Mamdani Model based Adaptive Neural Fuzzy Inference System and its Application

Yuanxuan Chai, Limin Jia, and Zundong Zhang

Abstract—Hybrid algorithm is the hot issue in Computational Intelligence (CI) study. From in-depth discussion on Simulation Mechanism Based (SMB) classification method and composite patterns, this paper presents the Mamdani model based Adaptive Neural Fuzzy Inference System (M-ANFIS) and weight updating formula in consideration with qualitative representation of inference consequent parts in fuzzy neural networks. M-ANFIS model adopts Mamdani fuzzy inference system which has advantages in consequent part. Experiment results of applying M-ANFIS to evaluate traffic Level of service show that M-ANFIS, as a new hybrid algorithm in computational intelligence, has great advantages in non-linear modeling, membership functions in consequent parts, scale of training data and amount of adjusted parameters.

Keywords—Fuzzy neural networks, Mamdani fuzzy inference, M-ANFIS

I. INTRODUCTION

Since 1992, Bezdek first proposed the concept of Computational Intelligence; in recent years CI has gained a widespread concern of many scholars emerging as a new field of study.

Computational intelligence actually uses the bionics ideas for reference, it origins from emulating intelligent phenomenon in nature. CI attempts to simulate and reappearance the characters of intelligence, such as learning and adaptation, so that it can be a new research domain for reconstructing the nature and engineering. The essence of CI is a universal approximator, and it has the great function of non-linear mapping and optimization. Its main branches are fuzzy logic (FL), artificial neural network (ANN), evolutionary computing (EC), etc.

Research on hybrid algorithms has been the hot issues in CI study. It is becoming less common to read about an application that uses just neural networks, or just evolutionary computation, or just fuzzy logic. There are many possibilities for combining the above mentioned technologies[1].

In large measure, fuzzy logic, neuro computing, and probabilistic reasoning are complementary, not competitive. It is becoming increasingly clear that in many cases it is advantageous to combine them. A case in point is growing number of “neurofuzzy” consumer products and systems that use a combination of fuzzy logic and neural-network techniques[2].

Hybrid tools developed using combinations of neural networks; evolutionary computation and fuzzy logic solve difficult problems, require relatively short development times, and are robust.

II. HYBRID ALGORITHMS ON CI

In recent years, computational intelligence has gained a widespread concern of many scholars emerging as a new field of study. In various fields of research and applications, more and more CI branches has made considerable progress, has become a hot research subject.

A. SMB Classification

The computational intelligence methods, although different from each other, share the property of being non-symbolic and operating in a bottom-up fashion, where structure emerges from an unordered begin and is different from the imposed method in AI[5].

CI mainly adopts Connectionism ideology and actually uses the bionics ideas for reference, it origins from emulating intelligent phenomenon in nature and is depicted in mathematics language. CI attempts to simulate and reappearance the characters of intelligence so that it can be a new research domain for reconstructing the nature and engineering. Bezdek consider that CI is based on data provided by the operator.
Computational Intelligence is a methodology class of high consistent in goal level and unique in technique level, which determines the characteristics of natural complementarity and integration among members of the CI. Combing CI branches is often able to give attention to different levels and perspectives in problem solving, and concerted by complementing each other which can achieve very good results. For complex systems solving problem in uncertain environment, any method can’t give the perfect answer to the exterior and interior of problem. In this background, only applying “cocktail therapy” similar to the biomedical application field can give a satisfactory answer to such a complex and difficult proposition. Mutual complementarity and integration of different CI branches provide strong support for complex system (decision-making, control, etc.) analysis and synthesis in uncertain environment. This is the original intention for the study on CI hybrid algorithm.

