surface upper boundary. The thermal stability of superposed porous and fluid layers has been studied by Nield [10], using linear stability analysis for an empirical interfacial condition at the fluid-porous interface suggested by Beavers and Joseph. Davis and Homsy [11] later study the effect of the surface deflection on the combined Benard-Marangoni problem. The thermal stability for different system of superposed porous and fluid regions has also been considered by Pillatsis et.al [12] and Taslim and Narusawa [13]. Perez-Garcia and Carneiro [14] have carried out a systematic study of the linear stability of the Benard-Marangoni convection with a deformable free surface. The effect of the internal heat generation on the Benard-Marangoni instability of a horizontal liquid layer with a deformable upper free surface was investigated by Ming-I Char and Ko-Ta Chiang [15]. The stability analysis is based on the linear stability theory and the resulting eigenvalue problem was solved by employing the fourth order Runge-Kutta-Gill method. Hennenberg et.al.[16] have considered a liquid saturated porous media in contact with air and subjected to an adverse gradient of temperature in the lower boundary is perfectly conducting. They have developed the model that can be described in terms of the Brinkman model. They solved the Brinkman approach over the whole saturated porous matrix and obtained a critical wave number which was highly dependent on the Darcy number. The linear stability analysis of Marangoni convection in a composite system comprised of an incompressible fluid-saturated porous layer underlying a layer of the same fluid is considered by Shivakumara and Krishna [17]. The upper fluid surface, free to the atmosphere, is considered to be deformable and subjected to temperature dependent surface tension.

The purpose of the present paper is primarily to examine the Marangoni convective instability in saturated porous medium with a deformable upper free surface which is heated from below. The linear stability theory and the normal mode analysis are applied and the resulting eigenvalue problem is solved exactly. The Darcy law and the Brinkman model are used to describe the flow in the porous medium and of interest are the effects of Crispation number; Cr, Bond number; Bo, and the Biot number; Bi.

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II. MATHEMATICAL FORMULATION

Consider a saturated isotropic porous matrix of thickness d and of infinite horizontal extent, heated from below.
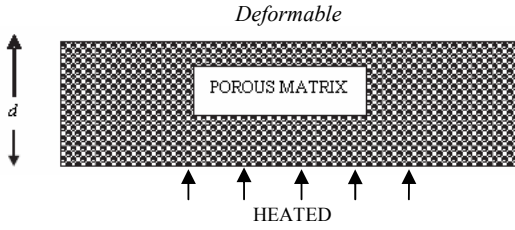
Deformable

Fig. 1 The porous layer heated from below

Its upper boundary is at a temperature T_0 and is in contact with a gaseous phase. The lower boundary is assumed to be a perfect conductor at a higher temperature $T_0 + \Delta T$. The free surface is assumed to be deformable. The saturated porous matrix is entirely described by the continuity, Brinkman momentum law and energy equations that are

$$\nabla \cdot \boldsymbol{V} = 0, \qquad (1)$$

$$\rho_l c_a \frac{\partial \boldsymbol{V}}{\partial t} = -\nabla P - \frac{\mu}{K} \boldsymbol{V} + \mu_{\mathrm{eff}} \nabla^2 \boldsymbol{V}, \qquad (2)$$

$$\rho_m c_m \frac{\partial T}{\partial t} + \rho_l c_l \boldsymbol{V} \cdot \nabla T = k_m \nabla^2 T, \qquad (3)$$

where $\boldsymbol{V} = (u, v, w)$ is the seepage velocity, ρ_m is the mean density, ρ_l is the clear liquid density, c_l is the specific solid heat capacity in the clear liquid, c_m is the specific solid heat capacity in the porous medium, k_m is the overall thermal conductivity of the porous medium, μ_{eff} is the effective saturated porous medium viscosity, c_a is the acceleration coefficient, P is the pressure, μ is the pure liquid viscosity and K the permeability of the porous matrix.

