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LQR Based PID Controller Design for 3-DOF Helicopter System

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Abstract—In this article, LQR based PID controller design for 3DOF helicopter system is investigated. The 3-DOF helicopter system is a benchmark laboratory model having strongly nonlinear characteristics and unstable dynamics which make the control of such system a challenging task. This article first presents the mathematical model of the 3DOF helicopter system and then illustrates the basic idea and technical formulation for controller design. The paper explains the simple approach for the approximation of PID design parameters from the LQR controller gain matrix. The simulation results show that the investigated controller has both static and dynamic performance, therefore the stability and the quick control effect can be obtained simultaneously for the 3DOF helicopter system.

Keywords—3DOF helicopter system, PID controller, LQR controller, modeling, simulation.

I. Introduction

The 3-DOF helicopter system is a benchmark laboratory model for theoretical study on helicopter controls and verification of the control algorithm. The helicopter is a complex mechanical system with strongly nonlinear characteristics and has open-loop unstable dynamics which make the control of such system a challenging task. The purpose of the control is to regulate desired pitch and roll positions as well as angular travel speed of the prototype 3DOF helicopter model. The 3-DOF helicopter system (see

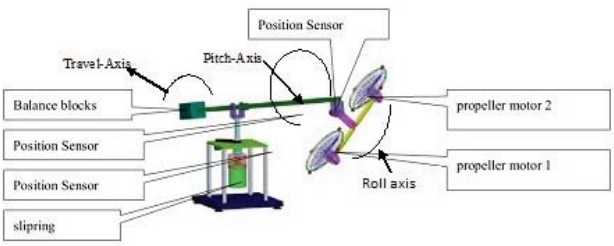

Fig. 1. 3-DOF Helicopter System [1]

Fig. 1) consists of base upon which an arm is mounted. The arm carries the helicopter body on one end and a balance block on the other end. The arm can roll about an pitch axis as well as swivel about a travel axis. Sensors that are mounted on these axes allow measuring the roll and travel of the arm. The helicopter body is mounted at the end of the arm and is free to swivel about a pitch axis. The pitch angle is measured via third sensor. Two motors with propellers mounted on the helicopter body can generate a force proportional to the voltage applied to the motors.The force generated by the propellers can cause the helicopter body to lift off the ground. The purpose of the balance block (counterweight) is to reduce the power requirements on the motors.

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The 3DOF helicopter system control involves linearization of the nonlinear dynamics about a set of pre-selected equilibrium conditions within the flight envelope. Based on the obtained linear models, classical single input single-output (SISO) techniques with a PID controller are widely used [2]–[5], etc. In another control approach, optimal tracking strategy using Linear quadratic regulator (LQR) control for helicopter model was proposed in [6]. Fuzzy and PID combined control used for an unmanned helicopter was discussed in [7]. Conventional PID controller's tuning methods seem inadequate for achieving better control performance because of bad tuning of parameters therefore these situations give strong motivation to LQR based PID control strategy for helicopter model.

In this article, LQR based PID controller design for 3DOF helicopter system is investigated. The paper begins with mathematical modeling of the 3DOF helicopter system and then controller design methodology is briefly discussed to deal with both performance and stability for the system. The design problem is then dealt with finding a LQR controller gain matrix, which gives a control solution. Finally an approximation method is suggested for finding the design parameters for PID controller from the obtained LQR controller gain matrix. The whole procedure involves selecting several parameters and the computation is simple, so it serves as a PID tuning method for 3DOF helicopter system. The simulation results show that the method is easy to use and the resulting PID controller has good time-domain performance and robustness of the system.

The rest of the article is organized as follows: In Section II, we present a mathematical modelling of helicopter system. Section III illustrates basic ideas and technical formulations for controller design. Section IV discuss about the results and simulation analysis and Section V concludes the paper with some remarks and conclusion.