Research on CI hybrid algorithms has never stopped and obtained good results in practical applications. In this area, FNN and FEC are as the main research issues. Fuzzy logic simulates human uncertainty understanding of the world; fuzzy inference reflects the human reasoning processes. Neural network tries to simulate the human brain’s neural structure and solve complex problems by learning and training. Combining fuzzy logic with neural networks has made human understanding of the objective world turn into a learning process, which is close to the mode of human thought process and has provided a strong theoretical and methodological support for us to understand and transform the nature. Evolutionary Computing simulates the evolution process of biology group, putting the human mind (knowledge) into this process give birth to FEC, which achieve the optimal solution faster and more effective.

This article introduces hybrid algorithm of Fuzzy Neural Network and puts forward the M-ANFIS model finally.

B. Fuzzy-Neural Network

Neural network structures can deal with imprecise data and ill-defined activities. However, the subjective phenomena such as reasoning and perceptions are often regarded beyond the domain of conventional neural network theory [6]. It is interesting to note that fuzzy logic is another powerful tool for modeling uncertainties associated with human cognition, thinking and perception. In fact, the neural network approach fuses well with fuzzy logic and some research endeavors have given birth to the field of “fuzzy neural networks” or “fuzzy neural systems”. Algorithms based upon this integration are believed to have considerable potential in the areas of expert systems, medical diagnosis, control systems, pattern recognition and system modeling.

Takagi and Hayashi [7] were pioneers in the study on fusion of fuzzy logic and neural networks. In their study, multi neural networks were used to represent an inference method, and a neural network was trained to identify nonlinear borders of subspaces in the antecedent, while others were trained to identify nonlinear equations in the consequent parts. It was reported that the training data for each rule should be prepared before learning. Lin, Lee[8-9] and Wang, Mendel[10-11] have also proposed FNN structures and models respectively.

rather than relying on "knowledge", which solve problems through establishment of connections by training. CI is data-based intelligence.

It can be found in many CI branches that neural networks reflect the high-level structure of the brain; fuzzy systems mimic low-level structure of the brain; evolution computing has many similar characteristics with biological population evolution. All these methods play a special role in certain area respectively, but there are some inherent limitations of them. Combine some methods and cross each other on Computational Intelligence has become the new direction of CI development.

Based on this consideration, domestic and foreign scholars put forward a variety of CI classification methods, the purpose is to further explore the nature of Computational Intelligence and develop hybrid algorithms on CI methods. As a result, establishment of the correct classification method and in-depth study on the computational mechanism and connection of each method will give an approach to combine these CI methods effectively, which provide the theoretical basis for establishment of a unified intelligence system design and optimization methodology.

From the comprehensive discussion on the definitions of Computational Intelligence and correct understanding the nature and simulation mechanism of CI, this paper introduces the Simulation-Mechanism-Based classification method for Computational Intelligence according to the collection of all the branches of CI and all the different classification methods. This method divides all the branches into three categories: Organic mechanism simulation class (OMS), Inorganic mechanism simulation class (IMS) and Artificial mechanism simulation class (AMS). For a variety of CI branches in each category, we can develop new hybrid algorithms through combing them among internal or inter-category according to the actual requirements.

SMB classification method and some of the hybrid algorithms are shown in figure 1.

![Fig. 1. SMB Classification and Composite Patterns](image-url)
The main steps of a fuzzy inference are realized in sequentially ordered layers of a neural network with architecture such that the weights can be adjusted in the network usually by means of a gradient descent algorithm [6]. Tuning of the membership function can be carried out with the learning capability of the FNN. It means that a fuzzy-neural model can be considered as a universal approximator because of its infinite approximating capability by training.

Neuro-fuzzy modeling is concerned with the extraction of models from numerical data representing the behavioral dynamics of a system. Such models have great function of prediction system behavior and system control[12].

Fuzzy production rules are extracted by considering the strength of connections and the contribution of inputs as the output of each unit. So if we deem the output of each neuron as the weights, such as the value of membership function, we can change the membership function through the change of the weights by NN. So we can extract the changed rules. A fuzzy inference structure was incorporated into a feed forward-type neural network. It was easy to extract fuzzy rules that described I/O relationships of a nonlinear system from the trained fuzzy neural network.