The variables are then nondimensionalized using $d, \varsigma d^2 / \alpha_m, \alpha_m / d, \Delta T, \mu \alpha_m / K$ as the units of length, time, velocity, temperature and pressure respectively. Using the dimensionless variables, the equations (1) – (3) are transformed to the following dimensionless form:

$$\nabla \cdot \boldsymbol{V} = 0, \qquad (4)$$

$$\gamma_a \frac{\partial \boldsymbol{V}}{\partial t} = -\nabla P - \boldsymbol{V} + \mathrm{Da}^{\mathrm{eff}} \nabla^2 \boldsymbol{V}, \qquad (5)$$

$$\frac{\partial \theta}{\partial t} = w + \nabla^2 \theta, \qquad (6)$$

where $\mathrm{Da}^{\mathrm{eff}} = \dfrac{\mu_{\mathrm{eff}}}{\mu} \dfrac{K}{d^2}$ and $\gamma_a = \dfrac{c_a \rho_f \alpha_m K}{\varsigma d^2 \mu}$.

The boundary conditions at the bottom are for rigid boundary conducting to temperature perturbations that are:

$$w = \theta = Dw = 0, \qquad (7)$$

which is evaluated at $z = 0$. The boundary conditions at z = 1, are

$$w = \frac{\partial \chi}{\partial t}, \qquad (8)$$

$$\left(\frac{\partial^2}{\partial z^2} - \nabla_h^2 \right) w + \mathrm{Ma}_p^{\mathrm{eff}} \nabla_h^2 (\chi - \theta) = 0, \qquad (9)$$

$$\frac{\partial \theta}{\partial z} + \mathrm{Bi}(\theta - \chi) = 0, \qquad (10)$$

$$\mathrm{Cr}\left[\frac{\partial}{\partial t} + \left(\frac{\partial^2}{\partial z^2} + 3\nabla_h^2 \right) \right] \frac{\partial w}{\partial z} + (\mathrm{Bo} - \nabla_h^2) \nabla_h^2 \chi = 0. \, (11)$$

$\mathrm{Ma}_p^{\mathrm{eff}}$ is the equivalent of a Marangoni number for the upper surface, defined as,

$$\mathrm{Ma}_p^{\mathrm{eff}} = \left[-\frac{\partial \sigma_m}{\partial T} \right] \frac{\Delta T d k_l}{\mu_{\mathrm{eff}} \alpha_l k_m}, \qquad (12)$$

where $\mathrm{Ma}_p^{\mathrm{eff}}$ is the product of the pure liquid Marangoni number by a quantity which is a function of the porosity ϕ and of the thermal conductivity of the clear liquid and the solid (see detailed in [16]). If f is a disturbance quantity, then following [16], and expressing this quantity as

$$f(x, y, z, t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dk_x dk_y f_k(z) \exp\left[i(k_x x + k_y y) + st \right], \, (13)$$

with $a = \sqrt{k_x^2 + k_y^2}$ is a wavenumber, equation (5) and (6) in dimensionless form become

$$\left((\gamma_a s + 1) - \mathrm{Da}^{\mathrm{eff}}(D^2 - a^2) \right)(D^2 - a^2) W(z) = 0, \quad (14)$$

$$(D^2 - (a^2 + s)) \theta(z) = -W(z), \qquad (15)$$

where $W(z)$ is the vertical variation of the z-velocity and $D = d / dz$. The dimensionless form boundary conditions (7) – (11) become

$$W = 0, \qquad (16)$$

$$\theta = 0, \qquad (17)$$

$$DW = 0, \qquad (18)$$

at $z = 0$ and

$$W = 0, \qquad (19)$$

$$D\theta + \mathrm{Bi}(\theta - \eta) = 0, \qquad (20)$$

$$(D^2 + a^2) W + \mathrm{Ma}_p^{\mathrm{eff}\, 2}(\theta - \eta) = 0, \qquad (21)$$

$$-\mathrm{Cr}(D^2 - 3a^2) DW + (\mathrm{Bo} + a^2) a^2 \eta = 0, \qquad (22)$$

at $z = 1$. The governing equations (14) and (15), subject to the boundary conditions (16) – (22), constitute an eigenvalue problem of order six can be solved exactly by setting $s = 0$, to obtain the equation relevant to the neutral stability.