II. Mathematical Modelling of Helicopter System Dynamics

Understanding the flight behavior has become essential to ensure control. Helicopter models are now well established [8],

[9] within the reach of many fields (academic and commercial purposes). Indeed, the ability to describe and explain the various phenomena involved and interacting in the helicopter dynamics has a large impact in practice. The aim of modeling is then to evaluate and control as soon as possible.

A. Pitch Axis Model

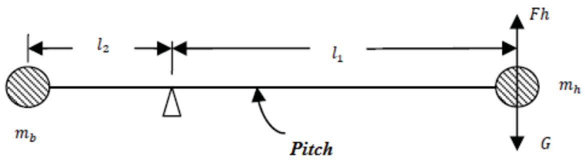

Fig. 2. Schematic Diagram for Pitch Axis Model

Consider the diagram in Fig. 2. Assuming the roll is zero, then the pitch axis dynamics of 3-DOF helicopter system can be modelled as:

$$\begin{aligned} J_e\ddot{\epsilon} &= l_1 F_h - l_1 G \\ &= l_1(F_1 + F_2) - l_1 G \\ &= K_c l_1 (V_1 + V_2) - T_g \\ \Rightarrow J_e\ddot{\epsilon} &= K_c l_1 V_s - T_g \end{aligned} \tag{1}$$

where ϵ is the pitch angle, $J_e = m_h l_1^2 + m_b l_2^2$ is the moment of inertia of the system about pitch axis, m_b is the mass of balance blocks, m_h is the total mass of two propeller motor, V_1 and V_2 are the voltages applied to the front and back motors resulting in force F_1 and F_2, K_c is the force constant of the motor-propeller combination, l_1 is the distance from the pivot point to the propeller motor, l_2 is the distance from the pivot point to the balance blocks, T_g is the effective gravitational torque due to the mass differential G about the pitch axis.

B. Roll Axis Model

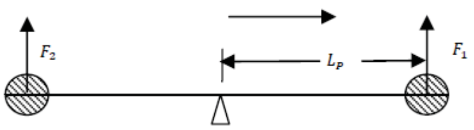

Fig. 3. Schematic Diagram for Roll Axis Model

Consider the diagram in Fig. 3. If the force generated by the left motor is higher than the force generated by the right motor, the helicopter body will be overturned clockwise. The roll axis dynamics of 3-DOF helicopter system can be modelled as:

$$\begin{aligned} J_p\ddot{p} &= F_1 l_p - F_2 l_p \\ &= K_c l_p (V_1 - V_2) \\ \Rightarrow J_p\ddot{p} &= K_c l_p V_d \end{aligned} \tag{2}$$

where p is the roll angle, J_p is the moment of inertia of the system about the roll axis, l_p is the distance from the roll axis to either motor.

C. Travel Axis Model

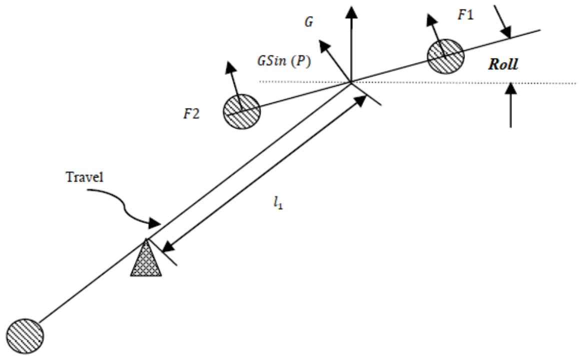

Fig. 4. Schematic Diagram for Travel Axis Model

When the roll axis is tilted and overturned, the horizontal component of G will cause a torque about that the travel axis which results in an acceleration about the travel axis. Assume the body has roll up by an angle p as shown in Fig. 4. The travel axis dynamics of 3-DOF helicopter system can be modelled as:

$$\begin{aligned} J_t\dot{r} &= -G\sin(p) l_1 \\ \Rightarrow J_t\dot{r} &= -K_p \sin(p) l_1 \end{aligned} \tag{3}$$

where J_t is the moment of inertia of the system about the travel axis, r is the travel rate in rad/sec, K_p is the force required to maintain the helicopter in flight and is approximately G, $\sin(p)$ is the trigonometric sin of the roll angle.