Jang proposed ANFIS that represented the Takagi-Sugeno-Kang model. This model will be used throughout for controller design and evaluation. ANFIS uses back propagation learning to determine premise parameters and least mean squares estimation to determine the consequent parameters[13]. This is referred to as hybrid learning. In the first or forward pass, the input patterns are propagated, and the optimal consequent parameters are estimated by an iterative least mean square procedure, while the premise parameters are assumed to be fixed; In the second or backward pass the patterns are propagated again, and in this epoch, back propagation is used to modify the premise parameters by the gradient descent algorithm, while the consequent parameters remain fixed. This procedure is then iterated until the error criterion is satisfied.

ANFIS model is a universal approximator which has the non-linear modeling and forecasting function.

III. M-ANFIS

This paper presents a class of adaptive neural network equivalent of Mamdani fuzzy inference system in its function, which named M-ANFIS. It means adaptive network based fuzzy inference system.

Neural network has the great function of dealing with imprecise data by training, while fuzzy logic can deal with the uncertainty of human cognition. The nature of these two methods is a universal approximator and they have the function of non-linear modeling. In fact, neural networks and fuzzy logic have fused very well. Fuzzy neural networks implement main steps of fuzzy inference in an ordered layers of a neural network with an architecture such that the weights to be adjusted in the network, which makes fuzzy inference more closer to actual situation by learning capability of NN.

FNN are widely used in a lot of areas. Jang has brought forward Sugeno fuzzy inference model-based ANFIS. This paper will introduce a Mamdani model based Adaptive Neural Fuzzy Inference System (M-ANFIS), which has greater superiority to ANFIS in expression of consequent part and intuitive of fuzzy reasoning. This model will reflect nature of CI much more. The details will be introduced in the following section.

A. Model Description

Two well-known Fuzzy rule-based Inference System are Mamdani fuzzy method and Tagaki-Sugeno (T-S) fuzzy method. Advantages of the Mamdani fuzzy inference system:

1. It’s intuitive. 2. It has widespread acceptance. 3. It’s well-suited to human cognition [14-16].

The T-S fuzzy inference system works well with linear techniques and guarantees continuity of the output surface[17-18]. But the T-S fuzzy inference system has difficulties in dealing with the multi-parameter synthetic evaluation; it has difficulties in assigning weight to each input and fuzzy rules. Mamdani model can show its legibility and understandability to the laypeople. The Mamdani fuzzy inference system shows its advantage in output expression and is used in this project.

To completely specify the operation of a Mamdani fuzzy inference system, we need to assign a function for each of the following operators:

1) AND operator (usually T-norm) for the rule firing strength computation with AND’ed antecedents
2) OR operator (usually T-conorm) for calculating the firing strength of a rule with OR’ed antecedents
3) Implication operator (usually T-norm) for calculating qualified consequent MFs based on given firing strength
4) Aggregate operator (usually T-conorm) for aggregating qualified consequent MFs to generate an overall output MF
5) Defuzzification operator for transforming an output MF to a crisp single output value

If AND operator and Implication operator is product, and aggregate operator is sum, defuzzification operator is centroid of area (COA)[19], we derive the following theorems. Advantages of applying such composite inference methods are that such Mamdani ANFIS model has the ability of learning because of differentiability during computation.

The sum-product composition provides the following theorems[13], see in Eq.1 and Eq.2. Final crisp output when using centroid defuzzification is equal to weighted average of centroids of consequent MFs, where:

$$\psi(r_i) = \omega(r_i) \times a$$

where, $\psi(r_i)$ is the weighted factor of $r_i$; $r_i$ is the ith fuzzy rule; $\omega(r_i)$ is the firing strength of $r_i$; $a$ is the area of the consequent MFs of $r_i$.

$$Z_{COA} = \frac{\int_Z \mu_{C_{r_i}}(z)dz}{\int_Z \mu_{C_{r_i}}(z)dz} = \frac{\omega_1 a_1 z_1 + \omega_2 a_2 z_2}{\omega_1 a_1 + \omega_2 a_2}$$

Where, $a_i$ and $z_i$ are the area and the center of the consequent $MF_{C_{r_i}}(z)$ respectively.
the membership function is the generalized bell function.

\[ \omega_1(x) = \frac{1}{1 + \left( \frac{(x - c_1)}{d_1} \right)^{2b_1}} \]  (4)

where \( \{b_i, c_i, d_i\} \) is the parameter set referred to as premise parameters.