III. METHOD OF SOLUTION

The resulting eigenvalue problem is solved exactly, in general, with $\mathrm{Ma}_p^{\mathrm{eff}}$ as an eigenvalue. Since equation (14) is independent of θ, it can be directly solved to get the general solution in the form

$$W(z) = A_1 \sinh(az) + A_2 \cosh(az) + A_3 \sinh(az\alpha) + A_4 \cosh(az\alpha) \tag{23}$$

where $A_1 - A_4$ are constants to be determined and $\alpha = \sqrt{\dfrac{1}{a^2 \mathrm{Da}^{\mathrm{eff}}}}$. The parameter α plays a crucial role. When the permeability K and the Darcy number, $\mathrm{Da}^{\mathrm{eff}}$ becomes infinite, then the parameter α is equal to one. Using the boundary conditions (16), (18) and (19) to solve equation (14), we obtain

$$W(z) = A_1\left[\sinh(az) + \Delta_1 \cosh(az) + \Delta_2 \sinh(az\alpha) + \Delta_3 \cosh(az\alpha)\right]$$

where

$$\Delta_1 = -\frac{1}{\alpha}\left(\frac{\alpha \sinh(a) - \sinh(a\alpha)}{\cosh(a) - \cosh(a\alpha)}\right),$$

$$\Delta_2 = -\frac{1}{\alpha}, \tag{24}$$

$$\Delta_3 = -\Delta_1.$$

The heat equation (15) has now to be solved defining their right-hand sides by the expressions given by equation (24). The solution obtained for θ using the boundary condition (17) is given by

$$\theta(z) = A\left\{\begin{array}{l}\left[-\mathrm{Da}^{\mathrm{eff}} + \dfrac{\alpha\beta(1)}{2a}z\right]\cosh(az) + \left[c^* - \dfrac{z}{2a}\right]\sinh(az) \\ + \mathrm{Da}^{\mathrm{eff}}\cosh(\alpha az) - \beta(1)\mathrm{Da}^{\mathrm{eff}}\sinh(\alpha az)\end{array}\right\}$$

$$\tag{25}$$

where $\beta(1) = \dfrac{\cosh(a) - \cosh(a\alpha)}{\alpha \sinh(a) - \sinh(a\alpha)}$ and two unknown quantities A and c^* remain to be calculated.

Now, we will use the last boundary conditions (20) and (21), to get the compatibility condition. From equation (20) and equation (25), after some obvious and tedious simplifications, we obtain

$$\left[a\cosh(a) + \delta_1\mathrm{Bi}\sinh(a)\right]\left[c^* - \frac{1}{2a}\right] =$$

$$\left\{\frac{-\alpha\beta(1)}{2a} + \delta_1\mathrm{Bi}\left[\delta_2 + \frac{1}{a^2}\left(\delta_2\mathrm{Bo} + 2a\alpha\,\mathrm{Cr}\,\beta(1)\right)\right]\right\}\cosh(a)$$

$$+\left\{a\delta_2 + \frac{1}{2a} + \frac{\delta_1\mathrm{Bi}}{a^2}\left[\mathrm{Bo}\left(\frac{1}{2a} - c^*\right) - 2a\mathrm{Cr}\right]\right\}\sinh(a)$$

$$+ \mathrm{Da}^{\mathrm{eff}}\left\{\begin{array}{l}a\alpha\beta(1) - \\ \delta_1\mathrm{Bi}\left[1 - \dfrac{1}{a^2\mathrm{Da}^{\mathrm{eff}}}\left(\begin{array}{l}a\alpha\,\mathrm{Cr}\,\beta(1)\left[\alpha^2 - 3\right] \\ -\mathrm{BoDa}^{\mathrm{eff}}\end{array}\right)\right]\end{array}\right\}\cosh(\alpha a)$$

