International Journal of Mechanical, Industrial and Aerospace Sciences
ISSN: 2517-9950
Vol:7, No:4, 2013

Magnetohydrodynamic Free Convection in a Square Cavity Heated from Below and Cooled from Other Walls

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***Abstract*—**Magnetohydrodynamic free convection fluid flow and heat transfer in a square cavity filled with an electric conductive fluid with Prandtl number of 0.7 has been investigated numerically. The horizontal bottom wall of the cavity was kept at T_h while the side and the top walls of the cavity were maintained at a constant temperature T_c with T_h>T_c. The governing equations written in terms of the primitive variables were solved numerically using the finite volume method while the SIMPLER algorithm was used to couple the velocity and pressure fields. Using the developed code, a parametric study was performed, and the effects of the Rayleigh number and the Hartman number on the fluid flow and heat transfer inside the cavity were investigated. The obtained results showed that temperature distribution and flow pattern inside the cavity depended on both strength of the magnetic field and Rayleigh number. For all cases two counter rotating eddies were formed inside the cavity. The magnetic field decreased the intensity of free convection and flow velocity. Also it was found that for higher Rayleigh numbers a relatively stronger magnetic field was needed to decrease the heat transfer through free convection.

***Keywords*—**Free Convection, Magnetic Field, Square Cavity, Numerical Simulation.

I. INTRODUCTION

FREE convection heat transfer in cavities occurs in many industrial applications such as electronic equipment, reactor insulation, ventilation of rooms, fire prevention, solar collector and crystal growth in liquids [1]. When the cavity is filled with an electric conductive fluid, with the help of a magnetic field the fluid flow and temperature distribution inside the cavity can be controlled. When the fluid is electrically conducting, by enforcing a magnetic field, the Lorentz force is generated which interacts with buoyancy force and reduces the velocities. In manufacturing industry, an external magnetic field is used for the better control of solidification and crystals growth which results in high quality manufactured products. As early as 1983, Oreper and Szekely [2] investigated effect of a magnetic field on free convection heat transfer in a rectangular enclosure and showed that the magnetic field strength is one of the most important parameters for crystal formation and suppresses the free convection currents. Ozoe and Maruo [3] conducted a numerical study on magnetohydrodynamic free convection fluid flow and heat transfer of a low Prandtl number fluid. They obtained correlations for the Nusselt number in terms of Rayleigh, Prandtl and Hartman numbers. In a numerical study Rudraiah et al. [4] investigated the effect of a transverse magnetic field on free convection heat transfer and fluid flow in a differentially heat rectangular with isothermal side walls and adiabatic horizontal walls. Their results showed that a circulating flow is formed with a relatively weak magnetic field. Moreover they found that with increasing the magnetic field, the convective heat transfer decreases. Al-Najem et al. [5] investigated numerically the effects of a transverse magnetic field free convection heat transfer and fluid flow in a tilted square enclosure with isothermal side walls and isolated horizontal walls. Mahmud and Faster [6] investigated the magnetohydrodynamic free convection and entropy generation for a square enclosure at low Hartman numbers. They found that the fluid velocity is reduced with increasing the value of the Hartman number. Krakov et al. [7] investigated numerically and experimentally effects of a uniform magnetic field on natural convection in a cubic enclosure. They found that a set of numerous convective structures exist in the cube. The problem of magnetohydrodynamic natural convection in an inclined enclosure differentially heated on adjacent walls was studied by Ece and Büyük [8]. Their results showed that the orientation and the aspect ratio of the enclosure and the strength and direction of the magnetic field have significant effects on the flow and temperature fields. Wang et al. [9] reported results of a numerical study on magnetohydrodynamic natural convection in a porous media filled square cavity. They used the Brinkman–Forchheimer extended Darcy model to solve the momentum equations, and the local thermal non-equilibrium (LTNE) models to solve energy equations for fluid and solid. They found that both the magnetic force and the inclination angle have significant effect on the flow field and heat transfer in porous medium. Kandaswamy et al. [10] studied numerically magnetohydrodynamic natural convection in a square cavity with partially thermally active side walls. They considered nine different combinations of the relative positions of the active portions. Their results showed that when the active portions are located at the middle of the side walls, maximum rate of heat transfer occurs. Moreover they found that the average Nusselt number decreases with an increase of Hartmann number and increases with an increase of Grashof number. Oztop et al. [11] investigate numerically the