Layer 2: Firing strength \( \omega_i \) is generated with product method.

\[ O_{2,i} = \omega_i = \mu_{A_i}(x) \times \mu_{B_i}(y), \quad i = 1, 2. \]  (5)

Layer 3: Implication layer

\[ O_{3,i} = \sum f_i = \sum \omega_i a_i, \quad i = 1, 2. \]  (6)

Layer 4: Aggregate operator is sum. The consequent parameters are determined by \( C_i \). If the consequent MF is trapezoidal membership function, each MF has 4 nonlinear parameters to be adjusted.

Layer 5: Defuzzification layer

\[ O_5 = f = D \circ O_4 \]  (14)

The crisp output \( f \) is achieved with the defuzzification method, COA(center of area).

\{b_i, c_i, d_i\} are premise parameters. The type of membership functions (MF) of the inputs are generalized bell functions, each MF has 3 nonlinear parameters; each consequent MF has 2 nonlinear parameters which are area and center of the consequent part. Totally, there is 16 parameters in this example.

A general M-ANFIS model can be expressed as Fig.3.

Rule 1: If \( x \) is \( A_1 \) and \( y \) is \( B_1 \), then \( Z = C_1 \);

Rule 2: If \( x \) is \( A_2 \) and \( y \) is \( B_2 \), then \( Z = C_2 \).
method and Sequential (Approximate) LSE Only method. The choice of above methods should be based on the trade-off between computation complexity and resulting performance.

In this paper, we apply the Gradient Descent for all model parameters modification and all these parameters are nonlinear parameters.

When there is adequate training data, we can achieve M-ANFIS model. We can also test the M model by checking data.

B. Weight Updating Formula

Weight updating formulas are very important for adjusting M-ANFIS model parameters. In this section, we conclude the weight updating formula for M-ANFIS model by discussing the general weight updating formula based on basic idea of back propagation in NN.

An adaptive network is a network structure whose overall input-output behavior is determined by a collection of modifiable parameters[13]. A feed forward adaptive network is a static mapping between its inputs and output spaces. Our goal is to construct a network for achieving a desired nonlinear mapping. This nonlinear mapping is regulated by a data set consisting of desired input-output pairs of a target system to be modeled: this data set is called training data set. The procedures that adjust the parameters to improve the network’s performance are called the learning rules.

A learning rule explains how these parameters (or weights) should be updated to minimize a predefined error measure. The error measure computes the discrepancy between the network’s actual output and a desired output[20-24]. The steepest descent method is used as a basic learning rule. It is also called backpropagation[20-21].

Our task is to minimize an overall error measure defined as:

$$E_p = \sum_{k=1}^{N(L)} (d_k - x_{i,k})^2 \quad (15)$$

where, $d_k$ is the $k$th component of the $p$th desired output vector and $x_{i,k}$ is the $k$th component of the predicted output vector produced by presenting the $p$th input vector to the network.

In order to minimize the overall error measure, we have to calculate gradient vector, which is defined as the derivative of the error measure with respect to the parameter variables.

In order to calculate gradient vector, error signal must be obtained, which is defined as the derivative of the error measure with respect to the output of a neuron. Once we obtain the gradient vector by chain rules, we can conclude the parameters updating formula for the whole network. The main idea is to go along (learn) against the direction of the gradient vector, and update the parameters by learning rules, eventually we can minimize the overall error measure for network output.