Remark 1: If the roll angle is zero, no force is transmitted along the travel axis. A positive roll causes a negative acceleration in the travel direction.

Remark 2: To design a speed controller for the travel axis, we consider the model (3) but for a position controller, we need to consider the travel differential equation as:$J_t\ddot{r} = -K_p\sin(p)l_1$.

D. State Space Model of 3 DOF Helicopter system

The simplified state space dynamic model of 3 DOF helicopter system relies on the following assumptions:

- All angles are sufficiently small so that the linear approximation is valid.
- The coupling dynamics is neglected.
- The gravitational torque T_g is neglected.
- The frictions forces are neglected.

Under the above assumptions, (1), (2) and (3) for the overall motion of the helicopter can be effectively reduced to these following equations:

$$\begin{cases} \ddot{\epsilon} = \frac{K_c l_1}{J_e} U_1 \\ \ddot{p} = \frac{K_c l_p}{J_p} U_2 \\ \dot{r} = \frac{G l_1}{J_t} p \end{cases} \tag{4}$$

where control inputs U_1 and U_2 are given by

$$\begin{cases} U_1 = \frac{V_s + V_d}{2} \\ U_2 = \frac{V_s - V_d}{2} \end{cases} \tag{5}$$

Remark 3: We calculate the input voltage U_1 and U_2 for each motor.
Finally, using (4), the sate space model of the helicopter is formulated as below:

$$\frac{d}{dt}\begin{bmatrix}\epsilon\\ p\\ \dot{\epsilon}\\ \dot{p}\\ r\\ \xi\\ \gamma\end{bmatrix} = \begin{bmatrix}0&0&1&0&0&0&0\\0&0&0&1&0&0&0\\0&0&0&0&0&0&0\\0&0&0&0&0&0&0\\0&\frac{Gl_1}{J_t}&0&0&0&0&0\\1&0&0&0&0&0&0\\0&0&0&0&1&0&0\end{bmatrix}\begin{bmatrix}\epsilon\\ p\\ \dot{\epsilon}\\ \dot{p}\\ r\\ \xi\\ \gamma\end{bmatrix} + \begin{bmatrix}0&0\\0&0\\ \frac{K_cl_1}{J_e}&\frac{K_cl_1}{J_e}\\ \frac{K_cl_p}{J_p}&-\frac{K_cl_p}{J_p}\\0&0\\0&0\\0&0\end{bmatrix}\begin{bmatrix}U_1\\U_2\end{bmatrix} \tag{6}$$

$$\begin{bmatrix}\epsilon\\ p\\ r\end{bmatrix} = \begin{bmatrix}1&0&0&0&0&0&0\\0&1&0&0&0&0&0\\0&0&0&0&1&0&0\end{bmatrix}\begin{bmatrix}\epsilon\\ p\\ \dot{\epsilon}\\ \dot{p}\\ r\\ \xi\\ \gamma\end{bmatrix} + \begin{bmatrix}0&0\\0&0\\0&0\end{bmatrix}\begin{bmatrix}U_1\\U_2\end{bmatrix} \tag{7}$$

where ϵ, p and r are the pitch angle, roll angle and travel rate of the 3-DOF helicopter system respectively and $\dot{\xi} = \epsilon$ and $\dot{\gamma} = r$.

TABLE I
PHYSICAL PARAMETERS OF 3-DOF HELICOPTER SYSTEM

Symbol	Physical unit	Numerical values
J_e	$Kg.m^2$	1.8145
J_t	$Kg.m^2$	1.8145
J_p	$Kg.m^2$	0.0319
G	N	4.2591
l_1	m	0.88
l_2	m	0.35
l_p	m	0.17
K_c	N/V	12

In order to obtain the linear 3-DOF state space model, we consider the physical parameter's value from Table I.