$$+ \delta_1\mathrm{Bi}\left[\beta(1) + \frac{1}{a^2\mathrm{Da}^{\mathrm{eff}}}\left(\mathrm{BoDa}^{\mathrm{eff}}\beta(1) + a\alpha\,\mathrm{Cr}\left[3 - \alpha^2\right]\right)\right]\sinh(\alpha a).$$

$$\tag{26}$$

Here $\delta_1 = \dfrac{a^2}{a^2 + \mathrm{Bo}}$ and $\delta_2 = \mathrm{Da}^{\mathrm{eff}} - \dfrac{\alpha\beta(1)}{2a}$. Using the boundary condition (21) and the properties of the hyperbolic

trigonometric functions and rearranging the terms, we obtain the explicit value of $\mathrm{Ma}_p^{\mathrm{eff}}$ as a function of the wave number; a, the Biot number; Bi, the Bond number; Bo, the Crispation number; Cr, and the Darcy number, $\mathrm{Da}^{\mathrm{eff}}$ is given by

$$\mathrm{Ma}_p^{\mathrm{eff}} = \frac{2\,a\,\overline{B}\,\overline{\alpha}^2\lambda_1}{a\,\alpha\,\overline{\alpha}\left[\lambda_2 - \lambda_3\right] + \overline{B}\lambda_4 + \lambda_5}, \tag{27}$$

where

$C = \cosh(a),\ S = \sinh(a),\ C_\alpha = \cosh(\alpha a),\ S_\alpha = \sinh(\alpha a)$,

$\overline{B} = a^2 + \mathrm{Bo}$, $\overline{\alpha} = \left(\alpha^2 - 1\right)$, $\varepsilon = \alpha^2 - 5$,

$\lambda_1 = \left(a\,C + S\,\mathrm{Bi}\right)\left(\alpha\,C_\alpha\,S - C\,S_\alpha\right)$,

$\lambda_2 = C\left(a^2\left\{2\,\varepsilon\,\mathrm{Cr} + 1\right\} + \mathrm{Bo}\right)$,

$\lambda_3 = C_\alpha\left(2a^2\varepsilon\,C^2\mathrm{Cr} + \overline{B}\right)$,

$\lambda_4 = \left(1 - C^2\right)S_\alpha\left(1 + 3\alpha^2\right) - S\alpha\left(1 - CC_\alpha\right)\left(3 + \alpha^2\right)$,

$\lambda_5 = 2a^3\mathrm{Cr}\left(CSS_\alpha\left\{\alpha^2\left(\alpha^4 - 4\alpha^2 + 1\right) + 2\right\}\right)$.

From equation (27), it is seen that the Marangoni number whose explicit value is highly dependent on α and is thus a function of the permeability K.

At finite a, when the Darcy number, $\mathrm{Da}^{\mathrm{eff}} \to \infty$, a clear fluid ϕ tends towards 1 and K which is dependent upon the layer width d, becomes infinite, then the problem (14) – (22) reduce to the problem studied by Wilson [13]. When α equal to one, equation (27) will produced the explicit Marangoni function for the conducting rigid wall. By applying the l'Hospital rule, we obtain

$$\lim_{\alpha \to 1}\mathrm{Ma}_p^{\mathrm{eff}} = \frac{8a\left\{aSC^2 + \mathrm{Bi}\left[C^3 - C - aS\right] - a^2C\right\}}{SC^2 - S + \left(8\,\mathrm{Cr}\,\delta_1 - 1\right)a^3C}. \tag{28}$$

The compatibility condition (28) produces for Darcy numbers, $\mathrm{Da}^{\mathrm{eff}}$ much larger than one, exactly the results derived by [13]. To verify our results, test computations have been performed the marginal stability curves obtained by (28) are plotted in Figure 3 and 5. As expected, the classical curve [13] is reproduced and the critical Marangoni number obtained from equation (28) are found to be in excellent agreement with those of [13].