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magnetohydrodynamic free convection in non-isothermally heated square enclosure. They observed that the heat transfer decreases with increasing Hartman number and decreasing amplitude of sinusoidal function. Pirmohammadi et al. [12] studied steady laminar free convection flow in presence of a magnetic field in an enclosure heated from left and cooled from right. The result showed that with increasing Hartman number the rate of convective transfer and the average Nusselt number increase. Nithyadevi et al. [13] using a numerical simulation investigated effect of time periodic boundary conditions on magnetohydrodynamic natural convection in a square cavity with partially heated and cooled side walls. They found that the flow and the heat transfer rate in the cavity are affected by the sinusoidal temperature profile and by the magnetic field at lower values of Grashof number. Moreover they found that the maximum rate of heat transfer occurs for the active portions located at the middle of the side walls. Very recently Mahmoodi and Talea'pour [14] investigated numerically magnetohydrodynamic free convection in a square cavity with hot left wall, cold top wall and insulated right and bottom wall. They found that a clockwise primary eddy is formed inside the cavity regardless the Rayleigh number and the Hartman number. Also they found that the magnetic field decrease the intensity of free convection and flow velocity.

In the present paper, free convection in a square cavity heated from blow and symmetrically cooled from other walls is investigated numerically. The governing equations are solved using the finite volume method. The SIMPLER algorithm is used to couple the pressure and velocity field. The effects of the Hartman number and the Rayleigh number on the fluid flow and heat transfer inside the enclosure are investigated. The results are presented in terms of streamlines and isotherms inside the cavity, vertical component of the velocity along the horizontal centerline of the cavity, local Nusselt number along the hot wall, and average Nusselt number of the hot wall.

II. Mathematical Modeling

A schematic geometry of the square cavity with boundary conditions considered in the present paper is shown in Fig. 1. The height and the width of the cavity are denoted by H. The bottom wall is kept at a constant high temperature T_h, while, the side walls and the top wall are kept at cold temperature T_c. The length of the cavity perpendicular to its plane is assumed to be long enough; hence the problem is considered two dimensional. The magnetic field of strength B_0 is applied parallel to x-axis. The cavity is filled with an electric conductive fluid with Pr = 0.7 that is considered Newtonian and incompressible. The fluid flow is assumed to be laminar. The thermophysical properties of the fluid are considered constant with the exception of the density which varies according to the Boussinesq approximation [15].

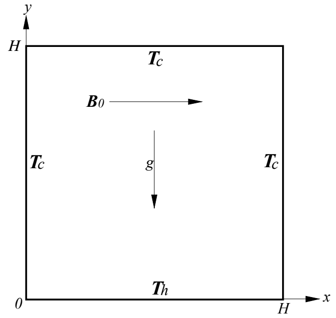

Fig. 1 Schematic view of the cavity with boundary conditions considered in the present paper

The continuity, momentum and energy equations for laminar, steady state, two-dimensional free convection with a magnetic field in x-direction, are as follow:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+\rho g\beta\left(T-T_c\right)-\sigma B_0^2 v \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

where u and v are the velocity components, p is pressure, ρ is the density, μ is the dynamic viscosity, β is the thermal expansion coefficient, σ is the electrical conductivity, B_0 is the magnitude of magnetic field, T is the temperature and α is the thermal diffusivity.

Using the following dimensionless parameters, the governing equations can be converted to the non-dimensional forms:

$$X=\frac{x}{l},Y=\frac{y}{l},U=\frac{uH}{\alpha},V=\frac{vH}{\alpha},P=\frac{pH^2}{\rho\alpha^2},\theta=\frac{T-T_c}{T_h-T_c}. \tag{5}$$

The non-dimensional forms of the governing equations are:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{6}$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+Pr\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \tag{7}$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+Pr\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+Ra\,Pr\,\theta-Ha^2\,Pr\,V \tag{8}$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2} \tag{9}$$

where Ra, Pr and Ha are the Rayleigh, Prandtl and Hartman numbers and are defined as:

$$Ra = \frac{g\beta(T_h - T_c)H^3}{\alpha\nu},\ Pr = \frac{\upsilon}{\alpha}, Ha = B_0 H\sqrt{\frac{\sigma}{\rho\upsilon}} \tag{10}$$

where υ is the kinematic viscosity. The effect of magnetic field into the momentum equation is introduced through the Lorentz force term, $\vec{J} \times \vec{B}$ that is reduced to $-\sigma B_0 v^2$ as shown by Mahmoodi and Talea'pour [14].