III. Controller Design

This section is dedicated to design three PID controllers to allow us to command a desired pitch position, roll angle and travel rate of 3-DOF helicopter system. In this work, PID controller's design parameters are approximated from the LQR controller gain matrix.

A. PID Equation

If ϵ_c, p_c and r_c are the desired pitch angle, desired roll angle and desired travel rate of helicopter system, we can express the PID controllers in the following form to meet closed loop expectations:
Pitch Control Equation:

$$V_s = K_{\epsilon p}(\epsilon - \epsilon_c) + K_{\epsilon d}\dot{\epsilon} + K_{\epsilon i}\int (\epsilon - \epsilon_c)\,dt \tag{8}$$

Roll Control Equation:

$$V_d = K_{pp}(p - p_c) + K_{pd}\dot{p} \tag{9}$$

Travel Control Equation:

$$p_c = K_{rp}(r - r_c) + K_{ri}\int (r - r_c)\,dt \tag{10}$$

Remark 4: In order to achieve a desired travel rate r_c, we design a controller to command a desired roll angle p_c.

B. LQR Controller

Linear-quadratic-regulator (LQR) is a part of the optimal control strategy [10] which has been widely developed and used in various applications. LQR design is based on the selection of feedback gains K such that the cost function or performance index J is minimized. This ensures that the gain selection is optimal for the cost function specified. An advantage of using the LQR optimal control scheme is that system designed will be stable and robust, except in the case where the system is not controllable.
In this design method, the system must be described by state space model:

$$\begin{cases}\dot{x} = Ax + Bu\\ y = Cx + Du\end{cases} \tag{11}$$

The performance index J is defined as:

$$J = \int_0^\infty (x^TQx + u^TRu)dt \tag{12}$$

Where Q is positive definite (or semi-positive definite) Hermitian or a real symmetric matrix. and R is positive definite Hermitian or real symmetric matrix. These two matrices Q and R are often known as weighting matrices.

The LQR optimal control principle [10] for the system defined in (11) is to determine matrix K that gives the optimal control vector

$$u(t) = -Kx(t) \tag{13}$$

such that performance index J is minimized.
The control gain matrix K is given by

$$K = R^{-1}B^TP \tag{14}$$

and P can be found by solving the continuous time algebraic Riccati equation:

$$A^TP + PA - PBR^{-1}B^TP + Q = 0 \tag{15}$$

The block diagram showing the LQR control system configuration is shown in Fig. 5.

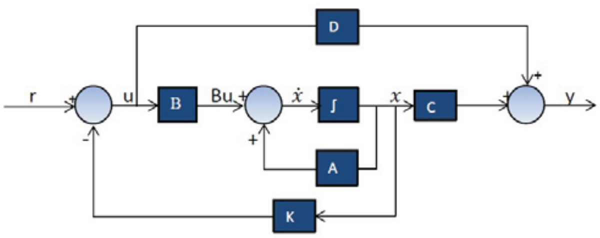

Fig. 5. LQR Control Systems Configuration

The crucial and difficult task in the LQR controller design is a choice of the weighting matrices. We generally select weighting matrices Q and R to satisfy expected performance criterion. The different Q and R values give a different system response. The system will be more robust to disturbance and the settling time will be shorter if Q is larger (in a certain range). But there is no straightforward way to select these weighting matrices and it is usually done through an iterative simulation process. In this article, we apply the Bryson's rule for weighting matrix selection and select the matrices Q and R for the 3-helicopter system in the following manner:

$$Q=\rho\begin{bmatrix}\frac{\alpha_1^2}{(\epsilon_{\max})^2}&0&0&0&0&0&0\\0&\frac{\alpha_2^2}{(p_{\max})^2}&0&0&0&0&0\\0&0&\frac{\alpha_3^2}{(\dot\epsilon_{\max})^2}&0&0&0&0\\0&0&0&\frac{\alpha_4^2}{(\dot p_{\max})^2}&0&0&0\\0&0&0&0&\frac{\alpha_5^2}{(r_{\max})^2}&0&0\\0&0&0&0&0&\frac{\alpha_6^2}{(\xi_{\max})^2}&0\\0&0&0&0&0&0&\frac{\alpha_7^2}{(\gamma_{\max})^2}\end{bmatrix}$$