We deliver the weight updating formula in M-ANFIS.

$$\Delta \omega_{ij} = -\eta \frac{\partial E}{\partial \omega_{ij}}$$

$$= -\eta \frac{\partial E}{\partial f_i} \frac{\partial f_i}{\partial \omega_{ij}}$$

$$= -\eta \varepsilon_i \frac{\partial f_i}{\partial \omega_{ij}} \quad (16)$$

$$\omega_{\text{next}} = \omega_{\text{now}} - \eta \frac{\partial E}{\partial \omega_{ij}} \quad (17)$$

where, $j < i$, that is $x_i = f_i(\sum \omega_{ij} x_j + \theta)$.

Error signal $\varepsilon_i$ starts from the output layer and goes backward layer by layer until the input layer is attained. The error signals of each node can be derived by error signals in previous layer nodes. In this formula, $\frac{\partial f_i}{\partial x_j}$ (one element in error signal calculation formula ) and $\frac{\partial f_i}{\partial \omega_{ij}}$ can be obtained by derivation. So we can modify the network parameters according to the upper formula.

If we assume that $\omega_{ij} = \frac{\partial f_i}{\partial x_j}$ or $x_j = \frac{\partial f_i}{\partial \omega_{ij}}$, the parameter updating formula for each node can be derived, and the weights in whole network can be updated. For example, $\Delta \omega_{ji} = -\eta \varepsilon_i x_j$.

The general weight-updating formula is

$$\Delta \omega_{ji} = -\eta (d_i - x_i) x_j X, \quad (18)$$

where, $\eta$ is the learning step, $d_i$ is the desired output for node $i$, $x_i$ is the real output for node $i$, $x_j$ is the input for node $i$, $X$ is a Polynomial, usually $(x_i \times (1 - x_i))$.

IV. EXPERIMENTS

In order to verify the validity of M-ANFIS model presented in this paper, we apply this M-ANFIS into the evaluation of traffic level of service. Through training and testing this model by historical sample data, the results indicate that this model has ability of mapping traffic sensors data to the value of LOS. In the mean time, it is illuminated that M-ANFIS model shows great superiority to ANFIS model according to the experiments results analysis between them. Sample data in this experiment are supported by Beijing traffic management bureau. All these data are obtained by experts confirm and are reliable.

A. Traffic Level-Of-Service Evaluation

In urban traffic systems, Level of service (LOS) is a quality measure describing operational conditions within a traffic stream, generally in terms of such service measures as speed and travel time, freedom to maneuver, traffic interruptions, and comfort and convenience.

Essentially, level of service reflects subjective judgments of drivers about the traffic condition. Now a day in Beijing, evaluation of LOS is a fundamental factor for daily traffic decision-making. Existed LOS evaluating methods, up to now, are all based on single input index by creating a hypothetical correlation between LOS value and one index, which can not reflect the true traffic condition reasonably. It’s very important
to establish multi-indices synthetic LOS evaluating method in LOS study.

Basic sensor data (such as volume, occupancy and speed) are obtained by detector and are effective for LOS synthetic evaluation. All these data are divided into training data and testing data. According to potential mapping relation between LOS and those three indices, we apply this M-ANFIS to LOS evaluation. Consequently, experiments by these sample data shows that the M-ANFIS model introduced in this paper provide theoretical basis and a new methodology for multi-inputs synthetic LOS evaluating.

B. M-ANFIS model

As stated above, Our goal is to construct mapping model between traffic sensor data (such as volume, occupancy and speed) and LOS, based on which, this paper puts forward the LOS evaluating method using M-ANFIS model. The method integrates the elementary features of M-ANFIS into traffic LOS evaluation, which has three inputs and one output with 6 MFs.

In the project of Level of Service, we can obtain the sensor data such as: speed, volume and occupancy. Level of service standards in consequent part can be determined by subjectivity and by analysis of historical data. But this method for grade is not exact; we must modify nonlinear borders of membership functions in the consequent part as well as the premise part.

In ANFIS, the output of each rule is a linear combination of input variables plus a constant term, and the final output is the weighted average of each rule's output. Level of service reflects the driver’s subjective feelings on the road; obviously the above model can’t reflect the true nature of LOS because of its linear output. M-ANFIS model is able to resolve this issue because grades of LOS in consequent part are expressed as membership function rather than a simple linear equation. M model reflects the true meaning of LOS and logic reasoning of drivers.