To computation of the rate of heat transfer, Nusselt number along the hot wall of the enclosure is used that is as follows:

$$Nu_{local} = -\left.\frac{\partial\theta}{\partial Y}\right|_{Y=0} \tag{11}$$

The average Nusselt number of the hot wall is obtained as follows:

$$Nu = \int_0^1 Nu_{local}\, dX \tag{12}$$

The non-dimensional boundary conditions for solving the governing (6-9) are:

$$\begin{cases} \text{On the hot wall:} & U = V = 0, \theta = 1 \\ \text{On the cold walls:} & U = V = \theta = 0 \end{cases} \tag{13}$$

III. Numerical Approach

The governing mass, momentum and energy equations written in terms of primitive variable are discretized using the finite volume method. The diffusion terms in the governing equations are approximated by a second order central difference scheme while a hybrid scheme which is a combination of central difference scheme and upwind scheme is applied to discretize convective terms. A staggered grid system, in which the velocity components are stored midway between the scalar storage locations, is used. In order to couple the velocity field and pressure in the momentum equations, the SIMPLER algorithm is used. The set of algebraic equations are solved iteratively using TDMA algorithm [16].

In order to validate the numerical procedure, the results obtained by the present code for a differentially heated square enclosure under a magnetic field are compared with the results of Pirmohammadi et al. [12] for the same problem. Table I shows a comparison between the average Nusselt number obtained by the present code with the results of Pirmoammadi et al. for different Rayleigh and Hartman numbers. As can be observed from the table, very good agreements exist between the two results.

TABLE I
COMPARISON BETWEEN THE AVERAGE NUSSELT NUMBERS OF PRESENT STUDY AND THOSE OF PIRMAHAMMADI ET AL.

Ra	*Ha*	*Nu*	
		Pirmahammadi et al. [12]	Present study
	0	2.29	2.289
10^4	50	1.06	1.061
	100	1.02	1.019
	0	4.62	4.631
10^5	25	3.51	3.507
	100	1.37	1.365

In order to determine a proper grid for the numerical simulation, a grid independence study is undertaken for magnetohydrodynamic free convection inside the cavity considered in Fig. 1 at $Ra = 10^6$ and $Ha = 50$. Four different uniform grids, namely, 10×10, 30×30, 60×60, and 120×120 are employed and for each grid size, average Nusselt number of the hot wall is obtained. Table II shows the average Nusselt number of the hot wall for different grids. As it can be observed from the table, a 60×60 uniform grid is sufficiently fine to ensure a grid independent solution. Based on these results, a 60×60 uniform grid is used for all the results to be presented in the following.

TABLE II
AVERAGE NUSSELT NUMBER OF THE HOT WALL FOR DIFFERENT GRID SIZES

Grid Size	10×10	30×30	60×60	120×120
Nu	8.7492	10.2395	13.1639	13.1712

IV. Results and Discussions

In this section, results of the numerical study on magnetohydrodynamic free convection fluid flow and heat transfer in a square cavity filled with an electric conductive fluid with $Pr = 0.7$ are presented. The results have been obtained for the Rayleigh number ranging from 10^3 to 10^6 and the Hartman number varying from 0 to 100. The results are presented in terms of streamlines and isotherms inside the cavity, the vertical velocity component along the horizontal midline of the cavity, the local Nusselt number along the hot wall, and the average Nusselt number of the hot wall.

Variation of streamlines and isotherms inside the cavity with Rayleigh number and Hartman number are shown in Figs. 2 and 3, respectively. As can be observed from the figures with existence of the symmetrical boundary conditions about the vertical centerline of the cavity, the flow and temperature fields are symmetrical about this line. As can be seen from the streamlines in the Fig. 2, a pair of counter-rotating eddies are formed in the left and right half of the cavity for all Rayleigh numbers and Hartman numbers considered. Each cell ascend through the symmetry axis, then faces the upper wall and moves horizontally and finally descends along the corresponding clod side wall.

At $Ra = 10^3$ and in the absence of the magnetic field ($Ha = 0$) the elliptic-shaped eyes of eddies are in the lower half of the cavity. With increase in Rayleigh number and buoyancy force, the eyes of the eddies move upward insofar as at $Ra = 10^5$, locate in the upper half of the cavity. At $Ra = 10^6$ the eyes are elongated along the height of the cavity. Conduction dominant heat transfer is observed form the isotherms in Fig. 3 at $Ra = 10^3$ and $Ha = 0$. With increase in Rayleigh number, the isotherms are condensed next to the side walls which means increasing heat transfer through convection. Formation of thermal boundary layers can be found from the isotherms at $Ra = 10^5$ and 10^6 and at $Ha = 0$.