$$=\begin{bmatrix}2&0&0&0&0&0&0\\0&0.2&0&0&0&0&0\\0&0&0.02&0&0&0&0\\0&0&0&0.02&0&0&0\\0&0&0&0&2&0&0\\0&0&0&0&0&0.02&0\\0&0&0&0&0&0&0.01\end{bmatrix} \quad (16)$$

$$R=\rho\begin{bmatrix}\frac{\beta_1^2}{(U_{1_{\max}})^2}&0\\0&\frac{\beta_2^2}{(U_{2_{\max}})^2}\end{bmatrix}=\begin{bmatrix}1&0\\0&1\end{bmatrix} \quad (17)$$

where:

- $\epsilon_{\max}, p_{\max}, \dot\epsilon_{\max}, \dot p_{\max}, r_{\max}, \xi_{\max}$ and $\gamma_{\max}$ represent the respective largest desired response for that component of the states.
- $U_{1_{\max}}$ and $U_{2_{\max}}$ are maximum acceptable control voltage inputs for actuator signal.
- $\sum_{i=1}^{7}\alpha_i^2=1$ and $\sum_{i=1}^{2}\beta_i^2=1$ are used to add an additional relative weighting on the various components of the state and control respectively.
- ρ is used as the last relative weighting between the control and state penalties which gives a relatively concrete way to discuss the relative size of Q and R and their ratio Q/R.

The synthesis of a state feedback controller K is obtained according to the LQR control system configuration shown in the Fig. 5. Using MATLAB code `K=lqr(A,B,Q,R)`, we obtain controller gain as follows:

$$K=\begin{bmatrix}1.0426&0.8661&0.4349&0.1534&1.0292&0.1000&0.0707\\1.0426&-0.8661&0.4349&-0.1534&-1.0292&0.1000&-0.0707\end{bmatrix} \quad (18)$$

C. PID Approximation

In this subsection PID controller's design parameter $K_p, K_i$and K_d are approximated from the LQR controller gain matrix (18).

First, we write the LQR optimal control law using (13) and (18) for the 3-DOF helicopter system as :

$$\begin{aligned}\begin{bmatrix}U_1\\U_2\end{bmatrix}&=-\begin{bmatrix}1.0426&0.8661&0.4349&0.1534&1.0292&0.1000&0.0707\\1.0426&-0.8661&0.4349&-0.1534&-1.0292&0.1000&-0.0707\end{bmatrix}\begin{bmatrix}\epsilon\\p\\\dot\epsilon\\\dot p\\r\\\xi\\\gamma\end{bmatrix}\\&=-\begin{bmatrix}k_{11}&k_{12}&k_{13}&k_{14}&k_{15}&k_{16}&k_{17}\\k_{21}&k_{22}&k_{23}&k_{24}&k_{25}&k_{26}&k_{27}\end{bmatrix}x(t)\end{aligned} \quad (19)$$

Analyzing carefully (19), we can obtain:

$$\begin{bmatrix}U_1\\U_2\end{bmatrix}=-\begin{bmatrix}k_{11}&k_{12}&k_{13}&k_{14}&k_{15}&k_{16}&k_{17}\\k_{11}&-k_{12}&k_{13}&-k_{14}&-k_{15}&k_{16}&-k_{17}\end{bmatrix}\begin{bmatrix}\epsilon\\p\\\dot\epsilon\\\dot p\\r\\\xi\\\gamma\end{bmatrix} \quad (20)$$

Now, the sum of the rows of (20) results in:

$$\begin{aligned}U_1+U_2&=-(2k_{11}\epsilon+2k_{13}\dot\epsilon+2k_{16}\xi)\\&=-\left(2k_{11}\epsilon+2k_{13}\dot\epsilon+2k_{16}\int\epsilon dt\right)\end{aligned} \quad (21)$$