IV. RESULT AND DISCUSSION

The criterion for the onset of Marangoni convection in a deformable saturated Benard-Marangoni one-layer porous system is investigated theoretically. The stability analysis of the Benard-Marangoni problem in porous media has been studied by Hennenberg et al. [16] and the linear stability analysis of the Benard-Marangoni problem in a layer of fluid with a deformable free surface has been studied by Perez-Garcia et al. [14]. To verify our numerical results, test computations have been performed and the critical Marangoni number allocation shows a good agreement with the results given in [14] and [16] which are listed in Table 1 and Table 2.

In Table 1, the critical values of Marangoni number and the corresponding critical wavenumber are shown for different

values of Da^{eff} and Cr. Our results are compared with those of Perez-Garcia [14] obtained using a numerical method in the absence of thermal buoyancy (Rayleigh number = 0). We note that the results compare well with each other and for Cr = 0, the critical Marangoni number is equal to 79.60688, a known value for the case of a single fluid layer in the absence of surface deflection at the free surface which discussed by [3]. As Da^{eff} decreases, the onset of Marangoni convection is increases. Although the value of the wavenumber increases as the value of Da^{eff} increase, the critical wavenumber is approaching zero as the Cr increase. It can be clearly seen that the decrease in the Da^{eff} number and Crispation number, will delay the onset of convection.

The critical values of Marangoni number for different values of Bi, Da^{eff} and Cr on the stability of the Marangoni convection in the case of Bo = 0, are shown in Table 2. The results of this analysis agree well with [16]. As Bi number increases, the value of critical Marangoni number increases quite rapidly. We also find that at each Bi, the critical Marangoni increases obviously as Da^{eff} increase from the value of 10^{-1} to 10^{-2}, but it is fairly insensitive to the increase of the value of the Crispation number. From the table, the system becomes more stable when the Biot number is increases.

Figures 2(a) and 2(b), respectively show the plots of $(Ma_p^{eff})_c$ and a_c as the function of Cr for different values of Da^{eff} and fixed values of Bo and Bi. From the figures, it may be inferred that an increase in the value of Cr is to decrease the value of $(Ma_p^{eff})_c$, and thus making the system more unstable. The reason is that an increase in Cr is to increase the deflection of the upper surface, which in turn promotes instability much faster. It is also seen that $(Ma_p^{eff})_c$ number attains a constant value at specific Da^{eff} number and at certain Cr, $(Ma_p^{eff})_c$ number decrease rapidly before attaining an asymptotic value with further increase in Cr.

The other physical parameter that we considered is the Bond number, Bo as shown in Figure 3. Contrast to the effect of Cr, increase in the value of Bo makes the system more stable. The reason for this may be attributed to the fact that an increase in Bo leads to an increase in the gravity effect, which keeps the upper surface flat against the effect of surface tension, which forms a meniscus on the free surface. Ma_p^{eff}, corresponding to the first minimal point of the zero wavenumber, is quite sensitive to the surface tensile of the upper surface and increases as Bo increases. But it is very indifferent to the value of Bo at the second minimal point of the finite wavenumber especially at Bo = 0.4 and Bo = 0.5, where the $(Ma_p^{eff})_c$ fixed at the same value.

Figure 4 shows a variation of Ma_p^{eff} with wavenumber a, for different values of Bi, in the case of Bo = 0, Cr = 0 and $Da^{eff} = 10^{-1}$. From the graph, an increasing of Bi, the medium becomes prone to stability.

The variation of Ma_p^{eff} with wavenumber a, for different values of Da^{eff}, in the case of Bo = 0.1, Bi = 2, Cr = 10^{-6} are shown in Figure 5. It can be seen clearly that the onset of convection started earlier for $Da^{eff} = 100$

compared with $Da^{eff} \leq -1$. This is because when the permeability K is large the resistance to flow becomes effectively controlled by the ordinary viscous resistance and in this case, the convection phenomenon is similar to that in a fluid layer. The lower the permeability K, the lower the Da^{eff} would be and the $(Ma_p^{eff})_c$ will increase.