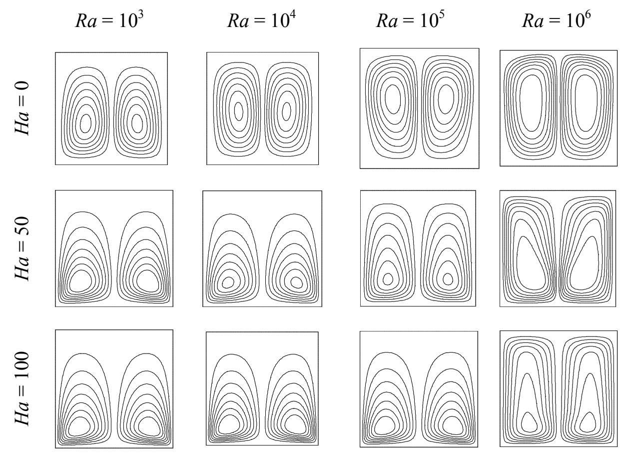

Fig. 2 Streamlines at different Rayleigh numbers and Hartman numbers

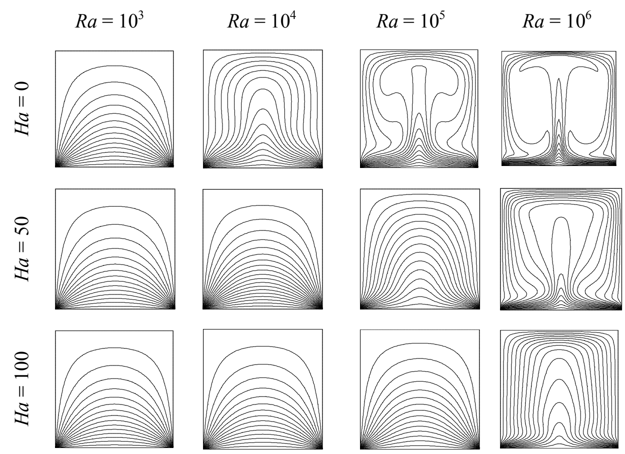

Fig. 3 Isotherms at different Rayleigh numbers and Hartman numbers

From the streamlines it is found that with increasing the Hartman number (increasing the strength of the magnetic field) the eyes of eddies move downward and close to the lower corners of the cavity. This phenomenon means decrease in the flow velocity with increasing Hartman number. From the isotherms it is observed that with increase in Hartman number, free convection is suppressed and heat transfer occurs mainly through convection. At $Ha = 50$ and for $Ra = 10^3$ and 10^4, the isotherms illustrate a pure conduction heat transfer, while at $Ra = 10^5$ a combination of conduction and weak free convection observed from the corresponded isotherms. As a result it can be said that with increase in the buoyancy force via increase in Rayleigh number, to decrease free convection, a stronger magnetic field is needed compared to the lower Rayleigh numbers. At $Ha = 100$ pure conduction is observed from the isotherms. A similar temperature distribution is observed for $Ra = 10^3$, 10^4 and 10^5 at $Ha = 100$.

Effects of the magnetic field on the flow and temperature fields at $Ra = 10^6$ are completely different from the previous results at lower Rayleigh numbers. At $Ha = 50$ the thickness of the thermal boundary layers decreases. At $Ha = 100$ the isotherms are parallel with the horizontal walls in the vicinity of these walls, while in the major portion of the cavity the isotherms are parallel with the side walls. From the streamlines it is observed that at $Ha = 50$, the centrally located eyes of eddies move downward and their shape converts from ellipse to a right triangle. At $Ha = 100$ the shape of the core of eddies converts to isosceles triangle and locates close to the hot bottom wall.

Variations of the vertical velocity component along the horizontal centerline of the cavity with the Rayleigh number and for $Ha = 0$ are shown in Fig. 4. It can be seen from the figure that the absolute value of maximum and minimum value of velocity increases with increasing the Rayleigh number (increasing the buoyant force).

Effect of the Hartman number on vertical component of the velocity along the horizontal midline of the cavity at $Ra = 10^6$ is shown in Fig. 5. As can be seen from the figure with increase in Hartman number motivates the flow velocity to decreases. A very slow fluid velocity occurs at $Ha = 100$. It is found that free convection heat transfer decreases with increase in fluid velocity via increasing the Hartman number.