The full state feedback results in a controller that feedback two voltages, therefore (21) can be written as:

$$V_s=-2k_{11}(\epsilon-\epsilon_c)-2k_{13}\dot\epsilon-2k_{16}\int(\epsilon-\epsilon_c)\,dt \quad (22)$$

Equations (8) and (22) have the same structure, this means that the gains we obtain from LQR design can still be used for the pitch PID controller. Thus, comparing (22) with (8), we can get the following pitch PID controller design parameters:

$$\begin{cases}K_{\epsilon p}&=-2k_{11}\\K_{\epsilon d}&=-2k_{13}\\K_{\epsilon i}&=-2k_{16}\end{cases} \quad (23)$$

Similarly, the difference of the rows of (20) results in:

$$\begin{aligned}&U_1-U_2=-2k_{12}p-2k_{14}\dot p-2k_{15}(r-r_c)-2k_{17}\gamma\\&\Rightarrow V_d=-2k_{12}p-2k_{14}\dot p-2k_{15}(r-r_c)-2k_{17}\int(r-r_c)\,dt\end{aligned} \quad (24)$$

Now, using PID equation (10) in (9), we can get,

$$V_d=K_{pp}p+K_{pd}\dot p-K_{pp}K_{rp}(r-r_c)-K_{pp}K_{ri}\int(r-r_c)\,dt \quad (25)$$

Equations (24) and (25) have exactly the same structure. The means that we can design an LQR controller and still maintain the same structure we used previously.

TABLE II
PID DESIGN GAINS VALUE

PID parameters	Relationship	Absolute Value
$K_{\epsilon p}$	$-2k_{11}$	2.0852
$K_{\epsilon d}$	$-2k_{13}$	0.8698
$K_{\epsilon i}$	$-2k_{16}$	0.2
K_{pp}	$-2k_{12}$	1.7322
K_{pd}	$-2k_{14}$	0.3069
K_{rp}	$-\frac{2k_{15}}{2k_{12}}$	1.1883
K_{ri}	$-\frac{2k_{17}}{2k_{12}}$	0.0816

On comparing (24) and (25), the feedback gains for the controller are obtained from the LQR gains as:

$$\begin{cases} K_{pp} &= -2k_{12} \\ K_{pd} &= -2k_{14} \\ K_{rp} &= -\frac{2k_{15}}{2k_{12}} \\ K_{ri} &= -\frac{2k_{17}}{2k_{12}} \end{cases} \tag{26}$$

The design we have performed, resulting in the controller gain values and it is shown in the Table II and got PID controller, PD controller and PI controller for pitch, roll and travel axis model respectively of 3DOF helicopter system.

Remark 5: Due to the minimum-phase requirement of a PID controller, the signs of the proportional gain, the integral gain and the differential gain must be the same. Therefore, absolute gain values are considered to avoid the problem of different signs.

IV. Simulation and Results Analysis

We now present the performance analysis for 3DOF helicopter feedback system through MATLAB simulation. Simulations are carried out for different axis: pitch, roll and travel axis. The control objective is to get good performances in dynamics as well as in statics. In the simulation, the reference signal for the pitch, travel and roll angles are changed between $0°$ to $20°$ to simulate the demands given by the pilot. Also sampling time and simulation time used are 0.1 and 60 seconds respectively.