In Fig.4, the evaluating process of LOS is illustrated.

![Fig. 4. Process of Mamdani ANFIS for LOS Evaluation](image)

Consequently, we construct the following model. See in Fig.5.

In this model, x, y, z represents the input, which is speed, volume, and occupancy. A1-A3 represents membership functions of speed; B1-B3 represents membership functions of volume; C1-C3 represents membership functions of occupancy. D1-D6 represents membership functions of LOS output. Rule’s format is:

**Rule i**: If x is A1 and y is B1 and z is C1, then LOS=A.

In this model, premise parameters are 27 and consequent parameters are 24, which are all nonlinear parameters. We adjust all these parameters in M-ANFIS based on the weights updating formula, which is shown in Eq.16 and Eq.17.

The training process takes 0.521 second and 450 steps. The mean square error is 0.00029106. The training errors are
shown in Fig.8.

The desired output and real output of M-ANFIS are in Fig.9.

Testing errors are in Fig.10. Average test error is 0.045327.

Indices of ANFIS and M-ANFIS are in TableI.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>M-ANFIS</th>
<th>ANFIS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter Number</td>
<td>51</td>
<td>135</td>
</tr>
<tr>
<td>Training Steps</td>
<td>450</td>
<td>1429</td>
</tr>
<tr>
<td>Training Error</td>
<td>0.00029106</td>
<td>8.3482e-006</td>
</tr>
<tr>
<td>Testing Error</td>
<td>0.045327</td>
<td>0.09148</td>
</tr>
<tr>
<td>Consume Time (s)</td>
<td>0.521</td>
<td>8.392</td>
</tr>
</tbody>
</table>

From the above table, we can conclude that M-ANFIS model is superior to ANFIS in amount of adjusted parameters, scale of training data ,consume time and testing error. Training error satisfies the requirements. It is clear that M-ANFIS is more effective subject to small-scale sample data. In the experiment, M-ANFIS, with 6 MFs in the consequent part, reflects the essence of traffic LOS precisely.

V. Conclusion

Methods from computational intelligence have made great contributions in all specific application domains. As we know, it is hybrid algorithms research that has become a big trend in computational intelligence.

According to the SMB classification, we illustrate the existed and uncovered patterns of hybrid algorithm research, which provides a innovating roadmap for proposing new hybrid algorithms. This SMB classification divides all the branches into three categories: Organic mechanism simulation class, Inorganic mechanism simulation class and Artificial mechanism simulation class. All existed hybrid algorithms are developed through combing different methods among internal or inter-category according to specific requirements. By surveying on the existed hybrid algorithms, we find that there are some famous kinds of hybrid algorithms called Fuzzy-neural network and fuzzy evolutionary computing. In FNN, ANFIS, introduced by R. Jang, is the most popular one.

In the process of fuzzy inference, ANFIS adopts a linear equation in consequent part, which can not exhibit human’s judgement reasonably. So, we propose the Mamdani model based adaptive fuzzy inference system (M-ANFIS),which has greater superiority in consequent part and intuitive of fuzzy reasoning. M-ANFIS is a universal approximator because of its infinite approximating capability by training. All parameters in M-ANFIS are nonlinear parameters which can be adjusted by learning rules discussed above. M-ANFIS model can show its legibility and understandability and exhibit the essence of fuzzy logic more clearly.

Finally, we use M-ANFIS into traffic LOS evaluation. The experimental results show that M-ANFIS model is superior to ANFIS in amount of adjusted parameters, scale of training data ,consume time and testing error. M-ANFIS, as a new hybrid algorithm in computational intelligence, has great function of non-linear modeling and forecasting. The work in this paper proposes a new hybrid algorithm called M-ANFIS model, as
well as provides an effective and efficient approach for hybrid algorithms study in CI.

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