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TABLE I CRITICAL VALUES OF MARANGONI NUMBER $\left(Ma_p^{eff}\right)_c$ AND THE CORRESPONDING CRITICAL WAVENUMBER A_c, FOR DIFFERENT VALUES OF DA^{EFF} AND CR ON THE STABILITY OF THE MARANGONI CONVECTION FOR BO = 0.1 AND BI = 0.

Cr	[14] Fluid		Present Eq. 26		$Da^{eff} = 10^{-1}$	
	M_c	a_c	$\left(Ma_p^{eff}\right)_c$	a_c	$\left(Ma_p^{eff}\right)_c$	a_c
0	79.607	1.99	79.60688	1.99	96.55991	2.43
10^{-6}	79.606	1.99	79.60580	1.99	96.55791	2.43
10^{-5}	79.596	1.99	79.59608	1.99	96.53960	2.43
10^{-4}	79.499	1.99	79.49898	1.99	96.35567	2.42
10^{-3}	66.667	0	66.6556	0	30.63800	0
10^{-2}	6.667	0	6.66680	0	3.06400	0
10^{-1}	0.667	0	0.66664	0	0.30640	0

Cr	Present					
	$Da = 10^{-2}$		$Da = 10^{-3}$		$Da^{eff} = 10^{-4}$	
	$\left(Ma_p^{eff}\right)_c$	a_c	$\left(Ma_p^{eff}\right)_c$	a_c	$\left(Ma_p^{eff}\right)_c$	a_c
0	355.4975	2.57	2490.357	3.03	21720.94	3.61
10^{-6}	355.3287	2.57	2469.370	2.97	1000.000	0
10^{-5}	353.8065	2.55	316.264	0	100.000	0
10^{-4}	100.00	0	31.626	0	10.00	0
10^{-3}	10.00	0	3.162	0	1.00	0
10^{-2}	1.00	0	0.316	0	0.10	0
10^{-1}	0.10	0	0.032	0	0.01	0

TABLE II CRITICAL VALUES OF MARANGONI NUMBER $\left(Ma_p^{eff}\right)_c$, FOR DIFFERENT VALUES OF BI, DA^{EFF} AND CR ON THE STABILITY OF THE MARANGONI CONVECTION FOR BO = 0.

Bi	[16] Cr = 0		Present Cr = 0	
	$Da^{eff}=10^{-1}$	$Da = 10^{-2}$	$Da^{eff}=10^{-1}$	$Da = 10^{-2}$
0	96.560	355.497	96.55991	355.4973
2	166.268	591.668	166.2953	591.6683
4	229.544	796.751	229.5418	796.7514
6	290.592	989.224	290.5913	989.2241
8	350.510	1174.452	350.5096	1174.4517
10	409.753	1354.904	409.7532	1354.9040

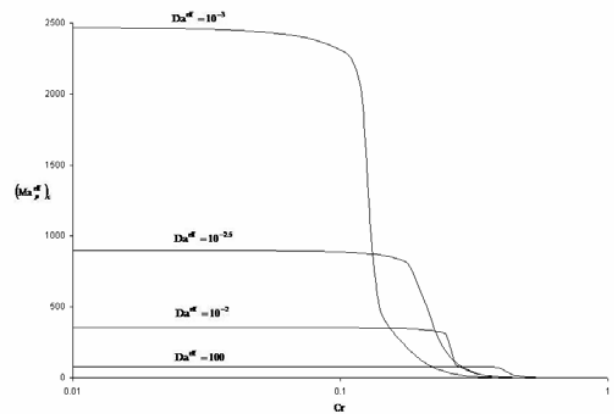| Bi	Present			
	$Cr = 10^{-6}$		$Cr = 10^{-4}$	
	$Da^{eff}=10^{-1}$	$Da = 10^{-2}$	$Da^{eff}=10^{-1}$	$Da = 10^{-2}$
0	96.55787	355.3262	96.35215	337.5594
2	166.2917	591.5195	166.2300	576.4552
4	229.5418	796.6037	229.5744	781.6695
6	290.5926	989.0756	290.7095	974.0292
8	350.5116	1174.302	350.7121	1159.1065
10	409.7559	1354.753	410.0402	1339.4086