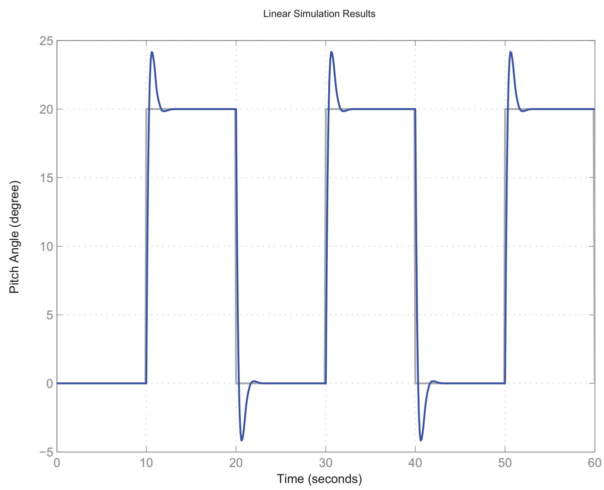

Fig. 6. PID control simulation of the pitch axis model

A. Pitch Axis Model Simulation

The control objective is to minimize the error between the desired pitch positions with the helicopter pitch axis position. The Fig. 6 illustrates the tracking curve of reference input with PID controller for pitch axis model of the 3DOF helicopter system. Simulation result shows that PID control for pitch axis has negligible overshoot and shorter settling time and is able to track the desired response, which is considerable system performance.

B. Roll Axis Model Simulation

The aim of this part is to design a controller that will allow us to command the roll movement. The tracking curve of the reference input signal to PD control of the roll axis model is shown in the Fig. 7. From the figure it can be seen that steady state have been completely obtained. The system has small overshoot which is settled within a fraction of a second.

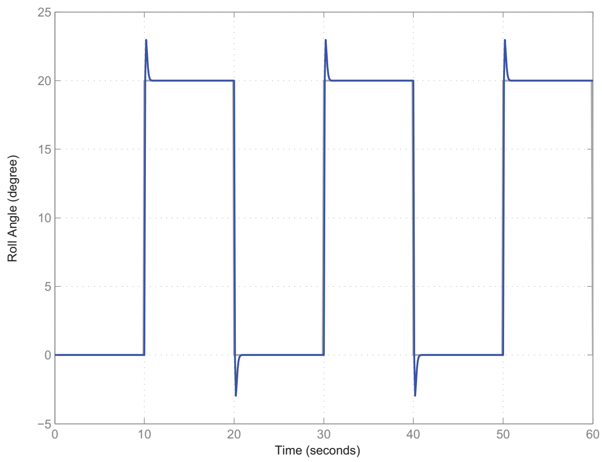

Fig. 7. PD control simulation of the roll axis model

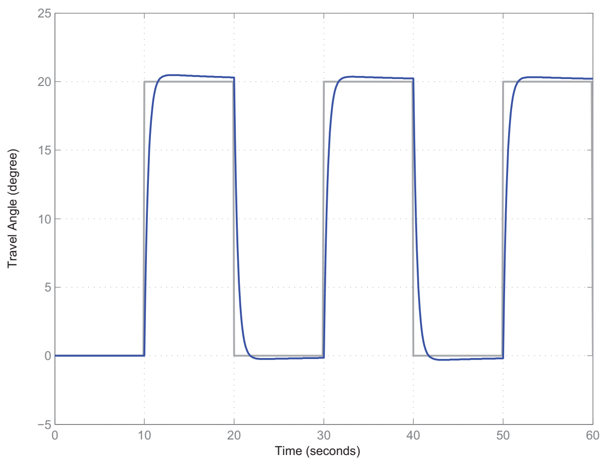

Fig. 8. PI control simulation of the travel axis model

C. Travel Axis Model Simulation

This section is dedicated to control of the travel rate of the helicopter by means of PI controller. The Fig. 8 shows the control curve of helicopter travel rate. It is cleared from the simulation result that PI control of the travel axis model is able to track desired response and has no overshoot. The simulation also shows that there is negligible steady state tracking error but overall tracking performance looks acceptable.

V. Conclusion

In this work, LQR based PID controller design for 3DOF helicopter system is investigated. The 3DOF helicopter dynamical equations along with state space model are first presented in the paper and then the basic ideas and technical formulations for designing a PID controller are briefly illustrated. This paper presents a simple approach for designing a PID controller based on the approximation of design parameters from the LQR controller gain matrix. The investigated controller has both static and dynamic performance, therefore the stability and the quick control effect can be obtained simultaneously. The robustness of the designed controller is superior in terms of less overshoot, short settling time and stable tracking of reference inputs.

Acknowledgements

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