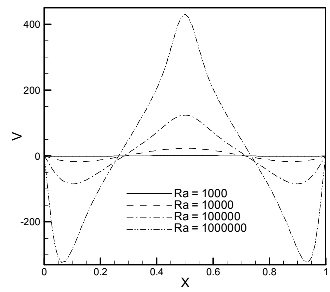

Fig. 4 Variation of the vertical velocity component along the horizontal centerline of cavity with Rayleigh number at $Ha = 0$

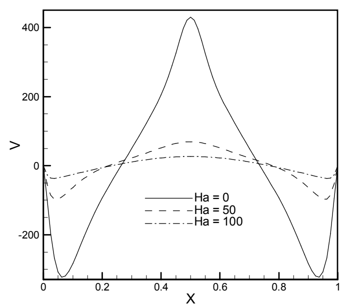

Fig. 5 Variation of the vertical velocity component along the horizontal centerline of cavity with Hartman number at $Ra = 10^6$

Variations of the local Nusselt number along the hot bottom wall with the Rayleigh number in the absence of the magnetic field (Ha = 0) are shown in Fig. 6. Owning to the symmetry in thermal boundary conditions, the local Nusselt number is symmetrical with respect to the vertical midline of the cavity. It can be seen from the figure that the local Nusselt number increases with the Rayleigh number in major portion of the hot wall. In the middle of the bottom wall the local Nusselt number equals to zero and does not change significantly with increase in the Rayleigh number.

Variations of the local Nusselt number along the hot wall of the cavity with the Hartman number at $Ra = 10^6$ are shown in Fig. 7. As can be observed from the figure in whole portion of the cavity the local Nusselt number decreases with increase in the Hartman number and the Lorentz force.

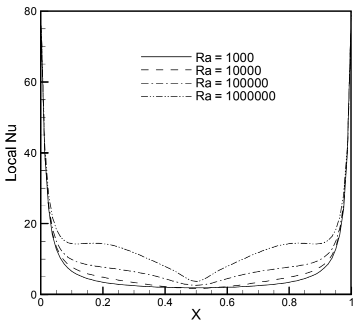

Fig. 6 Variation of the local Nusselt number along the hot wall of cavity with Rayleigh number at Ha = 0

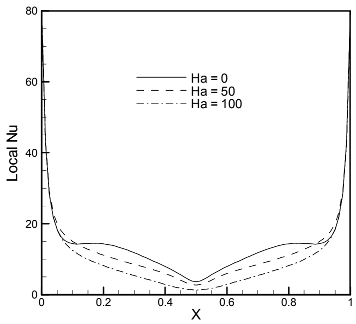

Fig. 7 Variation of the local Nusselt number along the hot wall of cavity with Hartman number at $Ra = 10^6$

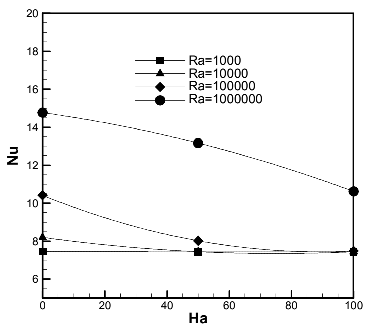

Fig. 8 Variation of the average Nusselt number of the hot wall of cavity with Hartman number at different Rayleigh numbers

Plot of the average Nusselt number of the hot wall as a function of Hartman number at different Rayleigh numbers is shown in Fig. 8. For a fixed Rayleigh number, with increase in the Hartman number the flow velocity decreases, the free convection is suppressed and finally the rate of heat transfer decreases. At Ha = 100 the average Nusselt number is equal for $Ra = 10^3$, 10^4 and 10^5, while at $Ra = 10^6$ a higher heat transfer rate occurs. At a constant Hartman number, with increase in Rayleigh number the buoyancy force increases and the heat transfer is enhanced. Therefore at high Rayleigh numbers, a relatively stronger magnetic field is needed to decrease the rate of heat transfer.

V. Conclusion

Using a numerical simulation based on the finite volume method, the Magnetohydrodynamic free convection fluid flow and heat transfer in a square cavity heated from below and cooled from other walls filled with an electric conductive fluid with Prandtl number of 0.7 was studied numerically. The numerical procedure was validated by comparing the average Nusselt number for a differentially-heated square enclosure obtained by the code with the existing results in the literature. Very good agreements were observed between them. Subsequently, a parametric study was performed and the effects of the Rayleigh number and the Hartman number on the fluid flow and heat transfer were investigated.

For all cases considered, two counter rotating eddies were formed inside the cavity regardless the Rayleigh and the Hartman number. The obtained results showed that the heat transfer mechanisms, temperature distribution and the flow characteristics inside the cavity depended strongly upon both the strength of the magnetic field and the Rayleigh number. Also it was found that using the longitudinal magnetic field results in a force (Lorentz force) opposite to the flow direction that tends to decrease the flow velocity. Moreover it was observed that, for low Rayleigh numbers, by increase in the Hartman number, free convection is suppressed and heat transfer occurs through conduction mainly.

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