Fig. 2(a) The effect of Cr on the stability of Marangoni convection for Bo = 0.2 and Bi = 0.
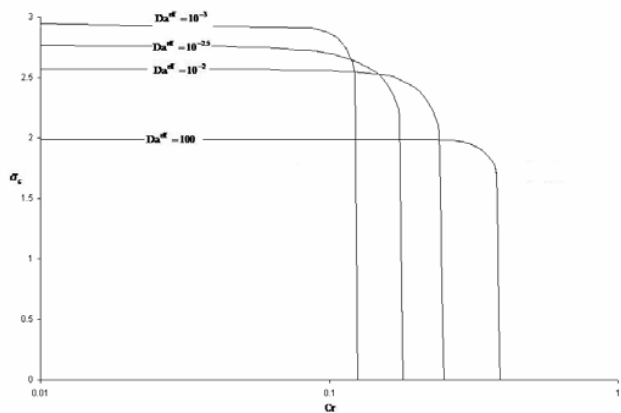

Fig. 2(b) The variation of a_c with Cr when Bo = 0.2 and Bi = 0 for a range of values of Da^{eff}.
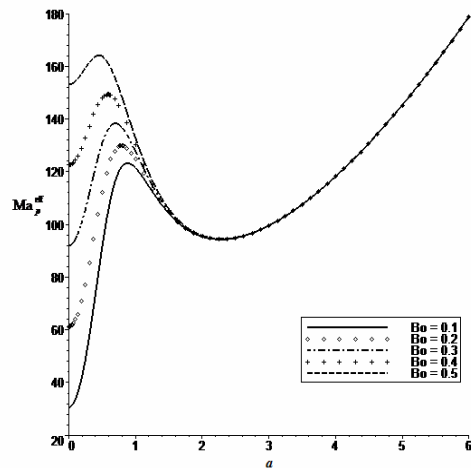

Fig. 3 The stationary neutral curves Ma_p^{eff} are plotted for several values of Bo on the Marangoni convection.
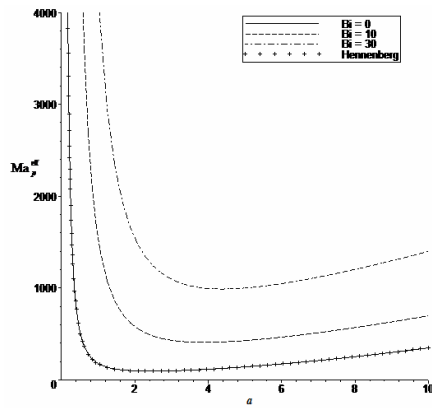
Fig. 4 Variation of Ma_p^{eff} with wavenumber a, for different
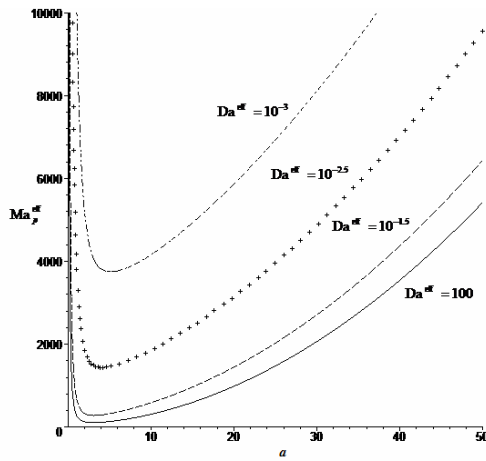values of Bi, in the case of $Da^{eff} = 10^{-1}$.

Fig. 5 Variation of Ma_p^{eff} with wavenumber a, for different
values of Da^{eff}, in the case of Bo = 0.1, Bi = 2, Cr=10^{-6}